Advanced Macroeconomics Special Note 4.

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IS-MP medium-run dynamics: Phillips curve, Taylor rule, and liquidity trap

This note presents and analyzes a simple Keynesian model of "short-to-medium run" dynamics under the assumption that monetary policy pursues a *Taylor rule*, also known as *inflation targeting*. This is a contra-cyclical monetary policy which uses the short-term nominal interest rate actively to counteract deviations of output and inflation from their structural and desired levels, respectively. To put it differently, through open-market operations the central bank let the money supply respond to business cycle fluctuations in inflation and possibly also output. In some form or another, this is nowadays the prevalent practice of central banks.

First a brief overview of two different views concerning monetary policy.

1 Two different views

A Taylor rule contrasts with *monetarist policy*. This term refers to a monetary policy where the central bank tries to maintain a constant but low growth rate in the money supply (be it M1, M2, or M3). In this way, according to monetarists (Milton Friedman and followers), monetary policy would not only restrain inflation but also safeguard stability (automatic and quick return to "full employment").

This view is profoundly questioned by macroeconomists of Keynesian conviction. The argument is that under a constant growth rate in the money supply, an adverse demand shock triggers both a *centripetal* force and a *centrifugal* force. On the one hand, the incipient recession reduces inflation, thereby making real money supply larger than otherwise. The result is a *reduced nominal* interest rate, whereby aggregate demand is *stimulated* − fine! On the other hand, by reducing inflation, also expected inflation is reduced. Through the resulting *higher expected real* interest rate than otherwise, aggregate demand is *dampened* − not so fine! Due to the inherent non-linearity in the money demand function (arising from the Zero Lower Bound), the lower the nominal interest rate has already become, the higher the risk that the centrifugal force dominates the centripetal force. This latter point has received increasing attention after the Zero Lower Bound has become a problem of urgent practical importance, first in Japan in the late 1990s, then in the Western World in the Great Recession 2008-?.

Now to the model.

2 Dynamic IS-MP model

Consider a closed economy. Let time be continuous. Ignore the time lag between output and aggregate demand.

2.1 The private sector

To avoid complicating the model with features of only secondary importance for the issue at hand, we assume that aggregate demand depends on only two endogenous variables, namely current aggregate income and the expected real interest rate faced by borrowing households and firms. At a given point in time we have

$$Y = D(Y, r^e, \eta), \qquad 0 < D_Y < 1, D_{r^e} < 0, D_\eta > 0. \tag{*}$$

Here Y is aggregate output, and r^e is the expected real interest rate faced by the ultimate private borrowers, the superscript e indicating expected value. Finally, η is a shift parameter on which aggregate demand depends positively.

For convenience we will base the analysis on a log-linear approximation to the equation (*). On both sides of (*) we take the total differential, to get $dY = D_Y dY + D_{r^e} dr^e + D_\eta d\eta$. Isolating the dY terms, we have

$$(1 - D_Y)dY = D_{r^e} dr^e + D_\eta d\eta.$$

Dividing through by $(1 - D_Y)Y$ gives

$$\frac{dY}{Y} = d \ln Y = \frac{D_{r^e}}{(1 - D_Y)Y} dr^e + \frac{D_\eta}{(1 - D_Y)Y} d\eta. \tag{**}$$

We assume the coefficients to dr^e and $d\eta$ are constants and name them $-\beta < 0$ and γ, respectively. Integrating on both sides of (**), we then get

$$\ln Y \equiv y = -\beta r^e + \gamma \eta + k,$$

where k is a constant.

Redefining our shift parameter to be $\mu \equiv \gamma\eta + k$ and letting time be explicit, we end up with

$$y_t = \mu - \beta r_t^e, \qquad \mu > 0, \beta > 0. \tag{1}$$

The shift parameter μ is an index of autonomous demand. It varies positively with any exogenous variable having the property that the higher its value, the higher is aggregate demand, everything else equal. So, the "degree of optimism" or "state of confidence" in the economy will affect the size of μ. In our main text we will interpret variations in μ as deriving from this source. government spending on goods and services will affect the size of μ. Alternatively, variation in μ could reflect variation in fiscal policy, an interpretation which we postpone to the concluding section.

Given the "short-to-medium-run" perspective of the model, it should embrace a *Phillips curve* of some sort. For simplicity we assume the simplest specification we can think of:

$$\dot{\pi}_t = \delta(y_t - y^*), \qquad \delta > 0, y^* > 0, \qquad \pi_0 \text{ given}, \tag{2}$$

where π_t in the inflation rate ($\equiv \dot{P}_t/P_t$), $y^* \equiv \ln Y^*$ is the NAIRU level of output, and δ measures the reaction speed.[1] The inflation rate thus speeds up or slows down according to whether output is above or below a certain level, $y^* \equiv \ln Y^*$, respectively. We may interpret this as reflecting a "wage-price spiral". In a boom ($y_t > y^*$) unemployment is low and workers' bargaining position strong. This results in fast nominal wage increases. Via firms' markup pricing fast inflation is induced. As long as the boom continues, faster and faster nominal wage and price increases ensue. In a slump ($y < y^*$) workers' bargaining position is weak and the spiral goes the opposite way.

When $y_t = y^*$, there is no internal pressure on inflation. For simplicity, y^* is assumed to be time independent, that is, the model abstracts from growth in labor force and technology. More to the point is that (2) indicates that the inflation rate is predetermined. So the inflation rate is sticky and can not jump. Inflation changes smoothly over time in response to the *output gap*, $y_t - y^*$.[2] The message of the Phillips curve (2) is that the output gap determines the *change* in inflation rather than the *level* of inflation.

To get further perspective on the Phillips curve (2), we may consider a standard

[1] Although strictly speaking, y is the log of output, we refer to y as "output" when there is no risk of confusion.

A reservation regarding the convenient assumption that δ is constant is made in Section 7.

[2] This seems to be in accordance with the empirics for industrialized economies without hyperinflation, cf. for instance Mankiw (2001).

expectations-augmented Phillips curve in discrete time:

$$\pi_t \equiv (P_{t+1} - P_t)/P_t = \delta(y_t - y^*) + \pi_t^e, \qquad t = 0, 1, 2, \ldots.$$

Now, assume inflation expectations are myopic: $\pi_t^e = \pi_{t-1}$. Then subtract π_{t-1} on both sides. We then get a discrete time analogue to (2).

Returning to our continuous time framework, let $\bar{\mu}$ ($> y^*$) be the value of the autonomous-demand parameter under "normal circumstances". By plugging this value and NAIRU output, y^*, into (1), we find the required value of r^e to be

$$\frac{\bar{\mu} - y^*}{\beta} \equiv \hat{r} > 0. \tag{***}$$

This interest rate level is sometimes called the "natural rate of interest", but we prefer the name *structural rate of interest* since it depends on autonomous demand, $\bar{\mu}$, which in turn depends on for instance fiscal policy. It is the real interest rate required for "full employment" (zero output gap) and stationary inflation under "normal circumstances" and fulfilled expectations.

The nominal interest rate, i_t, on short-term government bonds is, within bounds, controlled by the central bank through open-market operations (see below). We will call i_t the *policy rate*. Given i_t and given the expected inflation rate, π_t^e, the expected real interest rate can be written

$$r_t^e = i_t + \omega - \pi_t^e, \tag{3}$$

where ω is the *spread* (also known as the interest differential). This is the difference between policy rate i_t and the nominal interest rate at which the non-bank public borrows in financial markets (we assume the bank lending rate and the rate on corporate bonds are the same).

In view of government bonds being practically risk-free (usually), while loans to the ultimate borrowers in the private sector are generally risky, the spread will generally be positive, although less than the structural interest rate. We will treat the spread as a quasi-parameter, i.e., as being directly determined by the shift parameter μ (within its relevant range) and nothing else:

$$\omega = \omega(\mu), \qquad \omega'(\mu) < 0, \qquad 0 < \omega(\bar{\mu}) < \hat{r}. \tag{4}$$

So, when the state of confidence shifts, the spread shifts in the opposite direction.[3]

[3] In the IS-BL model of Short Note 3, the spread is "fully endogenous", measured as the difference between the two separate endogenous variables i_L and i_B in that model.

We assume that households, firms, and the central bank have the same expectations. So there is only one π_t^e in the economy. Until further notice, we do not want to be specific about how this expectation is determined, be it rational or adaptive.

2.2 The central bank

The central bank pursues a certain *inflation target*, $\hat{\pi}$. In addition, we simplifying assume that the central bank, owing to its accumulated experience, knows y^*, the structural interest rate, $\hat{r}$, and the "normal" spread $\omega(\bar{\mu})$. Through open-market operations the central bank then establishes its policy rate as the maximum of the "desired level" and nil:

$$\begin{aligned} i_t &= \max\left[0, \hat{\imath} + \alpha_1(y_t - y^*) + \alpha_2(\pi_t^e - \hat{\pi})\right], \\ \text{where } \hat{\imath} &\equiv \hat{r} - \omega(\bar{\mu}) + \hat{\pi}, \quad \hat{\pi} > 0,\ \alpha_1 \geq 0,\ \alpha_2 > 1. \end{aligned} \tag{5}$$

This is an example of a *Taylor rule.* As long as the zero lower bound (ZLB) on the nominal interest rate is not binding, the central bank adjusts the policy rate, i_t, depending on the current output gap, $y_t - y^*$, and expected excess inflation, $\pi_t^e - \hat{\pi}$. Thereby the expected real interest rate, r_t^e, in the economy is raised or lowered depending on whether a dampening or stimulation of aggregate demand is called for. The limiting case $\alpha_1 = 0$, such that the policy rate does not at all respond directly to the output gap is included as a special case of the Taylor rule. The imperative $\alpha_2 > 1$ is known as the *Taylor principle.* It ensures that an increase in π_t^e results in a larger increase in i_t so as to raise r_t^e and thereby dampen output demand.

We see that the policy rate is such that when the output and inflation gaps ($y_t - y^*$ and $\pi_t^e - \hat{\pi}$, respectively) are nil and circumstances are "normal", then the expected real interest rate equals the structural rate. Indeed, under these circumstances (3) gives

$$r_t^e = \hat{\imath} + \alpha_1(y_t - y^*) + \alpha_2(\pi_t^e - \hat{\pi}) = \hat{r} - \omega(\bar{\mu}) + \hat{\pi} + 0 + 0 + \omega(\bar{\mu}) - \pi_t^e = \hat{r}.$$

If the economy is in recession and the recession is deep enough, the targeted nominal short-term rate implied by the Taylor rule could be negative. Then the zero lower bound indicated by (5) becomes binding, and the actual nominal short-term rate, i_t, stays at nil for some time. Further increases in the money supply can not bring i below 0 because agents prefer holding cash at zero interest rather than short-term government bonds (or demand deposits in banks) at negative interest. Well, strictly speaking, the lower bound is slightly below zero because the alternative to holding bonds or demand deposits is holding cash which gives zero interest but involves costs of storing, insuring, and transporting.

We shall consider the described economy under two alternative scenarios, one where the ZLB is not binding and one where it is binding. The variables α_1, α_2, y^*, $\hat{\pi}$, and μ are independent exogenous variables. There are six endogenous variables in our system: $y_t, r_t^e, i_t, \pi_t^e, \pi_t$, and the quasi-fixed interest spread ω. So far we have only one differential equation, (2) and four static equations, namely (1), (3), (4), and (5); the remaining equations are just preliminaries or define shorthands for combinations of exogenous variables. The lacking element in the model is a specification of how expectations are formed. Below, we shall consider different approaches to this problem.

Empirically there are signs that central banks prefer to "smooth" the time path of the interest rate, letting the policy rate be a weighted average of the rate in the previous period and the current "pure" Taylor-rule value, i_t^T, given from (5). Thereby $i_t = \max\left[0, \rho i_{t-1} + (1-\rho) i_t^T\right]$. In the present exposition we do not integrate this.

3 Short-run equilibrium when the ZLB is not binding

Combining (1) and (3), equilibrium in the output market can be written

$$y_t = \mu - \beta(i_t + \omega(\mu) - \pi_t^e), \tag{IS}$$

which we rewrite as

$$i_t = \frac{\mu - y_t}{\beta} - \omega(\mu) + \pi_t^e. \tag{IS'}$$

Assuming the zero lower bound on the interest rate is not binding, the Taylor rule gives

$$i_t = \hat{\imath} + \alpha_1(y_t - y^*) + \alpha_2(\pi_t^e - \hat{\pi}) \equiv \alpha_0 + \alpha_1 y_t + \alpha_2 \pi_t^e, \tag{MP}$$

where MP stands for monetary policy.

3.1 The IS-MP cross

At any given point in time, t, there are historically given expected and actual inflation rates, π_t^e and π_t, respectively. So for fixed t, the combinations of y_t and i_t that are consistent with equilibrium in the output market are given by the equation (IS). In Fig. 1 these combinations are depicted as the downward-sloping IS curve. Although this curve, as well as the MP curve, is here a straight line (due to log-linearization), we shall stick to the standard terminology and speak of both as "curves".

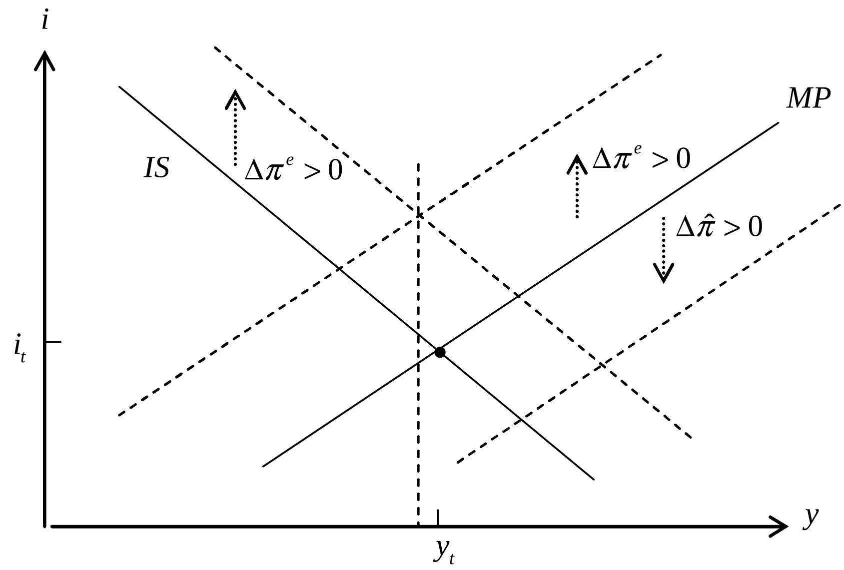

Figure 1: Short-run equilibrium at the IS-MP cross at time t for given π_t^e, $\hat{\pi}$, and μ ($\alpha_1 > 0$).

The upward-sloping MP curve in Fig. 1 represents the combinations of y_t and i_t that are consistent with the Taylor rule (MP). The point of intersection between the IS and MP curves represents the short-run equilibrium, (y_t, i_t), at time t.

Fig. 1 also indicates that for greater expected inflation, both the IS curve and the MP curve move upwards. For a given $\Delta\pi^e$, the MP curve features the largest upward shift in view of $\alpha_2 > 1$ (compare (MP) and (IS')). Hence the new equilibrium value of y_t will be *smaller* than the old. This feature is a first indication of the contra-cyclical role of the Taylor rule. It anticipates what the dynamic analysis below will unfold.

The economic logic behind this result is the following. On the one hand, the higher expected inflation *tends* to reduce the expected real interest rate and thereby stimulate output demand. On the other hand, following the Taylor rule the central bank counteracts this by a rise in the policy rate i_t, indeed a rise *larger* than that of expected inflation. So, in response to the higher expected inflation the central bank effectively *raises* the expected real interest rate. Thereby output demand, hence output, is dampened and the undesired higher inflation averted. If instead the central bank had kept the policy rate unchanged, actual output would have increased and thereby stimulated actual inflation.

In view of the linearity of the model, we get an explicit solution for the short-run equilibrium value of y. Inserting (MP) into (IS) gives

$$y_t = \mu - \beta\left[\hat{\imath} + \alpha_1(y_t - y^*) + \alpha_2(\pi_t^e - \hat{\pi}) + \omega(\mu) - \pi_t^e\right].$$

Isolating y_t, we can thus write

$$y_t = x - \theta \pi_t^e, \tag{AD}$$

where

$$x = \frac{\mu - \beta\left(\hat{\imath} - \alpha_1 y^* + \omega(\mu) - \alpha_2 \hat{\pi}\right)}{1 + \beta\alpha_1} \equiv x(\mu), \tag{6}$$

$$\theta \equiv \frac{\beta(\alpha_2 - 1)}{1 + \beta\alpha_1} > 0. \tag{7}$$

The quasi-parameter x shifts when the autonomous demand parameter μ shifts, but is otherwise constant. It measures the level of *aggregate demand in case expected inflation is nil.*

The relationship (AD) tells us that, given the autonomous demand parameter μ, and thereby given the spread, $\omega(\mu)$, aggregate demand and thus output is – through monetary policy – determined by expected inflation. More precisely: the *strong* response (inherent in $\alpha_2 > 1$) of monetary policy to expected inflation determines aggregate demand such that output ends up *depending negatively* on expected inflation. This is first indication that the Taylor rule is a contra-cyclical monetary policy and seems promising for stability.

We call the relationship (AD) the *aggregate demand curve* of the economy. As a preparation for dynamic analysis, we will consider a graphical illustration.

3.2 The AD curve under "normal circumstances"

Fig. 2 depicts in the (y, π^e) plane the AD curve under "normal circumstances", i.e., when confidence is "normal" and thus results in an autonomous demand level equal to $\bar{\mu}$. Then the AD curve reads

$$y_t = x(\bar{\mu}) - \theta \pi_t^e. \tag{$\overline{\text{AD}}$}$$

As long as $\mu = \bar{\mu}$, the AD curve is fixed and the economy must be at some point on this curve (line), depending on the current expected rate of inflation.

As already noted, it is the Taylor rule's $\alpha_2 > 1$ which ensures the *negative* slope of the AD curve. A more "passive" monetary policy, keeping i_t constant or allowing only a modest response to a rise in expected inflation, would make the AD curve positively sloped, cf. (7). This would make stability of the economy precarious, as alluded to in Section 1.

The situation depicted in Fig. 2 is one where at time 0, $\pi^e = \pi_0^e < \hat{\pi}$, cf. the point A in the figure. The corresponding equilibrium output, y_0, is higher than NAIRU output

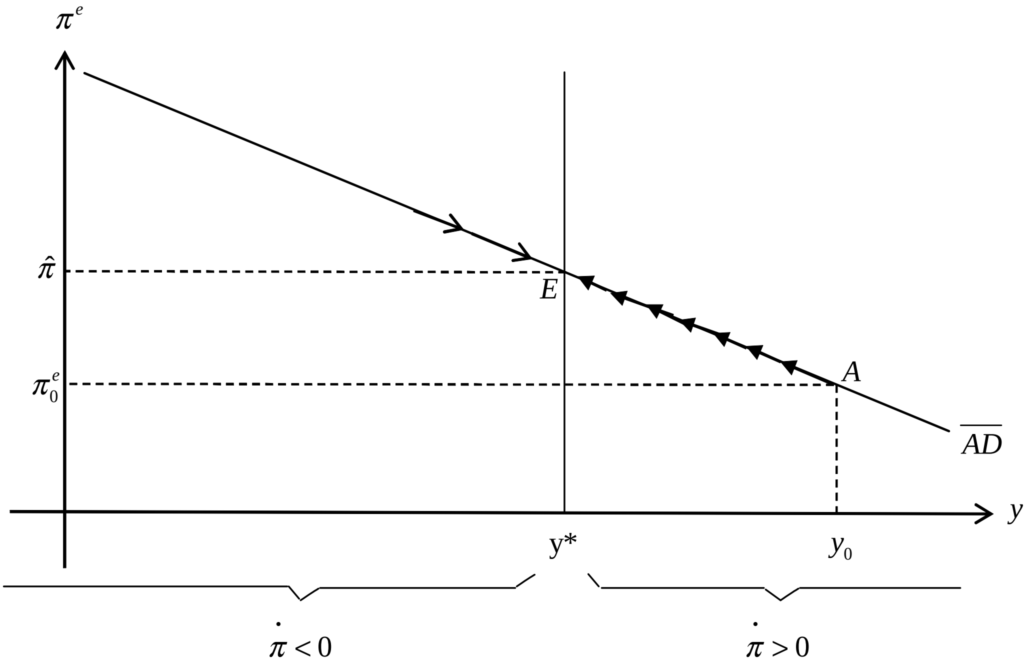

Figure 2: $\overline{\text{AD}}$ is the AD curve under "normal circumstances", i.e., when $\mu = \bar{\mu}$.

as indicated on the horizontal axis. So initially the economy is in a boom. This might seem paradoxical since the initial expected inflation is relatively low. But it is exactly this low expected inflation that invites a slack monetary policy, implying a low expected real interest rate, hence a high level of aggregate demand. Indeed, by (MP) we have

$$r_t^e = i_t + \omega(\bar{\mu}) - \pi_t^e = \hat{\imath} + \alpha_1(y_t - y^*) + \alpha_2(\pi_t^e - \hat{\pi}) + \omega(\bar{\mu}) - \pi_t^e, \tag{8}$$

where $\partial r_t^e / \partial \pi_t^e = \alpha_2 - 1 > 0$. So, in spite of the conceptual relationship, $r^e \equiv i + \omega(\mu) - \pi^e$, the real interest rate depends, everything else equal, *positively* on the expected inflation as a result of the Taylor rule. The low expected real interest rate needed to get high aggregate demand will be concomitant with the low expected inflation rate via a very low policy rate, i_t.

The arrows in Fig. 2 are explained below.

4 Dynamics when the ZLB is not binding

We shall here characterize the time path of y_t under the assumption that the zero lower bound, ZLB, does not become binding. We start with the easiest case, the benchmark case of rational expectations which here means *perfect foresight* with respect to the inflation rate.

4.1 Dynamics under perfect foresight

Assuming perfect foresight, we have

$$\pi_t^e = \pi_t \quad \text{and} \quad r_t^e = i_t + \omega(\bar{\mu}) - \pi_t \equiv r_t \qquad \text{for all } t.$$

Then the equation ($\overline{\text{AD}}$) reduces to

$$y_t = x(\bar{\mu}) - \theta \pi_t, \tag{9}$$

or, by inverting,

$$\pi_t = \frac{x(\bar{\mu})}{\theta} - \frac{1}{\theta} y_t.$$

Because $\pi_t^e = \pi_t$, we may interpret the (y, π^e) plane in Fig. 2 as an (y, π) plane. The shown AD curve in Fig. 2 is still a valid representation of the economy as long as the state of confidence is unchanged so that $\mu = \bar{\mu}$. Depending on the predetermined initial inflation rate, π_0, the economy must at time 0 be at the corresponding point, (y_0, π_0), on the AD curve.

The initial boom depicted in Fig. 2 is a state of affairs which, in view of the Taylor rule, requires *low* initial (expected and actual) inflation. Via the Phillips curve the boom induces *rising inflation* over time. And because the inflation coefficient, α_2, in the Taylor rule is above 1, rises in the inflation rate prompt even greater rises in the nominal interest rate. The result is a *rising real interest rate.* This gradually dampens aggregate demand and output. Monetary policy is thus in its "tightening mode". We get a leftward adjustment along the $\overline{\text{AD}}$ curve from the initial point A in Fig. 2 towards the steady-state point E. At this point the system is "at rest".

Let us instead imagine that the historically inherited inflation rate is relatively high, i.e., $\pi_0 > \hat{\pi}$. The corresponding equilibrium output, y_0, is then below NAIRU output. Also this may seem a paradoxical situation since the initial inflation is relatively high. A low level of output requires a low level of aggregate demand which in turn requires a high expected real interest rate. This is exactly what the monetary policy in this situation brings about. When the inflation rate is above its steady state level, $\hat{\pi}$, monetary policy chooses a nominal interest rate even more above *its* steady state level, due to the policy parameter α_2 exceeding 1. A high real interest rate and thereby low aggregate demand is the result.

In response to the high inflation, monetary policy has thus brought about a recession. Via the Phillips curve, the recession brings about *falling inflation.* With a policy parameter α_2 less than one, this would result in a rising real interest rate and thus reinforce the recession. But with $\alpha_2 > 1$, it is ensured that the nominal interest rate is lowered *more* than the inflation rate so that a *falling real interest rate* is the result. Monetary policy is here in its "relaxing mode". Aggregate demand and output are gradually stimulated in the rightward process along the AD curve towards the steady state, E.

As a conclusion, if autonomous demand remains at "normal", the economy settles down in steady state at the point E in Fig. 2. So, in contrast to the traditional *static* AS-AD model with an AD curve in the (y, P) plane, in the present model we have an AD curve in the (y, π) plane. A convenient feature of this AD curve is that it does not change its position or slope during the adjustment process. Instead, the dynamic adjustment of the economy takes place in a movement *along* the AD curve (at least if actual and expected inflation coincide).

The dynamics of the economy can also be depicted in the $(\pi, \dot{\pi})$ plane. This is shown in Fig. 6 and 7 in the appendix.

4.2 Dynamics under adaptive expectations

The specification considered of the Taylor rule assumes that monetary policy is forward-looking and responds to anticipated inflation rather than actual inflation. Under perfect foresight this is of course immaterial.

But what can we say in the absence of perfect foresight? First, our Taylor rule will still ensure that equilibrium output at a given point in time is determined uniquely for a given expected inflation rate as in the equation (AD). As long as autonomous demand equals $\bar{\mu}$, Fig. 2 is still applicable.[4]

Second, the ensuing dynamics can be described as follows. At time 0, there is a historically given expected inflation rate, π_0^e. Suppose $\pi_0^e < \hat{\pi}$. Then, through the Taylor rule the corresponding initial aggregate demand is high and y_0 therefore above NAIRU output. Whether or not the actual inflation rate initially differs from the expected, the situation triggers, through the Phillips curve (2), a *rising* actual inflation rate. Expected inflation seems thereby likely to rise as well, in which case the economy represented by the point (y_t, π_t^e) will again move up the AD curve in Fig. 2. As long as $y > y^*$, actual inflation will also rise further although the speed may deviate from that of expected

[4] This would not be true if we had *actual* inflation entering the Taylor rule instead of expected inflation. In that case, aggregate demand would become a function of both expected inflation, via (IS), and actual inflation, via the Taylor rule. We would then need a three-dimensional diagram, which is beyond the scope of this lecture note.

inflation. Intuitively, if no new shocks occur, over time the economy will again settle down in steady state at E in Fig. 2, where both π_t^e and π_t will equal $\hat{\pi}$.

This conjectured stability property definitely holds if we specify expectations to be formed according to the adaptive expectations formula,

$$\dot{\pi}_t^e = \lambda(\pi_t - \pi_t^e), \qquad \lambda > 0. \tag{10}$$

Inserting ($\overline{\text{AD}}$) into the Phillips curve (2), we get

$$\dot{\pi}_t = \delta(\frac{\bar{\mu} - \beta(\alpha_0 + \omega(\bar{\mu}) - \beta(\alpha_2 - 1)\pi_t^e}{1 + \beta\alpha_1} - y^*). \tag{11}$$

Hereby we have a system of two linear differential equations in two endogenous variables, π_t^e and π_t, both of which are predetermined. The steady state is $(\hat{\pi}, \hat{\pi})$ and is globally asymptotically stable. That is, for arbitrary initial values, π_0^e and π_0, the solution, (π_t^e, π_t), converges to the steady state for $t \to \infty$.[5]

5 An adverse demand shock

We return to the assumption of rational expectations, here *perfect foresight*.

Suppose that up until time t_1, the economy is in steady state with $\pi = \hat{\pi}$ and the autonomous demand parameter μ has its "normal" value, $\bar{\mu}$, so that aggregate demand in case of zero inflation is $x = x(\bar{\mu})$. Then, unexpectedly, a fall in the general state of confidence occurs so as to shift μ to the level $\mu' < \bar{\mu}$. The background for this adverse demand shock could be a financial crisis in the aftermath of a bursting housing price bubble. The interest spread now rises to $\omega(\mu') > \omega(\bar{\mu})$, which prompts a reduced x, at least for a while.

We first consider the case where the ZLB does not become binding.

5.1 Restoration when the shock is "minor"

Suppose the adverse demand shock is "minor". It shifts the AD curve down to the new position, indicated by AD' in Fig. 4. Immediately after the shock the economy shifts its

[5] This follows by calculating the Jacobian matrix of the right-hand sides of (10) and (11). We get that the trace equals $-\lambda$ and the determinant equals $[\delta\beta(\alpha_2 - 1)\lambda] / (1 + \beta\alpha_1)$. Thereby the trace is negative and the determinant positive (again $\alpha_2 > 1$ is decisive). This is both necessary and sufficient for a two-dimensional linear dynamic system, where both variables are predetermined, to be globally asymptotically stable, cf. Sydsæter et al. (2008, p. 244).

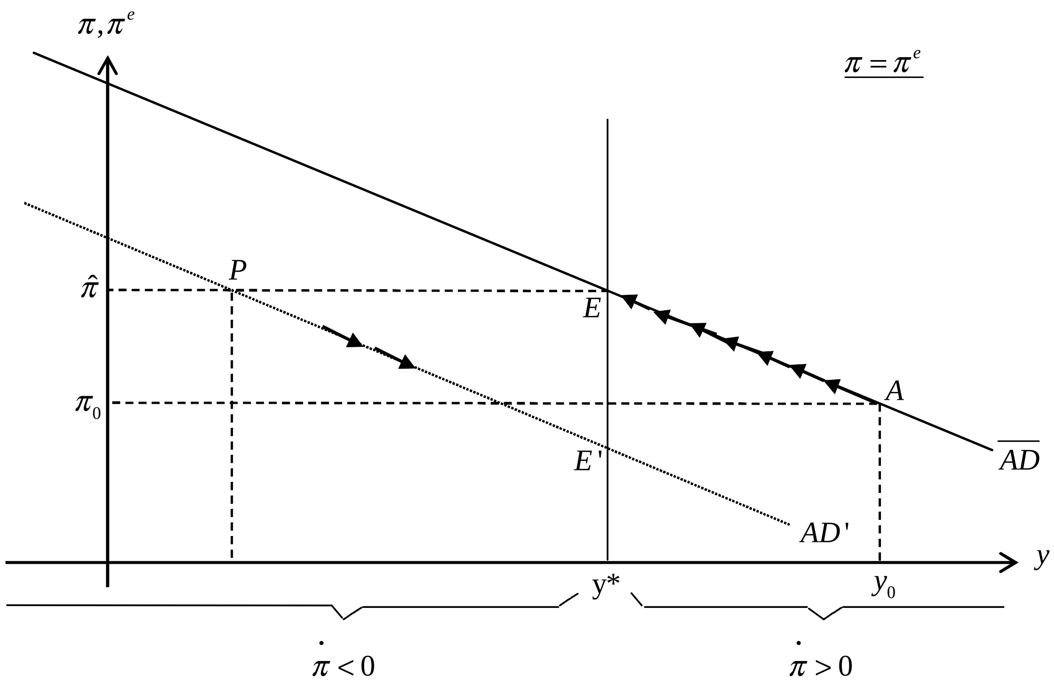

Figure 3: $\overline{\text{AD}}$ is the AD curve under "normal circumstances", i.e., when $\mu = \bar{\mu}$.

position from the point E to the point P in the figure. The implied recession activates the Taylor rule, both via the output gap (if $\alpha_1 > 0$) and, possibly with a delay, via low expected and actual inflation generated by the Phillips curve in response to $y < y^*$. That is, over time the economy travels down the new AD curve, AD', towards a new (quasi-)steady state, E'. So the recession is not lasting. This new steady state has "full" employment, but low inflation and hence low policy rate. This state is conditional on no repair of confidence taking place (hence the qualifier "quasi-").

It may seem more plausible that during the adjustment process, after a while, the experience of a gradual upturn restores confidence. As a crude representation of this, we imagine that a complete restoration of confidence takes place in a discrete jump at time $t_2 > t_1$. So, for $t \geq t_2$, equation ($\overline{\text{AD}}$) with the old $x = x(\bar{\mu})$ is again valid. In Fig. 3 the restoration of confidence shifts the aggregate demand curve back to its original position, $\overline{\text{AD}}$, and the position of the economy to the point A. Instead of settling down at y^*, the economy thus experiences a boom with $y > y^*$. Then inflation begins to rise through the Phillips curve and monetary policy gradually dampens demand and output through the Taylor rule. Over time the economy moves up the AD curve and approaches the old steady-state point E.

The corresponding dynamics in the $(\pi, \dot{\pi})$ plane for $t \geq t_2$ is depicted in Fig. 7 of the appendix.

5.2 Deep recession if the ZLB becomes binding

Suppose again that up until time t_1, the economy is in steady state with $\pi = \hat{\pi}$. Then a *large* adverse demand shock occurs so that the right-hand side of (5) becomes negative. Then the ZLB immediately becomes *binding* and instead of the desired negative interest rate being realized, we have $i_{t_1} = 0$.[6] We maintain the assumption that expected and actual inflation coincides.

According to the equation (IS), immediately after the shock aggregate output is therefore

$$y_{t_1} = \mu' - \beta(0 + \omega(\mu') - \hat{\pi}) < y^*.$$

Owing to the binding ZLB, the nominal interest rate remains at nil for some time. So conventional monetary policy based on adjusting the interest rate does not work − the economy is in a *liquidity trap*. Through the Phillips curve the recession triggers a falling inflation rate. As i_t cannot go negative, the real interest rate *rises,* whereby aggregate demand and output are further reduced, thus sustaining the tendency for the inflation rate to fall. This increases the real interest rate further. A *vicious spiral* is unfolding.

In algebra, for $t \geq t_1$ we have

$$\begin{aligned} y_t &= \mu' - \beta r_t = \mu' - \beta(0 + \omega(\mu') - \pi_t) = \mu' - \beta\omega(\mu') + \beta\pi_t, & (12) \\ \dot{\pi}_t &= \delta(y_t - y^*) = \delta(\mu' - \beta\omega(\mu') + \beta\pi_t - y^*) = \delta(\mu' - \beta\omega(\mu') - y^*) + \delta\beta\pi_t < 0. & (13) \end{aligned}$$

So, for $t \geq t_1$ both output and inflation will be falling and r_t rising. The recession becomes a depression and there will be no recovery unless either *other* monetary policies or fiscal policies are introduced. Alternatively, the crisis may last until (outside the model) the capital stock has been worn down enough − and new innovation possibilities have mounted up enough − to generate a new upturn with rising capital investment and construction activities.

The condition that the lower bound is binding can be represented by a particular area, the *liquidity trap region,* in the (y, π) plane of Fig. 4. In view of (5), the boundary of the liquidity trap region is given by the equation

$$\hat{\imath} + \alpha_1(y - y^*) + \alpha_2(\pi - \hat{\pi}) = 0. \tag{14}$$

[6] When the full-blown financial crisis late in 2008 unfolded, the policy rate in the US and several other countries was quickly reduced to $[0.00 - 0.25)$. In the US this interval remained in force for seven years (Dec. 2008 - Dec. 2015).

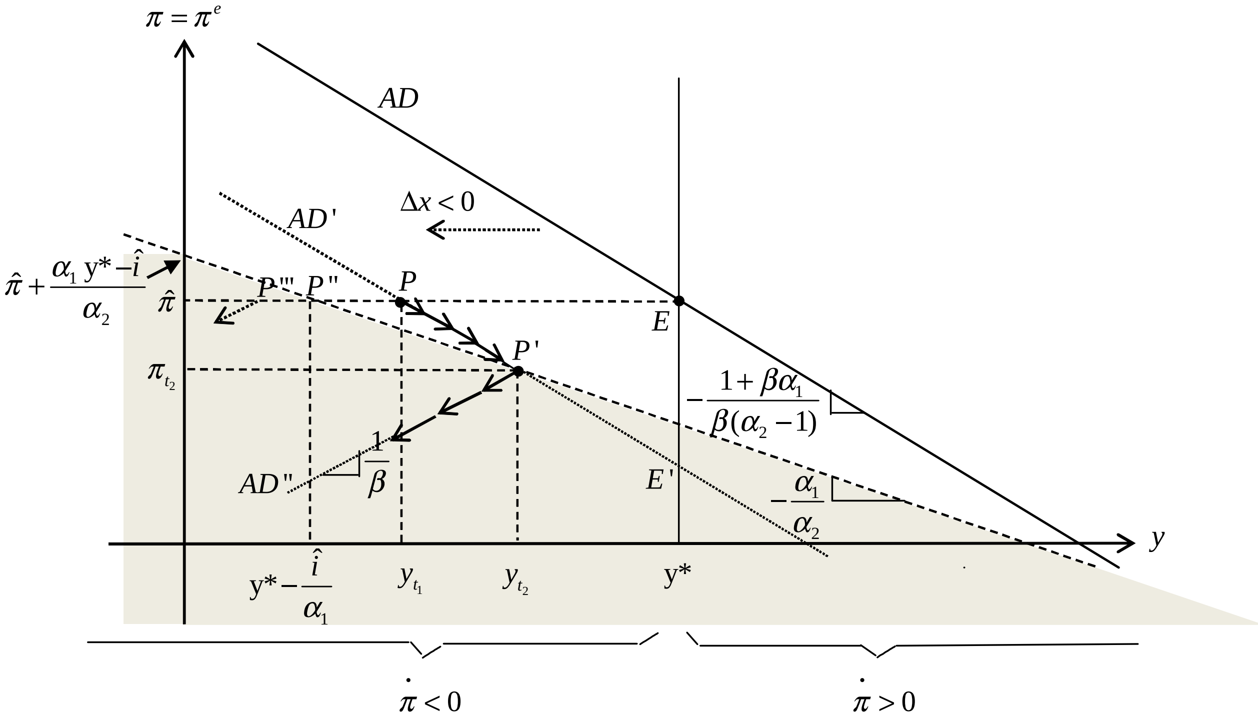

Figure 4: A large demand shock causes the liquidity trap to be operative when the economy hits the point P'. The case $\alpha_1 > 0$.

Rearranging, the boundary of the liquidity trap region thus is

$$\pi = \frac{\alpha_1 y^* + \alpha_2 \hat{\pi} - \hat{\imath}}{\alpha_2} - \frac{\alpha_1}{\alpha_2} y. \tag{15}$$

Comparing the absolute slope of the boundary of the trap region, α_1/α_2. with the slope of the AD and AD' curves, we see that the former is smaller than the latter, as also indicated in Fig. 4.

There are two cases to consider: $\alpha_1 > 0$ and $\alpha_1 = 0$.

5.2.1 The case $\alpha_1 > 0$: Monetary policy responds directly to both gaps

The shaded area in Fig. 4 represents the liquidity trap region for the case $\alpha_1 > 0$, where the boundary of the liquidity trap region is downward sloping. The point P indicates the position of the economy immediately after the adverse demand shock. In the text above we implicitly assumed that P were at P" or to the left of P", say at P"'. In this case the lower bound is immediately operative and forces the economy to move South-West in the diagram as indicated at the point P"'.

As Fig. 4 is drawn, however, P" is to the left of the point P, implying that the lower bound is not immediately operative. Nevertheless, in the process of lowering the nominal interest rate more than inflation falls, monetary policy hits the zero lower bound, at time $t_2 > t_1$, cf. the point P' in Fig. 4. From then on the economy is governed by (12) and (13). The movement is South-West along the *positively* sloped branch, AD", of the total *kinked aggregate demand* curve AD'-AD" in the diagram. Along the positively sloped branch the vicious spiral unfolds with output demand and output falling owing to a rising real interest rate caused by a continuing fall in the inflation rate due the low level of output while there is no longer a falling nominal interest rate.

There is empirical evidence that when the price inflation has become low, it tends to be more and more sticky downwards; similarly with wage inflation (see Hendry and ??, 2013). This *may* end the vicious spiral but does not reverse it. Inflation, π, *may* go negative, which amounts to *deflation,* as we saw under the Great Depression in the 1930s.[7] Also y may go negative. This might seem absurd, but is not, since y is really the *logarithm* of output.

In Fig. 8 of the appendix is depicted what the vicious spiral looks like in the $(\pi, \dot{\pi})$ plane.

A benchmark case* Let us consider the question: How large is the minimum adverse demand disturbance, measured by the change in x, needed to bring the economy immediately into the liquidity trap region? That is, when will the point P in Fig. 4 coincide with point P" on the boundary of the liquidity trap region?

At the point P" we have $\pi^e = \pi = \hat{\pi}$. The associated output level is, by (14), easily found to be

$$y = y^* - \frac{\hat{\imath}}{\alpha_1}.$$

For y_{t_1} to equal this value, we must, in view of (AD), have

$$y_{t_1} = x(\mu') - \theta\hat{\pi} = y^* - \frac{\hat{\imath}}{\alpha_1} = x(\bar{\mu}) - \theta\hat{\pi} - \frac{\hat{\imath}}{\alpha_1},$$

because y^* satisfies (AD) with $\mu = \bar{\mu}$ and $\pi_t^e = \hat{\pi}$. As $\theta\hat{\pi}$ cancels out, we find

$$x(\bar{\mu}) - x(\mu') = \frac{\hat{\imath}}{\alpha_1} \tag{16}$$

[7] At the time of writing (fall 2015), the European central Bank (ECB), facing an inflation rate in the Eurozone down at 0.3 percent on an annual basis, conducts *quantitative easing* (see Section 6 below) in its attempt to stop the vivious spiral and avoid the Eurozone ending up in deflation.

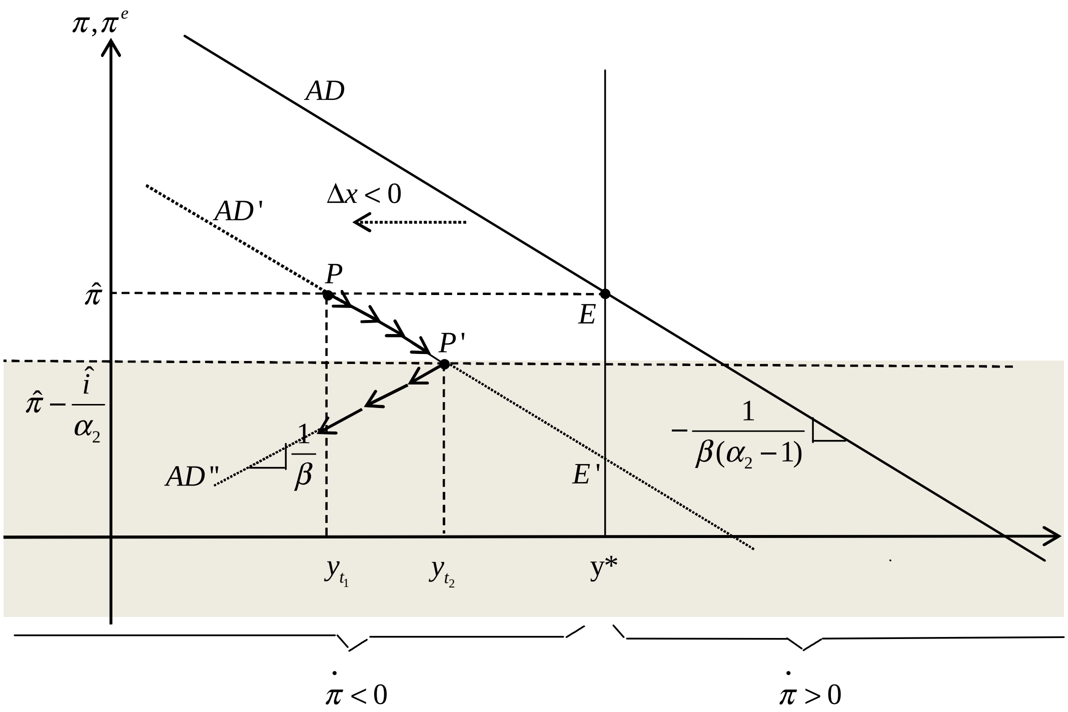

Figure 5: A large demand shock causes the liquidity trap to be operative when the economy hits the point P'. The case $\alpha_1 = 0$.

This is the minimum adverse demand disturbance (drop in x) needed to immediately bring the economy into the liquidity trap region. If the adverse demand disturbance is at least as large as this value, the lower bound becomes binding immediately at time t_1.

We see that the policy coefficient α_1 to the output gap in the the Taylor rule plays a role here. Given the inflation target, $\hat{\imath}$, we have that the larger is α_1, the smaller is the required Δx (considering the Taylor rule formula (5), this is no surprise). Everything else equal, this speaks for choosing a small α_1. Nevertheless, as we shall now see, even $\alpha_1 = 0$ is no guarantee for not ending up in a liquidity trap.

5.2.2 The case $\alpha_1 = 0$: Monetary policy only responds directly to expected inflation

When $\alpha_1 = 0$, the boundary of the liquidity trap region is horizontal. The shaded area in Fig. 5 represents the region in this case. Again the point P indicates the position of the economy immediately after the adverse demand shock at time t_1. By inspection of the figure, since by assumption $\hat{\imath} > 0$, P is necessarily situated above the liquidity trap region. Anyway, falling inflation sets in. In the process of lowering the nominal interest rate even more than inflation falls, monetary policy *may* hit the lower bound during the adjustment. As the figure is drawn, this is what happens at some point in time, t_2, cf. the point P' in Fig. 5. From then on the vicious spiral unfolds and the economy moves South-West along the *positively* sloped branch of the kinked aggregate demand curve AD'-AD" in the diagram.

6 Policy options

We have studied the dynamic interaction between aggregate demand, an "accelerationist" Phillips curve, and monetary policy following a Taylor rule.

Vis-a-vis small demand disturbances of the economy, the Taylor rule works well and tend to stabilize the economy around the "full" employment steady state.

For large adverse demand shocks, the economy may end up in a liquidity trap. In that case an unchanged Taylor rule cannot hinder a vicious circle to arise, leading into prolonged depression.

One policy option is to *raise the inflation target* and thereby inflation expectations. A central bank trying to follow that route may run into credibility problems, however.

Alternative or supplementary policy options are situated outside conventional monetary policy (short-term interest rate policy).

One possibility is *expansionary fiscal policy*. When the economy is in a liquidity trap, fiscal policy multipliers tend to be high. This is so for several reasons. One reason is that there will be no financial crowding out as long as the central bank wants its policy rate to be as low as possible. Another reason is that the economic situation which has triggered the liquidity trap is likely to also be a situation where involuntary unemployment is high.

Another possibility is so-called *quantitative easing* (QE). This can take several forms. The central bank may offer credit to financial intermediaries (banks, mutual funds, mortgage credit companies, insurance firms, etc.) on more gentle conditions than usually. And it may try directly to reduce the spread, ω, by buying long-term government bonds and other assets in the market.

Another form of QE is "helicopter money" as Milton Fridman called it. This is fiscal policy in the form of income transfers to the private sector directly financed by money issue.

7 Discussion

The model, as it stands, makes it appear that the central bank has a strong grip on the economy outside ZLB. Probably stronger than in reality. One circumstance behind this is of course that the model contains no stochastic elements. A second circumstance is that perfect knowledge of the NAIRU and the structural interest rate are strong assumptions, not likely to be fulfilled in practice. Also the assumption that output *immediately* adjusts to the demand changes prompted by changes in the policy rate seems too strong.

The assumed version of the Phillips curve is quite brute. In particular it tends to exaggerate the deflationary pressure in a liquidity trap. The constancy of the reaction speed δ (and perhaps also of y^*) in (2) is not in accordance with the empirical evidence. Hendry and ?? (2013) find that when the inflation rate has become low, it tends to be more and more sticky downwards. According to Stock and Watson (2010), once a slump has lasted 11 quarters at the same rate, no matter how high, unemployment loses its downward pressure on inflation.

Finally two *terminological remarks*:

1. Recognizing the serious limitations of the static AS-AD model in the (Y, P) plane, well-known from many textbooks in the past, several newer textbooks, e.g. Jones (2015), now use the label AS-AD for dynamic models in the (Y, π) plane. To avoid confusion, it may be better to use a label like *dynamic IS-MP model* or *dynamic AD-AS-MP model.*

2. Be aware that in the elder literature "output gap" usually meant $y^* - y$, while in recent literature the opposite meaning, $y - y^*$, has become quite established.

8 Appendix: What the dynamics look like in the $(\pi, \dot{\pi})$ plane

In this appendix we illustrate the dynamics in an alternative way, namely in the $(\pi, \dot{\pi})$ plane rather than the (y, π) plane. We stick to the case of perfect foresight: $\pi_t^e = \pi_t$ for all t.

Dynamics when the lower bound is not binding. After substitution of $(\overline{\text{AD}})$ into the Phillips curve, we have

$$\dot{\pi}_t = \delta(x(\bar{\mu}) - \theta\pi_t - y^*). \tag{17}$$

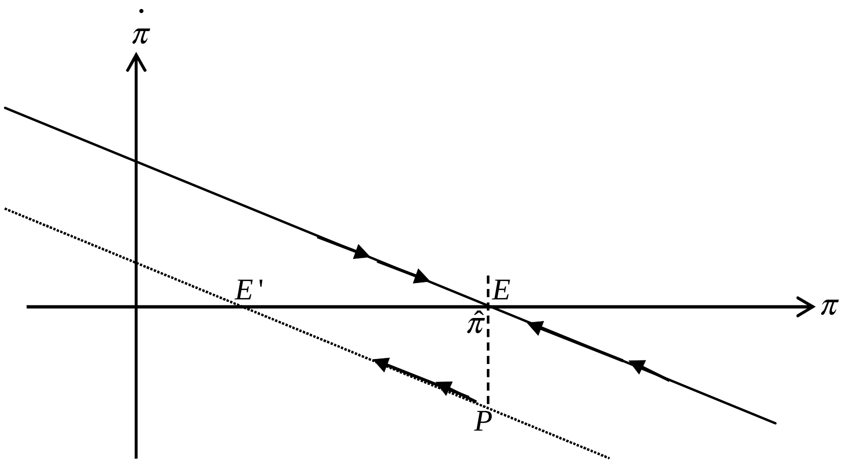

Figure 6:

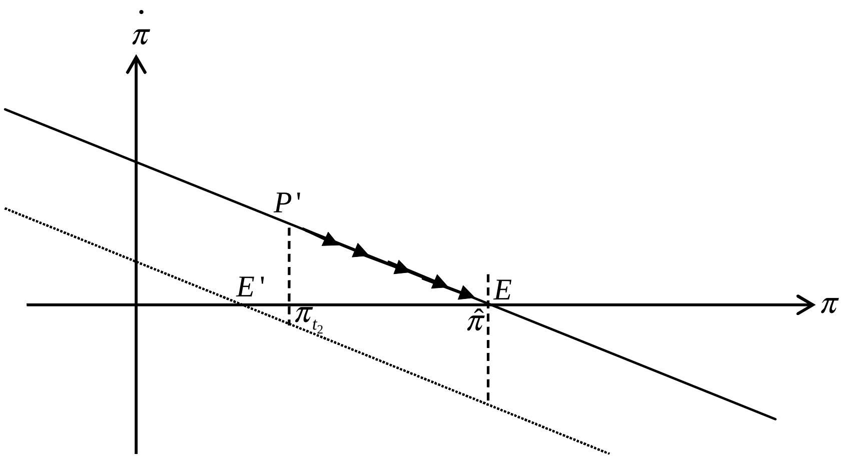

Figure 7:

The graph of this relationship is shown in Fig. 6 as the downward-sloping solid line in the figure. As long as $\mu = \bar{\mu}$, the economy must be at some point on this line. If $\pi < \hat{\pi}$, π will be growing towards $\hat{\pi}$, while if $\pi > \hat{\pi}$, π will be falling towards $\hat{\pi}$. Over time the economy moves along the line representing equation (17) until steady state at the point E is "reached".

Fig. 6 also illustrates the consequence of an adverse demand shock, disturbing an economy initially in steady state. The shock leads to a downward shift of the line representing (17). The position of the economy shifts from the point E to the point P, representing recession. Hereafter, there is a gradual fall in inflation ($\dot{\pi} < 0$) which, by the monetary policy, is accommodated by a faster fall in the nominal interest rate so as to lower the real interest rate.

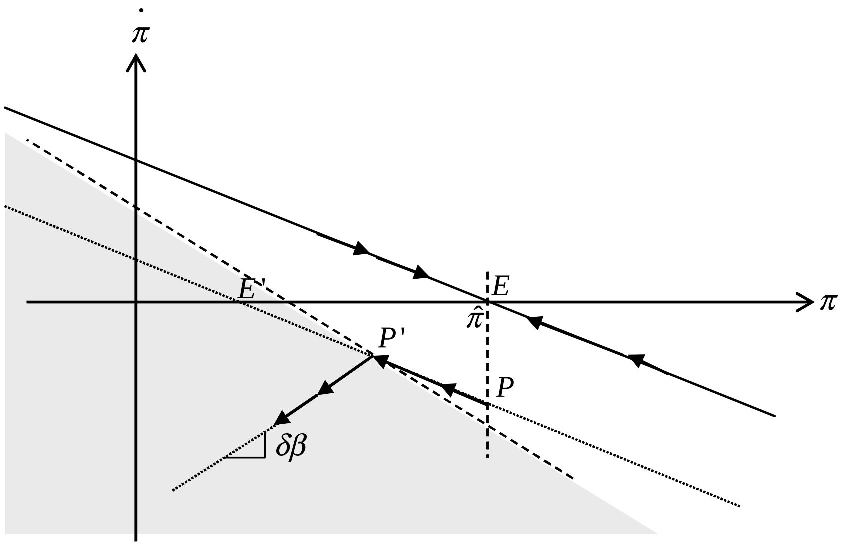

Figure 8:

An ensuing restoration of confidence at time t_2 and the implied dynamics is illustrated in Fig. 7. The favorable restoration of confidence shifts the line representing (17) back to its original position. The resulting boom triggers a gradual rise in inflation. Through the monetary policy the nominal interest rate rises even faster, thereby gradually raising the real interest rate. The boom is thus dampened and the economy is gradually brought back to the original steady state, E.

Dynamics when the lower bound is binding. As we saw in Fig. 4, if the adverse demand shock at time t_1 is large enough, the economy may immediately after the shock be at point P in that figure and then, after some time, enter the liquidity trap region at point P' where the vicious spiral takes over. Fig. 8 shows the corresponding evolution in the $(\pi, \dot{\pi})$ plane.

9 References

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