Advanced macroeconomics Short Note 3.
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## The bank-lending channel: The IS-BL model

In this note we consider the IS-BL model by Bernanke and Blinder (1988), "BL" being an abbreviation of "bank loans". The model aims at clarifying the monetary transmission mechanism in an economy where commercial banks offer checkable deposits to households and grant long-term bank loans to ultimate borrowers (households and non-bank firms). It is shown that the transmission of monetary policy takes two routes, the well-known interest rate channel and the bank lending channel.

Small and medium-sized firms are generally unable to issue bonds and equity shares for the centralized financial capital markets. Hence they are dependent on banks for external finance; indeed, in many countries bank loans are the main source of external finance. ${ }^{1}$

Fig. 1 portrays a stylized financial system. Households' financial saving is their income minus spending on durable as well as non-durable goods and services. The financial saving is partly channeled directly through centralized bond and stock markets to the ultimate users (government and other households and firms) and partly channeled indirectly to the ultimate users through financial intermediaries, the commercial banks.

## 1 The IS-BL model

A closed economy is considered. There is a public sector with a government in charge of fiscal policy and a partly independent central bank, the latter controlling the monetary base. The model gives no details about the credit market imperfections lying behind the existence of financial intermediaries, the commercial banks. We may think of asymmetric information and moral hazard problems between lender and borrower.

One-period bonds are issued by the government and traded in a centralized auction market. The private sector consists of the commercial banks and the "non-bank general

[^0]

Figure 1: A stylized financial system.
public" (households and non-bank firms). From now "banks" means the commercial banks. The banks act as competitive financial intermediaries which pool a significant part of households' financial savings, placed in demand deposit accounts. The pooled deposits are then used to invest in government bonds and to offer long-term loans to households and production firms (the non-bank firms) in a "customer credit market" with limited contract enforcement. The bank loans should be thought of as variable-rate loans. The deposits earn no interest (as an approximation for a "low" deposit interest rate). The model ignores the equity market and assumes that firms finance their investment partly by withheld profit, partly by bank loans. The model also ignores so-called investment banks that make up a major part of what is known as the "shadow banking system". ${ }^{2}$

The IS-BL model is static in the sense that only one period is considered. Notation is indicated in Table 1.

[^1]Table 1. List of main variable symbols.
$i_{B}=$ nominal bond interest rate,
$i_{L}=$ nominal bank lending interest rate (the "lending rate"),
$D=$ deposits of the non-bank private sector (a stock, earns no interest),
$\rho=$ required reserve-deposit ratio, $\rho \in[0,1)$, exogenous,
$M_{0}=$ monetary base,
$m m=$ the money multiplier,
$M_{1} \equiv \mathrm{~mm} \cdot M_{0}=$ money supply in the sense of demand deposits in commercial banks (a stock, earns no interest),
$E \equiv M_{0}-\rho D=$ excess reserves (a stock, like required reserves earning no interest), $L^{s}=$ supply of bank loans ("credit supply", a stock),
$\sigma=$ a shift parameter measuring perceived riskiness of offering bank loans,
$B=$ nominal value of the stock of government bonds held by the private sector,
$B_{b}=$ nominal value of government bonds held by the banks,
$B_{n}=$ nominal value of government bonds held by the non-bank public,
$W \equiv M_{0}+B=$ aggregate nominal financial wealth of the private sector,
$P=$ price level, exogenous, $P=1$,
$Y=$ real aggregate output,
$G=$ real government spending on goods and services, a policy parameter, $\tau=$ "fiscal tightness" (shift parameter).

Contrary to the simple IS-LM model, here $L$ refers to bank loans, not liquidity demand. The subscript $b$ stands for commercial banks, the subscript $n$ for the non-bank private sector, also called the "non-bank public". All deposits are fully liquid in the sense of being demand deposits (i.e., checkable deposits). The superscripts $s$ and $d$ signify supply and demand, respectively. We ignore currency. So the monetary base is identical to the stock of bank reserves. In studies like the present one where there is a distinction between directly granted credit and intermediated credit, it is common to speak of the latter as just "credit" and the "bank lending channel" as just the "credit channel".

The balance sheet ${ }^{3}$ of the central bank (CB) is given in Table 2.

Table 2. Balance sheet of the central bank (CB).

| Assets | Liabilities |
| :--- | :--- |
| $\bar{B}-B=$ value of gov. bonds held by CB | Currency (for simplicity here $=0$ ) <br> Deposits held by commercial banks <br> (here $\left.=M_{0}=\rho D+E\right)$ |
| Gold | Net worth |
| Total | Total |

[^2]The aggregate balance sheet of the commercial banks is shown in Table 3.

Table 3. Merged balance sheet of the commercial banks.

| Assets | Liabilities |
| :--- | :--- |
| $M_{0}=\rho D+E=$ reserves (deposited in CB) | $D=$ liquid deposits held by the |
| non-bank public |  |
| Vault currency (here $=0$ ) | Long-term debt (for simplicity here $=0$ ) |
| $B_{b}=$ loans to the non-bank public of gov. bonds held by <br> $B_{b}$ <br> commercial banks | Net worth (for simplicity here $=0$ ) |

On the assets side, in addition to reserves we have two interest-bearing assets, bank loans and government bonds. In the limiting case where these assets are perfect substitutes, the model becomes a simple IS-LM model.

The fact that the major part of the assets of the banks are long-term, hence comparatively illiquid, while the major part of the liabilities are short-term, gives rise to recurrent historical episodes of bank runs. The term bank run refers to situations where many depositors, fearing that their bank will be unable to repay their deposits in full and on time, simultaneously try to withdraw their deposits. This is the reason that since the Great Depression of the 1930s, in developed countries deposits in "ordinary banks" are typically protected by government deposit insurance up to a certain limit. To be enrolled in this kind of insurance, the banks must comply with a set of regulations.

Throughout the analysis, the variables $\rho, \sigma, \bar{B}, G$, and $P$ are exogenous, given the short time horizon of the model. For simplicity, $P=1$. The expected inflation rate is considered exogenous and, for simplicity, equal to zero. Financial wealth of the private sector, $W \equiv M_{0}+B$, is exogenous as well. Until further notice, also the monetary base, $M_{0}$, is exogenous in the sense of the CB using $M_{0}$ as its policy instrument. The CB can change $M_{0}$ through an open market operation whereby $\Delta M_{0}=-\Delta B$.

### 1.1 The supply of bank loans and broad money

From the balance sheet of the commercial banking sector in Table 3 follows the identity

$$
\begin{equation*}
M_{0}+L^{s}+B_{b} \equiv D \tag{1}
\end{equation*}
$$

We subtract required reserves, $\rho D$, on both sides of (1) to get

$$
M_{0}-\rho D+L^{s}+B_{b} \equiv E+L^{s}+B_{b} \equiv(1-\rho) D, \quad 0 \leq \rho<1
$$

where $E$ is excess reserves. So far this is just accounting, saying that disposable deposits, $(1-\rho) D$, on the liability side make up excess reserves, $E$, loans, $L^{s}$, and government bonds, $B_{b}$, on the asset side.

Given $(1-\rho) D$, how are the components of the triple $\left(E, L^{s}, B_{b}\right)$ determined? Regarding $E$, we assume that to dispose of sufficient "cash" (be sufficiently liquid), the banks generally have to hold some excess reserves. The desired amount of excess reserves depends negatively on the opportunity cost, the bond interest rate, $i_{B}$, forgone. Because of their comparatively high degree of liquidity, government bonds make up a close substitute for reserves. Denoting the desired fraction of disposable deposits held as excess reserves $e\left(i_{B}\right) \in[0,1]$, we have

$$
\begin{equation*}
E=e\left(i_{B}\right)(1-\rho) D, \tag{2}
\end{equation*}
$$

where $E / D$ is known as the "excess reserves ratio", and

$$
e\left(i_{B}\right)\left\{\begin{array}{l}
=0 \text { for } i_{B} \geq \bar{\imath}_{B},  \tag{3}\\
\in(0,1) \text { and } e^{( }\left(i_{B}\right)<0 \text { for } 0<i_{B}<\bar{\imath}_{B}, \\
\text { is set-valued (i.e., indeterminate) for } i_{B}=0 .
\end{array}\right.
$$

The upper bound, $\bar{\imath}_{B}$, above which no excess reserves are held, will for our purposes be treated as exogenous and "large" so as to not be binding. There is a zero-lower bound for $i_{B}$ because agents prefer holding money at zero interest rather than bonds at negative interest. As indicated by (3), at the zero-lower-bound banks are indifferent between holding excess reserves or government bonds.

How is the supply of bank loans determined? The fraction, $\ell\left(i_{B}, i_{L}, \sigma\right) \in[0,1]$, of disposable deposits used as bank loans is assumed to depend positively on the interest rate obtainable on these and negatively on the opportunity cost, the interest rate on bonds. That is, ${ }^{4}$

$$
L^{s}=\ell\left(i_{B}, i_{L}, \sigma\right)(1-\rho) D, \quad \ell_{i_{B}}^{\prime}<0, \ell_{i_{L}}^{\prime}>0, \ell_{\sigma}^{\prime}<0 \text { as long as } \ell\left(i_{B}, i_{L}, \sigma\right) \in(0,1) .
$$

Indeed, offering bank loans is less attractive the higher is $i_{B}$. And it is more attractive the higher is $i_{L}$. A precise no-arbitrage condition from the banks' perspective between what

[^3]is held as bank loans and what is held as government bonds is absent. This is due to heterogeneity and unknown risk of default within the stock of bank loans. The negative dependency of the supply of loans on the shift parameter $\sigma$ reflects that given the lending rate, $i_{L}$, less loans will be offered the higher the perceived riskiness.

The remaining part of disposable deposits, $(1-\rho) D-\left(E+L^{s}\right)$, is placed in government bonds.

We will concentrate on "normal circumstances" where both $e\left(i_{B}\right)$ and $\ell\left(i_{B}, i_{L}, \sigma\right)$ belong to the interior of $[0,1]$. That is, we assume "non-crisis circumstances" where $i_{B}>0$, and $\sigma$ is not "too high".

Since the model ignores currency, the monetary base, $M_{0}$, equals bank reserves held in the CB. The money supply, $M_{1}^{s}$, available for the non-bank private sector, equals deposits, $D$. The inverse of the money multiplier, $m m$, thus equals the reserve-deposit ratio:

$$
\begin{equation*}
\frac{1}{m m}=\frac{M_{0}}{M_{1}^{s}}=\frac{M_{0}}{D}=\frac{\rho D+E}{D}=\rho+e\left(i_{B}\right)(1-\rho) \equiv \frac{1}{m m\left(i_{B}\right)} \tag{4}
\end{equation*}
$$

where the fourth equality follows from (2). The money multiplier is thereby seen to be a function, $m m\left(i_{B}\right)$, of the interest rate on government bonds. From (4), and the fact that $e^{\prime}\left(i_{B}\right)<0$ in (3), follows that $m m^{\prime}\left(i_{B}\right)>0$. In words, a higher bond interest rate implies a higher money multiplier because a higher bond interest rate motivates the bank to hold less excess reserves. Moreover, in view of $\rho<1$, and as long as $0<e\left(i_{B}\right)<1$ (which is the case under "normal circumstances"), we have $m m\left(i_{B}\right)>1$. To summarize, the money supply can be written

$$
\begin{equation*}
M_{1}^{s}=D=m m\left(i_{B}\right) M_{0}>M_{0}, \quad m m^{\prime}\left(i_{B}\right)>0 \tag{5}
\end{equation*}
$$

### 1.2 The demand bank loans and $\mathrm{M}_{1}$-money

The balance sheet of the non-bank private sector (households and production firms) is given in Table 4. Aggregate nominal financial wealth of the non-bank private sector is $W$.

Table 4. The balance sheet of the non-bank private sector.

| Assets | Liabilities |
| :--- | :--- |
| $D=$ bank deposits | $L^{d}=$ bank loans |
| Currency in circulation (here $=0)$ <br> $B_{n}=$ value of gov. bonds held by non-bank public | $W=$ net worth |
| Total | $=$ Total |

The demand function for bank loans (a stock) is assumed given by

$$
L^{d}=L\left(Y, i_{B}, i_{L}\right), \quad L_{Y}^{\prime}>0, L_{i_{B}}^{\prime}>0, L_{i_{L}}^{\prime}<0
$$

The demand derives from households' and firms' need for finance to purchase consumption and investment goods, respectively. The positive dependency on $Y$ captures that higher current income and employment stimulates consumption, directly as well as indirectly through raising expected future income and thereby the state of confidence. To smooth consumption, some households may need credit. Similarly, from the perspective of production firms, a higher current level of economic activity, $Y$, may signal higher aggregate demand in the near future ("good times are underway"), thus making investment in increased capacity profitable.

The negative dependency of the demand for bank loans on the lending rate, $i_{L}$, reflects that bank loans are less attractive the higher the interest cost. Finally, a liability in the form of a bank loan is likely to be more tolerable the higher is $i_{B}$. This is because the borrower may place temporary excess liquidity in government bonds.

The demand for deposits in the banks derives from liquidity being needed for transactions. In addition, deposits offer a convenient book-keeping arrangement and a measure against theft. The stock of checkable deposits willingly held by the non-bank private sector is given by the money demand function

$$
M_{1}^{d}=M\left(Y, i_{B}\right), \quad M_{Y}^{\prime}>0, M_{i_{B}}^{\prime}<0
$$

Money demand thus depends partly on aggregate economic activity (due to the transactions motive) and partly on the opportunity cost of holding money, the interest rate on government bonds. As the exogenous price level, $P$, is set to $1, P$ is not visible.

In view of the balance sheet constraint

$$
\begin{equation*}
M_{1}^{d}+B_{n}=L^{d}+W \tag{6}
\end{equation*}
$$

the demand for government bonds coming from the non-bank private sector is given as the residual: $B_{n}=L^{d}+W-M_{1}^{d}$.

Since the banks are ultimately owned by households, $W$ equals the aggregate nominal financial wealth of the private sector as a whole and consists of the monetary base and the value of outstanding government bonds, $B$ :

$$
\begin{equation*}
W \equiv M_{0}+B \tag{7}
\end{equation*}
$$

## 2 General equilibrium

There are three asset markets and an output market. There is abundant capacity in the output market and the nominal price level is rigid in the short run so that output is demand determined. Behind the scene employment adjusts to the demand for labor.

### 2.1 Equilibrium in the asset markets

The three asset markets to consider are the money market, the market for bank loans, and the bonds market. Interest rates adjust so as to generate equilibrium in all three asset markets. Since assets are stocks (quantities at a given point in time), we should think of the asset markets as being open at a given moment of time, that is, either at the beginning or the end of the current period. To fix ideas, we choose the beginning of the period.

Considering the two first-mentioned asset markets, we have $M_{1}^{s}=M_{1}^{d}$, that is,

$$
\begin{equation*}
m m\left(i_{B}\right) M_{0}=M\left(Y, i_{B}\right), \tag{MM}
\end{equation*}
$$

and $L^{s}=L^{d}$, that is,

$$
\begin{equation*}
\ell\left(i_{B}, i_{L}, \sigma\right)(1-\rho) m m\left(i_{B}\right) M_{0}=L\left(Y, i_{B}, i_{L}\right) \tag{BL}
\end{equation*}
$$

Regarding the third market, the bond market, let $x$ denote the number of one-period bonds issued by the government at the beginning of the period (recall that the government debt has to be refinanced each period). Let each bond offer a payoff of 1 unit of money at the end of the period. If $v>0$ denotes the market price of a bond at the beginning of the period, the effective interest rate $i_{B}$ is given by the equation

$$
v=1 /\left(1+i_{B}\right) .
$$

The market value of the whole government debt at the beginning of the period is $B \equiv v x$.
Considering the demand side, the banks' demand a quantity of bonds equal to $x_{b}=$ $B_{b} / v$ and the non-bank private sector demands $x_{n}=B_{n} / v$. Market clearing in the bond market requires

$$
x=x_{b}+x_{n} .
$$

This is equivalent to

$$
B \equiv v x=v x_{b}+v x_{n} \equiv B_{b}+B_{n} .
$$

So equilibrium in the bonds market is present if and only if

$$
\begin{equation*}
B=B_{b}+B_{n} . \tag{8}
\end{equation*}
$$

This equilibrium condition holds automatically, when the two other asset markets are in equilibrium. This is the principle known as "Walras' law for stocks" and is an implication of the balance sheet constraint of the private sector. This balance sheet constraint is given by (7) combined with (6):

$$
\begin{equation*}
M_{1}^{d}-L^{d}+B_{n}=W \equiv M_{0}+B \tag{9}
\end{equation*}
$$

Indeed,

$$
\begin{aligned}
B & \equiv W-M_{0}=B_{n}+M_{1}^{d}-L^{d}-M_{0} \quad(\text { by }(9)) \\
& =B_{n}-L^{s}+M_{1}^{s}-M_{0}=B_{n}-L^{s}+D-M_{0} \quad(\text { by }(\mathrm{MM}),(\mathrm{BL}), \text { and }(5)) \\
& =B_{n}-L^{s}+B_{b}+L^{s}=B_{n}+B_{b} \quad(\text { by }(1)),
\end{aligned}
$$

thus confirming that the bond market clears, i.e., (8) holds, if the money market and the market for bank loans clear.

### 2.2 The output market

The asset market equilibrium conditions should be combined with equilibrium in the market for output.

The expected inflation rate is by assumption nil. So the nominal interest rates, $i_{B}$ and $i_{L}$, on bonds and bank loans, respectively, are at the same time real interest rates. We assume that the sum, $\mathfrak{D}$, of private consumption and investment can be written

$$
\mathfrak{D}=C\left(Y^{p}, i_{B}, i_{L}, W\right)+I\left(Y, i_{B}, i_{L}\right), \quad 0<C_{Y^{p}} \leq C_{Y^{p}}+I_{Y}<1,
$$

where the partial derivatives of both the consumption and the investment function w.r.t. the two interest rates are negative (see (YY) below). Here $Y^{p}=Y-(\tau+T(Y))$ is disposable private income, $\tau$ being a shift parameter ("fiscal tightness") and $T(Y)$ being the "automatic" net tax revenue, $0 \leq T^{\prime}(Y)<1$.

Interpreting $\mathfrak{D}$ as a function, equilibrium in the output market can be expressed on the compact form
$Y=\mathfrak{D}\left(Y, W, i_{B}, i_{L}, \tau\right)+G, \quad 0<\mathfrak{D}_{Y}^{\prime}<1, \mathfrak{D}_{W}^{\prime}>0, \mathfrak{D}_{i_{B}}^{\prime}<0, \mathfrak{D}_{i_{L}}^{\prime}<0,-1<\mathfrak{D}_{\tau}^{\prime}=-C_{Y^{p}}<0$.

We now have three equations, (MM), (BL), and (YY), and three endogenous variables, $Y, i_{B}$, and $i_{L}$.

## 3 Analysis

Let us derive a graphical representation of the economy in the ( $Y, i_{B}$ ) plane in analogy of the standard illustration of IS-LM equilibrium.

### 3.1 Derivation of the MP curve

For given $M_{0}$, equilibrium in the money market provides a monetary policy curve. Indeed, the equation (MM) defines $i_{B}$ as an implicit function of $Y$ and $M_{0}$ :

$$
\begin{equation*}
i_{B}=i_{M P}\left(Y, M_{0}\right) . \tag{MP}
\end{equation*}
$$

The graph of this function in the $\left(Y, i_{B}\right)$ plane for fixed $M_{0}$ is our monetary policy curve, in brief, the MP curve. The slope of the MP curve equals the partial derivative of the (MP) function w.r.t. $Y$ and can be found by taking the total differential on both sides of (MM):

$$
m m\left(i_{B}\right) d M_{0}+M_{0} m m^{\prime}\left(i_{B}\right) d i_{B}=M_{i_{B}}^{\prime} d i_{B}+M_{Y}^{\prime} d Y
$$

By setting $d M_{0}=0$ and reordering, we find

$$
\begin{equation*}
{\frac{\partial i_{B}}{\partial Y_{\mid M P}}}=\frac{M_{Y}^{\prime}}{M_{0} m m^{\prime}\left(i_{B}\right)-M_{i_{B}}^{\prime}}>0 \tag{10}
\end{equation*}
$$

cf. Fig. 2. The MP curve is thus positively sloped. The curve depicts the combinations of $i_{B}$ and $Y$ that for a given $M_{0}$ are consistent with money market equilibrium. The positive slope reflects that, for a given $M_{0}$, the higher transactions-motivated demand for money, induced by a higher level of economic activity, results in initial excess supply of bonds, thereby lowering their price and so raising the interest rate, $i_{B}$. As $i_{B}$ is raised, the money multiplier goes up because banks become more eager to turn excess reserves into interest bearing assets.

In passing we note that by setting $d Y=0$ in (MM'), while allowing an increase in the monetary base of size $d M_{0}$ and reordering, we find

$$
\begin{equation*}
\frac{\partial i_{B}}{\partial M_{0 \mid M P}}=\frac{-m m\left(i_{B}\right)}{M_{0} m m^{\prime}\left(i_{B}\right)-M_{i_{B}}^{\prime}}<0 . \tag{11}
\end{equation*}
$$

As long as the bond interest rate has not yet changed, the effect of an expansion of the monetary base is an excess supply of money and excess demand for bonds. But thereby the price of bonds goes up which amounts to a fall in the bond interest rate, $i_{B}$, thus driving the money multiplier up and money demand down until equilibrium in the money market is obtained, given the hypothetical unchanged level of output. In Fig. 2 below this tells us that a higher $M_{0}$ shifts the MP curve downwards and thereby, everything else equal, stimulates output demand. This "channel" for the influence of money supply changes on the economy is called the (bond) interest rate channel and represents a mechanism also known from the simple IS-LM model.

We shall now see there is an additional channel, the bank lending channel.

### 3.2 Derivation of the IS curve

The description of equilibrium in the bank loans and output markets is a little more cumbersome. First, consider the bank loan market. The equilibrium condition, (BL), gives the lending rate as an implicit function of $Y, i_{B}, \sigma$, and $M_{0}$ :

$$
\begin{equation*}
i_{L}=f\left(Y, i_{B}, \sigma, M_{0}\right) \tag{12}
\end{equation*}
$$

The partial derivatives can be found by taking the total differential on both sides of (BL):

$$
\begin{align*}
& (1-\rho)\left[\ell\left(i_{B}, i_{L}, \sigma\right)\left(m m\left(i_{B}\right) d M_{0}+M_{0} m m^{\prime}\left(i_{B}\right) d i_{B}\right)\right. \\
& \left.+m m\left(i_{B}\right) M_{0}\left(\ell_{i_{B}}^{\prime} d i_{B}+\ell_{i_{L}}^{\prime} d i_{L}+\ell_{\sigma}^{\prime} d \sigma\right)\right]  \tag{13}\\
= & L_{i_{B}}^{\prime} d i_{B}+L_{i_{L}}^{\prime} d i_{L}+L_{Y}^{\prime} d Y .
\end{align*}
$$

We find the partial derivative of $f$ w.r.t. $Y$ by setting $d i_{B}=d \sigma=d M_{0}=0$ and reordering:

$$
\begin{equation*}
f_{Y}^{\prime}=\frac{d i_{L}}{d Y}=\frac{L_{Y}^{\prime}}{(1-\rho) m m\left(i_{B}\right) M_{0} \ell_{i_{L}}^{\prime}-L_{i_{L}}^{\prime}}>0 . \tag{14}
\end{equation*}
$$

So, given $i_{B}, \sigma$, and $M_{0}$, a higher $Y$ induces a tendency for the lending rate to rise. The reason is that the induced higher transaction demand for money raises the demand for bank loans. On the other hand this higher demand for bank loans is held at bay by this very increase in the lending rate.

We find the partial derivative of $f$ w.r.t. $i_{B}$ by setting $d Y=d \sigma=d M_{0}=0$ in (13) and reordering:

$$
\begin{equation*}
f_{i_{B}}^{\prime}=\frac{d i_{L}}{d i_{B}}=\frac{L_{i_{B}}^{\prime}-(1-\rho)\left[\ell\left(i_{B}, i_{L}, \sigma\right) M_{0} m m^{\prime}\left(i_{B}\right)+m m\left(i_{B}\right) M_{0} \ell_{i_{B}}^{\prime}\right]}{(1-\rho) m m\left(i_{B}\right) M_{0} \ell_{i_{L}}^{\prime}-L_{i_{L}}^{\prime}}>0 . \tag{15}
\end{equation*}
$$

The positivity is imposed by assuming, as Bernanke and Blinder (1988) do, that $m m^{\prime}\left(i_{B}\right)$ is "not too large". The intuitive explanation that this assumption is needed to get a positive derivative $d i_{L} / d i_{B}$ is as follows. On the one hand, given $Y, \sigma$, and $M_{0}$, a higher bond interest rate raises the value of the option to place temporary excess liquidity in bonds, thus making a high bank lending rate more tolerable. Moreover, from the banks' perspective a higher bond interest rate makes it attractive to invest more in bonds and offer less bank loans ( $\ell_{i_{B}}^{\prime}<0$ ). This means upward pressure on the lending rate also from the supply side. On the other hand there is a partly offsetting influence coming from the induced rise in the money multiplier along with the rise in the bond interest rate. The imposed assumption is that this influence is only partly offsetting.

The partial derivative of $f$ w.r.t. $\sigma$ is found by setting $d Y=d i_{B}=d M_{0}=0$ in (13) and reordering:

$$
\begin{equation*}
f_{\sigma}^{\prime}=\frac{d i_{L}}{d \sigma}=\frac{-(1-\rho) m m\left(i_{B}\right) M_{0} \ell_{\sigma}^{\prime}}{(1-\rho) m m\left(i_{B}\right) M_{0} \ell_{i_{L}}^{\prime}-L_{i_{L}}^{\prime}}>0 \tag{16}
\end{equation*}
$$

This derivative is positive because a higher perceived riskiness associated with offering bank loans reduces the supply of bank loans. Given the demand for bank loans, the lending interest rate thereby becomes higher.

Finally, the partial derivative of $f$ w.r.t. $M_{0}$ is found by setting $d Y=d i_{B}=d \sigma=0$ in (13) and reordering:

$$
\begin{equation*}
f_{M_{0}}^{\prime}=\frac{d i_{L}}{d M_{0}}=\frac{-(1-\rho) \ell\left(i_{B}, i_{L}, \sigma\right) m m\left(i_{B}\right)}{(1-\rho) m m\left(i_{B}\right) M_{0} \ell_{i_{L}}^{\prime}-L_{i_{L}}^{\prime}}<0 . \tag{17}
\end{equation*}
$$

An increase in the monetary base through an open-market purchase of bonds thus lowers the bank lending rate. The mechanism is that the inflow of central bank money allows the banks to increase profitable lending and at the same time maintain reserves at the desired level. In turn, the raised supply of bank loans lowers the lending rate - and thereby stimulates aggregate demand and output. This mechanism is called the bank lending channel.

Now consider the equilibrium condition (YY) for the output market. Substituting (12) into (YY) gives

$$
\begin{equation*}
Y=\mathfrak{D}\left(Y, i_{B}, f\left(Y, i_{B}, \sigma, M_{0}\right), \tau\right)+G \tag{IS}
\end{equation*}
$$

where the partial derivatives of $\mathfrak{D}$ are reported in (YY). Instead of the standard IS equation we thus arrive at an IS equation the position of which depends both on the supply of base money, $M_{0}$, and the perceived riskiness of offering bank loans. The IS equation
defines $i_{B}$ as an implicit function of $Y, \sigma, M_{0}, \tau$, and $G$ :

$$
\begin{equation*}
i_{B}=i_{I S}\left(Y, \sigma, M_{0}, G, \tau\right) \tag{IS’}
\end{equation*}
$$

The graph of this function in the $\left(Y, i_{B}\right)$ plane for fixed $\sigma, M_{0}, G$, and $\tau$, defines our $I S$ curve.

To be able to calculate (among other things) the slope of the IS curve, we first take the total differential on both sides of (IS):

$$
\begin{equation*}
d Y=\mathfrak{D}_{Y^{p}}^{\prime} d Y+\mathfrak{D}_{i_{B}}^{\prime} d i_{B}+\mathfrak{D}_{i_{L}}^{\prime}\left(f_{Y}^{\prime} d Y+f_{i_{B}}^{\prime} d i_{B}+f_{\sigma}^{\prime} d \sigma+f_{M_{0}}^{\prime} d M_{0}\right)+\mathfrak{D}_{\tau}^{\prime} d \tau+d G \tag{18}
\end{equation*}
$$

By setting $d \sigma=d M_{0}=d \tau=d G=0$ and reordering, we get

$$
\frac{\partial i_{B}}{\partial Y_{\mid I S}}=\frac{1-\mathfrak{D}_{Y}^{\prime}-\mathfrak{D}_{i_{L}}^{\prime} f_{Y}^{\prime}}{\mathfrak{D}_{i_{B}}^{\prime}+\mathfrak{D}_{i_{L}}^{\prime} f_{i_{B}}^{\prime}}<0
$$

where $f_{Y}^{\prime}$ from (14) can be inserted. This formula gives the slope of the IS curve which is thus negative, cf. Fig. 2.

The interpretation of the IS curve is that it depicts the combinations of $i_{B}$ and $Y$ that, for given $\sigma, M_{0}, \tau$, and $G$, are consistent with equilibrium in both the market for bank loans and the output market. The IS curve is negatively sloped because a rise in the bond interest rate, $i_{B}$, both directly and indirectly, via the associated increase in the lending rate, cf. (15), reduces aggregate demand (via reducing consumption and investment). In contrast to a standard IS curve, the position of this IS curve depends not only on the fiscal policy parameters $\tau$ and $G$, but also on the supply of base money, $M_{0}$, and the perceived riskiness, $\sigma$, of offering bank loans. By setting $d Y=d \sigma=d \tau=d G=0$ in (18), we find that a higher $M_{0}$ shifts the IS curve upwards. By setting $d Y=d M_{0}=d \tau=d G=0$, we find that a higher $\sigma$ shifts it downwards.

### 3.3 General equilibrium

In general equilibrium both the output market, the money market, and the bank loan market (and thereby also the bond market) clear. The equilibrium is given as the point where the MP and IS curves in Fig. 2 intersect. Since an upward-sloping MP curve and a downward-sloping IS curve can only intersect once, a solution to the model, $\left(Y, i_{B}\right)$, is unique. Assuming existence of a solution, we can thus write $Y$ and $i_{B}$ as implicit functions of the exogenous variables we are interested in:


Figure 2: The IS-MP cross (for fixed $M_{0}, \sigma$, and $G$ ). A higher $\sigma$ shifts the IS curve to the stippled position.

$$
\begin{align*}
Y & =g\left(\sigma, M_{0}, \tau, G\right)  \tag{19}\\
i_{B} & =h\left(\sigma, M_{0}, \tau, G\right) \tag{20}
\end{align*}
$$

By construction, the equilibrium is a Keynesian equilibrium. The partial derivatives of the solutions for $Y$ and $i_{B}$, respectively, w.r.t. $\sigma, M_{0}, \tau$, and $G$, can be found by using Cramer's rule on the system consisting of (MM) and (IS). If we are only interested in the sign of the effects, "curve shifting" in Fig. 2 is in many cases sufficient.

### 3.3.1 Crisis

Suppose an economic crisis is under way and that an increased riskiness of supplying bank loans is perceived. The MP curve in Fig. 2 is unaffected, cf. the equation (MP). The IS curve is shifted downwards because a higher $\sigma$, for fixed $Y$ and $i_{B}$, induces a higher $i_{L}$, cf. (16). At a given output level, $Y$, equilibrium in the output market then requires a lower $i_{B}$ to compensate for the higher $i_{L}$, cf. equation (YY). The conclusion is that the IS curve will now intersect the MP curve South-West from the old equilibrium. So both $i_{B}$ and $Y$ will be lower in the new equilibrium. These responses of $i_{B}$ and $Y$ imply a dampening feedback on the "initial" rise in $i_{L}$. But they do not eliminate the latter rise. This is because the dampening feedback only exists to the extent that the net effect on
$i_{L}$ of the higher $\sigma$ is positive. The final outcome is thus an increased spread: higher bank lending rate and lower interest rate on government bonds. A concomitant phenomenon is a reduced money multiplier. In summary:

$$
\left.\sigma \uparrow \Rightarrow i_{L} \uparrow \Rightarrow i_{B} \downarrow \text { (for fixed } Y\right) \Rightarrow\left\{\begin{array}{c}
i_{L}-i_{B} \uparrow \text { (increased spread), } \\
m m\left(i_{B}\right) \downarrow \text { (reduced money multiplier) }
\end{array}\right.
$$

These traits describe well what happened in the U.S. both in the first years of the Great Depression in the 1930s and when the full-scale financial and economic crisis we call the Great Recession broke out in 2008-2009. The comparatively risk-free interest rates, like those on government bonds, fell while risky interest rates, like those on consumer loans (car loans etc.) and corporate bonds, rose.

## 4 Policy

Monetary policy: Suppose $M_{0}$ is increased through an open market operation, implying $\Delta M_{0}=-\Delta B>0$. This affects output through two channels.

The credit channel: The banks are now able to offer more bank loans, and so the lending rate decreases, cf. (17). This stimulates demand in view of $\mathfrak{D}_{i_{L}}^{\prime}<0$. Thereby the IS curve is shifted to the right, and output is raised for given $i_{B}$.

The interest rate channel: For the increased money supply, $M_{1}$, to be willingly held by the public, the bond rate, $i_{B}$, must fall for given $Y$, cf. (11). The MP curve thus moves South-East. For unchanged $i_{B}$ this allows a higher level of output.

The total effect of the monetary expansion on output is thus unambiguously positive, and on the lending rate the effect is unambiguously negative. The total effect on the bond interest rate is ambiguous.

Fiscal policy: Although a rise in $G$ (not accompanied by a change in $M_{0}$ ) will automatically raise tax revenue, this may not be enough to avoid a budget deficit. There will thereby be a larger supply of government bonds next period. Feedback effects from this are ignored in this simple model. The case of a fully financed fiscal expansion can be analyzed by the method used for the simple IS-LM model in Chapter 21.3.

An alternative version of the model would consider $i_{B}$ as the monetary policy instrument (as long as the zero-lower-bound is not binding) and then let $M_{0}$ adjust endogenously.

Allowing corner solutions - desired excess reserves $E=0$ for instance - the model should be extended to include credit rationing. See Blinder (1987). Stiglitz and Weiss (1981) study the microeconomics of credit rationing from an incomplete-information perspective.

In Exercise Problem X. 2 the reader is asked to apply this model for a series of economic questions.

Banks runs Bank runs as a self-fulfilling prophesy are studied in Diamond and Dybvig (1983), Diamond (2007), and Gertler and Kiyotaki (2015). In view of government regulation and the institution of deposit insurance since the 1930s, bank runs on ordinary banks are no longer common, but phenomena similar to bank runs, driven by self-fulfilling expectations, may occur vis-a-vis financial intermediaries in the "shadow banking system", like investment banks and mutual funds (think of Northern Rock, London, Sept. 2007, and Lehman Brothers, New York, Sept. 2008).

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[^0]:    ${ }^{1}$ See for instance Mishkin (2007, p. 182) and Kashyap and Stein (1994).

[^1]:    ${ }^{2}$ The investment banks do not offer deposit accounts, and they are subject to significantly less public regulations than "ordinary banks".

[^2]:    ${ }^{3}$ A balance sheet account shows the status (stock of assets and liabilities at a given point in time). An operations account shows the deliveries and uses per period (flows).

[^3]:    ${ }^{4}$ In the present model, variables may have several subscripts indicating the specific economic interpretation of the variable. To avoid confusion, we shall therefore add a prime, " '", when considering partial derivatives. The partial derivatives of a function $z=f(x, y)$ are thus denoted $f_{x}^{\prime}$ and $f_{y}^{\prime}$, respectively.

