Written exam for the M. Sc. in Economics Summer 2006

Economic Growth (Videregående vækstteori)

June 7, 2006

Four hours. No auxiliary material

To be answered in Danish or English[1]

The weighting of the problems is:

Problem 1: 10 %, Problem 2: 30 %, Problem 3: 50 %, Problem 4: 10 %.

Problem 1. In a magazine on science the following data was reported:

World income per capita relative to income per capita in the US: 1952-96

Year	*Percent*
1952	13.0
1962	13.3
1972	13.0
1982	13.8
1992	15.1
1996	17.7

Source: Knowledge, Technology, & Policy 13, no. 4, 2001, p. 52.
Remark. Countries' per capita income are weighted by population as a fraction of the world population.

a) Briefly, discuss this data relative to the concept of σ convergence or divergence and relative to your knowledge of the importance of weighting by population size.

b) Give a short list of mechanisms that could in principle explain the data above.

Problem 2. Consider a closed market economy with education in private schools. Under perfect competition the representative firm chooses capital input, K^d, and labour input, L^d, in order to maximize profit, given the production function

$$Y = F(K^d, hL^d),$$

where Y is output, h is average human capital and F is a neoclassical production function with constant returns to scale.

[1] You are also allowed to write in Swedish or Norwegian.

a) Given h and the aggregate supplies of capital, K, and labour, L, respectively, determine the real rental rate, R, for capital and the real wage, w, per unit of labour time in equilibrium.

Aggregate output (= aggregate income) is used for consumption, C, investment, I_K, in physical capital and investment, I_H, in human capital,[2] i.e.,

$$Y = C + I_K + I_H.$$

The dating of the variables is suppressed where not needed for clarity. A dot over a variable denotes the time derivative, e.g., $\dot{x} \equiv dx/dt$, where t denotes time. The stocks of the two kinds of capital change according to $\dot{K} = I_K - \delta K$ and $\dot{H} = I_H - \delta H$, respectively, where $H \equiv hL$. We have, for simplicity, assumed that the depreciation rates (decay rates) are the same for both kinds of capital.

The representative household (family) has infinite horizon and consists of L members, where $L = L_0 e^{nt}$, $n \geq 0$, $L_0 > 0$. Each family member supplies inelastically one unit of labour per time unit. Let θ and ρ be positive constants, where $\rho > n$. Let $a \equiv$ per-capita financial wealth, $c_t \equiv C_t/L_t$, $r \equiv$ the real rate of interest and $i \equiv I_H/L$.

The representative household chooses a path $(c_t, i_t)_{t=0}^{\infty}$ to maximise

$$\begin{aligned}
U_0 &= \int_0^{\infty} \frac{c_t^{1-\theta} - 1}{1-\theta} e^{-(\rho-n)t} dt \qquad \text{s.t.} && (1)\\
c_t &\geq 0, \; i_t \geq 0, && (2)\\
\dot{a}_t &= (r_t - n)a_t + w_t - c_t - i_t, \qquad a_0 \text{ given}, && (3)\\
\dot{h}_t &= i_t - (\delta + n)h_t, \qquad h_0 > 0 \text{ given}, && (4)\\
\lim_{t\to\infty} a_t e^{-\int_t^{\tau}(r_s - n)ds} &\geq 0, && (5)\\
h_t &\geq 0 \text{ for all } t. && (6)
\end{aligned}$$

b) Briefly interpret this decision problem, including the parameters θ and ρ.

c) Use the Pontryagin maximum principle to find the first-order conditions for an interior solution.

d) Derive from the first-order conditions the Keynes-Ramsey rule and a no-arbitrage equation showing a relationship between $\hat{w} \equiv w/h$ and r.

Assume now, for simplicity, that the aggregate production function is Cobb-Douglas:

$$Y = AK^{\alpha}(hL)^{1-\alpha}, \qquad A > 0, 0 < \alpha < 1,$$

e) Determine the real rate of interest in equilibrium at time t.

[2] That is, educational activity is a part of aggregate output.

Suppose parameters are such that $\dot{c}/c > 0$ and U_0 is bounded.

f) The no-arbitrage equation from d) (which is needed for an *interior* solution to the household's decision problem) requires a specific value of K/H to obtain. Determine this value and explain what happens to begin with if the historically given K/H ratio in the economy differs from it; and explain what happens in the long run.

Problem 3. Consider a closed market economy with L utility maximizing households. Each household supplies inelastically one unit of labour per time unit. There are two production sectors, the "basic-goods sector" and the "innovative sector". For convenience we call the two sectors Sector 1 and Sector 2, respectively. There is no physical capital in the economy. Households' financial wealth consists of shares in monopoly firms in Sector 2, which supplies specialized intermediate goods. These goods are input in Sector 1, where the firms operate under perfect competition. Also the labour market has perfect competition. All firms are profit maximisers. Generally variables are dated implicitly. A dot over a variable denotes the time derivative.

Firm i ($i = 1, 2, ..., M$) in Sector 1 has the production function

$$Y_i = AL_i^{1-\alpha}\sum_{j=1}^{N}(x_{ij})^{\alpha}, \qquad A > 0,\ 0 < \alpha < 1.$$

Here Y_i, L_i and x_{ij} denote output of the firm, labour input and input of intermediate good j, respectively ($j = 1, 2, ..., N$).

In Sector 2 R&D activity occurs. New "technical designs", that is, blueprints for making new specialized intermediate goods are invented. Ignoring indivisibility problems, we assume that the number of new technical designs invented in the economy per time unit can be written

$$\dot{N} = \mu R, \qquad \mu > 0,\ \mu \text{ constant},$$

where R denotes the aggregate R&D cost (per time unit) in terms of basic goods. For simplicity it is assumed that inventions can go in so many directions that the likelihood of different agents chasing and making the same invention is negligible.

After an invention has been made, the inventor begins supplying the new intermediate good. To begin with the inventor has a monopoly over the production and sale of the new good (say by concealment of the new technical design). But sooner or later imitators find out how to make very close substitutes (it is difficult to codify the technical aspects of the inventions, hence patents do not give effective protection and are in any case only of limited duration). There is uncertainty as to how long the monopoly position of an inventor lasts. We assume the erosion of monopoly power can be described by a Poisson process. That is, if T denotes the remaining lifetime of monopoly j, then the probability that $T > \tau$ is $e^{-p\tau}$, where $p > 0$ is a given Poisson "arrival rate" (the same for all monopolies). Further, the cessations of the different monopolies are stochastically independent. N is "large" so that by holding shares in many different firms, the households face no risk.

Aggregate output of basic goods, $Y \equiv \sum_{i=1}^{M} Y_i$, is used partly for consumption, $C \equiv cL$, partly for input in R&D activity and partly for input in the production of specialized intermediate goods. Once invented, an intermediate good of type j costs one basic good as input and nothing else (the same for all j). Hence we have

$$Y = C + R + X,$$

where $X \equiv \sum_j \sum_i x_{ij}$.

a) As long as inventor j (firm j in Sector 2) is still a monopolist, the earned profit per unit of time is $\pi_j = (\frac{1}{\alpha} - 1)X^m \equiv \pi^m$, where $X^m = LA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}$. Explain this result (you don't have to derive it formally).

b) As described above, sooner or later inventor j loses the monopoly. When this happens, intermediate good j becomes competitive, i.e., it is supplied in the amount $X^c = L(\alpha A)^{\frac{1}{1-\alpha}}$. Explain this result and comment on the relative size of X^c and X^m.

The market value of monopoly j at time t can be written

$$V(t) = E_t \int_t^{\infty} \pi_j(\tau) e^{-\int_t^{\tau} r(s)ds} d\tau = \pi^m \int_t^{\infty} e^{-\int_t^{\tau}(r(s)+p)ds} d\tau,$$

where r is the real rate of interest on safe loans.

c) Explain this result, either just in words or by deriving it formally.

d) Find r in an equilibrium with $\dot{N} > 0$; compare with what r would be in case of no erosion of monopoly power. *Hint:* In equilibrium with $\dot{N} > 0$, $V(t)$ satisfies a simple relation. This can be combined with the no-arbitrage condition $(\pi^m + \dot{V} - pV)/V = r$.

Suppose the households, all alike, have an intertemporal utility function with infinite horizon and a constant rate of time preference $\rho > 0$. The instantaneous utility function has elasticity of marginal utility equal to a constant $\theta > 0$.

e) Find the rate of growth of c in an equilibrium with $\dot{N} > 0$ (an informal argument, based on your general knowledge, is acceptable). In case you need to introduce restrictions on some parameters in order to ensure positive growth and/or bounded utility, do it. Let the growth rate of c be denoted γ_c. Compare with what the growth rate of c would be in case of no erosion of monopoly power. Comment.

Let N^c denote the number of intermediates that have become competitive and let $N^m \equiv N - N^c$.

f) Output by firm i in Sector 1 can now be written

$$Y_i = AL_i \left[(N - N^c)(\frac{x_i^m}{L_i})^\alpha + N^c(\frac{x_i^c}{L_i})^\alpha \right].$$

What is the economic logic behind this result?

g) Further, $x_i^m/L_i = X^m/L$ and $x_i^c/L_i = X^c/L$. Why?

h) Find an expression for aggregate output of basic goods as a function of L, N and N^c. Comment.

i) It can be shown that N^c/N approaches a constant $(N^c/N)^*$ over time, that this constant is $p/(\gamma_N + p)$, where $\gamma_N \equiv \dot{N}/N$, and that $\gamma_N = \gamma_c$ in steady state. Briefly explain the intuition behind these three features.

j) How does the size of p affect steady state growth? Comment.

k) Use the answers to h) and i) to find the solution for aggregate output of basic goods in steady state. Compare with what output would be in case of no erosion of monopoly power. Comment.

ℓ) The model - and some of the above results - illustrate dilemmas in antitrust policy and patent legislation. Explain.

Problem 4. *Short questions.*

1. “The Ramsey model predicts that for countries with similar structural characteristics, the further away from its steady state a country is, the higher is its per capita growth rate.” True or not true? Comment.

2. “The three-sector model by Kongsamut et al. (2001) explains structural change by differences in income elasticity of demand.” True or not true? Explain.

3. “In the Mincerian approach to human capital formation an individual’s human capital is assumed to be proportional to time spent in education.” True or not true? Explain.

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