Written exam for the M. Sc. in Economics 2008-II

Economic Growth (Videregående vækstteori)

Master's Course

June 13, 2008

(4-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is: Problem 1 10%, Problem 2 50%, Problem 3 30%, and Problem 4 10%.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 *Short questions*

a) "Relatively homogeneous groups of countries, such as for example the 12 old EU member countries, tend to experience income convergence in the sense that the standard deviation of income per capita across the countries diminishes over time." True or not true as an empirical statement? Why?

b) In 1960 per capita GDP in South Korea and Philippines were almost the same. Over the period 1960-1990 the average annual growth rate of per capita GDP was in South Korea 6.7 percent and in Philippines 1.5 percent. Give a brief account of what kind of explanation the Alesina-Rodrik model suggests. Briefly, assess the explanation.

c) "In a reduced-form AK model with a representative household and productive public services with congestion, a welfare-maximizing government has to implement a policy that reduces the per capita growth rate." True or false? Why?

Problem 2 Consider the "social planner" in an economy described by the simple increasing variety model in the B & S textbook. The social planner's criterion function is the same as that of the representative household. The dynamic problem faced by the social planner is to choose $(c_t, X_t)_{t=0}^{\infty}$ so as to:

$$\max U_0 = \int_0^{\infty} \frac{{c_t}^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \qquad \text{s.t.}$$

$$c_t > 0,\ X_t \geq 0,$$

$$\dot{N}_t = \frac{1}{\eta}(Y_t - X_t - c_t L),\ \text{where } Y_t = A{X_t}^{\alpha}(N_t L)^{1-\alpha} \text{ and } N_0 \text{ is given,} \qquad (*)$$

$$N_t \geq 0 \text{ for all } t \geq 0. \qquad (**)$$

The notation is: c = per capita consumption, X = aggregate input of specialized intermediate goods, N = number of different intermediate goods types (varieties), N "large", Y = output of the manufacturing sector. The remaining variables are constant parameters and satisfy the inequalities $\theta > 0$, $\rho > 0$, $\eta > 0$, $L > 0$, $A > 0$, and $0 < \alpha < 1$. In (*) indivisibilities are ignored and N is regarded as a continuous and differentiable function of time t.

a) Briefly interpret the parameters.

b) The production function of individual firms in the manufacturing sector is

$$Y_i = A\left(\sum_{j=1}^{N} {x_{ij}}^{\alpha}\right) L_i^{1-\alpha}, \qquad (***)$$

where i refers to an arbitrary firm i in the sector (to save notation, the time index is suppressed). The aggregate production function given in (*) is based on the condition $X = NX_{SP}$, where X_{SP} is the aggregate input of intermediate goods of type j $(j = 1, 2, ..., N)$. Explain the link from (***) to (*).

c) Assuming there is an interior solution, derive a candidate solution to the social planner's dynamic problem.

d) As one of the first-order conditions you should find $\partial Y/\partial X = 1$ or $X/Y = \alpha$. What is the economic intuition behind this condition?

e) Write down required parameter restrictions for existence of an optimal solution with growth.

f) Check whether your candidate solution satisfies the Mangasarian sufficient conditions for an optimal solution. For a problem like this, with two control variables and one state variable, the Mangasarian sufficient conditions are:

   1. The Hamiltonian is jointly concave in the control and state variables.
   2. There is for all $t \geq 0$ a non-negativity constraint on the state variable.
   3. The candidate solution satisfies the transversality condition $\lim_{t\to\infty} N_t\lambda_t e^{-\rho t} = 0$, where λ_t is the adjoint variable in the current-value Hamiltonian.

g) Comment on the social planner's allocation in relation to the allocation in the laissez-faire market economy where markets are competitive with the exception of the markets for specialized intermediate goods. At these markets the suppliers have a monopoly on their specific intermediate good due to infinitely-lived patents (that are free of charge).

h) Now consider a government that attempts to obtain the social planner's allocation in a decentralized way. The government pays a subsidy at constant rate, σ, to purchases of intermediate goods such that the net price of intermediate good j is $(1-\sigma)P_j$, where $P_j = 1/\alpha$ is the price set by the monopolist supplier of good j. The government finances this subsidy by taxing consumption at the constant rate τ. Derive the required value of the subsidy rate σ.

i) Derive the required value of the consumption tax rate τ. Is the suggested policy sufficient to implement the social planner's allocation? Comment.

j) Briefly, assess the model from a theoretical and empirical point of view.

Problem 3 Here we shall consider a series of three model frameworks for a closed economy with physical and human capital, Model 1, Model 2, and Model 3. The aggregate production function for the manufacturing sector is the same in all three models:

$$Y = AK^{\alpha}(\pi L)^{1-\alpha}, \qquad A > 0, \quad 0 < \alpha < 1, \tag{1}$$

where Y is output, K is capital input, L is labor input, and π is a measure of labor productivity. Time is continuous, but the dating of the variables is implicit. Manufacturing output is used for consumption, C, and investment, I :

$$Y = C + I, \tag{2}$$

In Model 1, total investment is composed of investment, I_K, in physical capital and investment, I_H, in human capital, respectively:

$$I = I_K + I_H. \tag{3}$$

Further, in Model 1 the stocks of the two kinds of capital, K and H, respectively, change according to

$$\dot{K} = I_K - \delta K, \tag{4}$$
$$\dot{H} = I_H - \delta H, \tag{5}$$

where δ is the depreciation rate or decay rate (for simplicity assumed to be the same for the two kinds of capital). Model 1 assumes $\dot{L}/L = n \geq 0$, a constant, and

$$\pi = h^\beta, \qquad 0 < \beta \leq 1, \tag{6}$$

where $h \equiv H/L$.

a) Is Model 1 technologically capable of generating endogenous growth (either fully endogenous or semi-endogenous)? It is enough to base the argument on general knowledge.

Model 2 shares equations (1), (2), (3), and (4) with Model 1, but replaces (5) and (6) by the Mincer hypothesis

$$\pi = e^{\psi s}, \quad \psi > 0, \tag{7}$$

where s is a measure of time spent in school by the "average citizen". In Model 2, $I_H \equiv 0$. With N denoting the population of age ≤ 65, Model 2 assumes

$$N \equiv N_t = N_0 e^{nt},$$

and that, by proper choice of measurement units, the relationship

$$L = (1 - s)N \tag{8}$$

holds approximately (n still ≥ 0 and constant).

b) Is Model 2 technologically capable of generating endogenous growth (either fully endogenous or semi-endogenous)? Give your argument.

Model 3 differs from Model 2 by regarding total factor productivity, A, in (1) as endogenous and by having a separate innovative sector with a Jones-style "invention production function":

$$\dot{A} = \mu A^\varphi L_A, \qquad \mu > 0, \varphi < 1,$$

where L_A is input of research labor. Consequently, L in (1) is replaced by L_Y, and (8) is replaced by

$$L_Y + L_A = L = (1 - s)N.$$

c) Assuming balanced growth in Model 3, derive the growth rate of $y \equiv Y/L$.

d) Is Model 3 technologically capable of generating endogenous growth (either fully endogenous or semi-endogenous)? Give your argument.

e) Without going into details about households' preferences, market structure and government sector in Model 3, suggest some policy instruments that might be used to provide a higher-situated y-path under balanced growth.

f) Briefly, assess the three model frameworks from a theoretical and empirical point of view.

Problem 4 *Short questions*

a) In relation to their model with productive public services in the form of pure public goods Barro and Sala-i-Martin write in the textbook: "The failure to [empirically] detect ....scale effects [on growth] likely means that most of the government's services do not have the nonrival character that is assumed in the model." Suggest an alternative explanation of the failure of the model to comply with the apparent absence of scale effects on growth, an explanation *maintaining* the nonrival character of the productive public service assumed in the model.

b) The huge pressure for migration from poor to rich countries has been explained in somewhat different ways. There is a Lucas-style explanation and a Romer-Jones-style explanation. Briefly compare.

c) In some innovation-based increasing variety models, an R&D subsidy is needed to implement the social planner's allocation, in others not. Give examples.

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