Economic Growth.

13.03.2008. Christian Groth

Problem set for the midterm paper

General information: you are encouraged to do the problem set together with fellow students (max four students per group). Readable handwriting is OK, but you may also use a computer. You may answer in English or Scandinavian as you prefer. During working on the problem set you are most welcome to consult me by e-mail, chr.groth@econ.ku.dk. I will try to answer within 24 hours or first-coming workday. You may also consult me at the office and in that case it is best to make an appointment by e-mail first.

In order to go in for the final written exam (four hours) at the end of the semester it is required that the term paper is accepted (which corresponds to a grade ≥ 2).

Time table

Thursday 13/03 2008 at 1 p.m. the midterm paper problem set is announced at the course website.

There is no lecture Monday 17/3, but I lecture Wednesday 2/4.

Wednesday 2/4 at 1 p.m. is last chance for consultation.

Deadline: At the latest Thursday 3/4 1 p.m. you deliver your paper (with readable date, name of the course, name of all the authors as well as the first six digits of their cpr.numbers) at the Study Office, Studiestraede 6. Opening hours 10-13.

Good luck!

The weights of the three problems are:

Problem 1: 15 %, Problem 2: 60 %, Problem 3: 25 %.

Problem 1 Find from the Penn World Table (use PWT 6.2) at http://pwt.econ.upenn.edu/ the following data and make a corresponding table:

Real GDP per capita (rgdpch) and Real GDP Chain per worker (rgdpwok) in Denmark in 1950, 1990 and 2003.

Real GDP per capita and Real GDP Chain per worker in India in 1950, 1990 and 2003.

Using the continuous time method (with continuous compounding) you should now answer the following questions:

a) Find the average annual growth rate of rgdpch between 1950 and 1990 and 1990 and 2003 in Denmark and India, respectively (four growth rates).

b) As a thought experiment, suppose India's average annual growth performance before 1950 (that is, in the total period back to "take-off") has been the same as between 1950 and 1990. And suppose the subsistence level is an annual per capita income of I$ 250 in 2000 constant prices. In what year[1] must the "take-off" (the first year at which sustained growth in per capita income occurs) have taken place?

c) Assuming India's growth performance continues to be like that between 1990 and 2003, how many years does it take for India's rgdpwok to be doubled? You should display your method.

d) Assuming both Denmark's and India's growth performance with respect to rgdpwok continue to be as between 1990 and 2003, how long time, reckoned from 1990, will it then take for India to catch up with Denmark?

e) Do you find it likely that the actual course of events will be (approximately) like that? Briefly say why or why not?

Problem 2. Consider a closed market economy with N profit maximizing firms, operating under perfect competition (N "large"). There is a representative household (family dynasty) with L members at time t. Assume $L = L_0 e^{nt}$, where n is constant, $n \geq 0$. Each household member supplies one unit of labour per time unit. Aggregate output is Y per time unit, and output is used for consumption, $C \equiv cL$, and investment

[1] Of course, this should not be taken literally. In this context one should rather talk of decades or more.

in physical capital K, i.e., $Y = C + \dot{K} + \delta K$, where $\delta \geq 0$ is the rate of physical decay of capital. Variables are dated implicitly. The initial value $K_0 > 0$ is given. There is a perfect market for loans at the real rate of interest r. There is perfect foresight.

The production function for firm i $(i = 1, 2, ..., N)$ is

$$Y_i = F(K_i, TL_i), \tag{1}$$

where F is neoclassical and has CRS. The variable T evolves according to

$$T = T_t = e^{xt} K_t^{\lambda}, \quad x \geq 0, 0 < \lambda \leq 1, \tag{2}$$

where x and λ are constants and $K_t = \sum_i K_{it}$.

a) Briefly interpret (1) and (2).

b) In general equilibrium, determine r and the aggregate production function at time t.

c) Assume $x > 0$ and $\lambda < 1$. Determine the rate of growth of Y and $y \equiv Y/L$ under balanced growth. *Hint:* use the proposition about equivalence of balanced growth and constancy of certain key ratios.

d) Comment on the model in relation to different types of endogenous growth.

From now, let $\lambda = 1$ and $x = n = 0$.

e) Assume that the representative household has infinite horizon, an instantaneous utility function with elasticity of marginal utility equal to a constant $\theta > 0$ and a constant rate of time preference w.r.t. utility, $\rho > 0$. Let $F_1(1, L) > \delta + \rho$. Determine the equilibrium rate of growth of c, k $(\equiv K/L)$ and y, respectively. In case, you need to introduce a restriction on some parameters, do it.

f) Comment on your result under e). What would happen if $n > 0$?

Now, introduce a government that wants to subsidize production at the constant rate $s > 0$ so that if firm i produces (and sells) Y_i, its revenue is $(1 + s)Y_i$.

g) Could there be good economic reasons for such a subsidy? Comment.

h) Provide an analysis of whether there is a level of the subsidy rate such that the social planner's allocation can in principle be implemented.

i) Suppose, the government always balances the budget, that it finances the subsidy by a consumption tax and that it has no other expenditures. Provide an analysis of whether there is a constant level of the consumption tax such that the social planner's allocation can be implemented. *Hint:* in the social planner's solution $c_t = (F(1, L) - \delta - \gamma_{SP})k_t$ for all $t \geq 0$.

j) Briefly, evaluate the two model versions considered.

Problem 3. *Short questions.*

a) "If there are constant returns with respect to physical capital and labour taken together, then, considering technical knowledge as a third production factor, there tends to be increasing returns w.r.t. to all three production factors taken together." True or false? Explain why.

b) "In models where technical knowledge is endogenous there is a built-in tendency for either weak or strong scale effects to arise (i.e., scale effects either on levels or growth)." True or false? Explain why.

c) "In models with productive pure public goods there is a built-in tendency for either weak or strong scale effects to arise (i.e., scale effects either on levels or growth)." True or false? Explain why.

d) Germany and Japan had a very high per-capita growth rate after the second world war (and up to the mid 1970s). "As predicted by neoclassical growth theory (Solow or Ramsey style), sooner or later the very high growth came to an end." Do you think this interpretation of the actual events has something to it or not? Explain why.

e) We consider a selection of "emerging economies". Half of the economies have $\tilde{k}_0 \equiv K_0/(T_0L_0) < \tilde{k}^*$ and the other half have $\tilde{k}_0 \equiv K_0/(T_0L_0) > \tilde{k}^*$. Suppose we observe β convergence for this selection of economies. Is there a sense in which a technological catching-up hypothesis seems a better approach to an explanation for this than the standard neoclassical closed economy model?

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