

Problem set for Homework 2

You are encouraged to solve the problem set together with fellow students (max. four students per group).¹ You may answer in English or Scandinavian as you prefer, either using a computer or readable handwriting. The maximum number of pages is 8.

The **deadline** for Homework 2 is March 18, at 12:00. The deadline is a must in the sense that your paper will not be considered if it arrives after deadline. You should hand in a printed (or hand written) version of your paper. Thus an attachment in an e-mail is not acceptable.

On the **front page** of your paper you write the following:

- 1) Full name of all the authors.
- 2) The first six digits of their cpr.numbers.
- 3) Date.
- 4) Name of the course and the teacher.

Place where you hand in your paper:

Økonomisk Institut, Studiekontoret (Mia Kildetoft), CSS Building 25, ground floor, office 25.0.09. Open: Thursday 10:00-12:00.

Workshop and consultation

The two Thursdays, March 11th and 18th, exercise classes are replaced by workshops with the teacher in the role as a consultant. The new class room, CSS 2.0.42, is at free disposal for those who want to sit there working on the problem set. Normally, I will not be present in the room but at my office (Building 26, sec. floor). In the time interval 8:15-10:00 these Thursdays, I will be available as a consultant, answering questions a.s.a.p., whether received by personal contact or e-mail.

¹If you do not know the other students, but would like to have one or two co-workers, I suggest you send me an e-mail about it. Then I might be able to match you with somebody else in the same situation.

I evaluate your paper (“accepted” or “not accepted”) and return it, with comments, the next Thursday where we all meet in class and discuss the solutions.

Good luck!

Christian Groth

The weights of the problems are:

Problem 1: 40%, Problem 2: 40%, Problem 2: 20%.

Problem 1 Consider a closed market economy with N profit maximizing firms, operating under perfect competition (N “large”). There is a representative household (family dynasty) with L_t members at time t . Assume $L_t = L_0 e^{nt}$, where n is constant, $n \geq 0$. Each household member supplies one unit of labor per time unit. Aggregate output at time t is Y_t per time unit. Output is used for consumption, $C_t \equiv c_t L_t$, and investment in physical capital K_t so that $\dot{K}_t = Y_t - C_t - \delta K_t$, where $K_0 > 0$ is given and $\delta \geq 0$ is the rate of physical decay of capital. The initial value $K_0 > 0$ is given. There is a perfect market for loans with short-term real interest rate r_t . There is no uncertainty (perfect foresight).

The production function for firm i ($i = 1, 2, \dots, N$) is

$$Y_{it} = F(K_{it}, T_t L_{it}), \quad (1)$$

where F is neoclassical and has CRS. The variable T_t evolves according to

$$T_t = e^{xt} K_t^\lambda, \quad x \geq 0, 0 < \lambda \leq 1, \quad (2)$$

where x and λ are constants and $K_t = \sum_i K_{it}$.

a) Briefly interpret (2).

Suppose each firm is small relative to the economy as a whole and perceives it has no influence on aggregate variables.

b) In general equilibrium, determine r and the aggregate production function at time t .

c) Assume $x > 0$ and $\lambda < 1$. Show that without knowing anything about household saving behavior we can determine the growth rate of Y and $y \equiv Y/L$ under balanced growth, assuming saving is positive. *Hint:* you may apply Proposition 1 from Lecture Note 6 about equivalence of balanced growth and constancy of certain key ratios.

d) What type of endogenous growth is the model as given in c) capable of generating?

From now, let $\lambda = 1$ and $x = n = 0$. Moreover, assume there is a Ramsey-style household sector with infinite horizon, a constant utility discount rate $\rho > 0$, and an instantaneous CRRA utility function with parameter $\theta > 0$. Finally, assume that $F_1(1, L) > \delta + \rho$ holds.

e) Determine the equilibrium growth rate for c and name it γ .

f) Determine the equilibrium growth rate for k ($\equiv K/L$) and y , respectively. *Hint:* show that the real interest rate must be a constant and that the aggregate production function is AK-style; then it is enough to refer to the general knowledge (from Lecture Note 8) that in such a case the transversality condition can only hold if k grows at the same rate as c .

g) Given the Ramsey-style setup, a certain additional parameter restriction is needed to ensure boundedness of the households' utility integral and thereby existence of general equilibrium. Write this restriction down. Why is the restriction needed?

h) What would happen if $n > 0$? Why?

Problem 2 In continuation of the last part of Problem 1 we still consider the case $\lambda = 1$ and $x = n = 0$ and the same Ramsey-style household sector. We imagine there is a social planner in the sense of an "all-knowing and all-powerful" central authority who can fully decide on the resource allocation within the constraints arising from technology and initial resources. Assume that the social planner has the same criterion function as the representative household.

a) Set up the social planner's problem and derive the first-order conditions.

b) Determine the implied optimal time path of c_t and name it γ_{SP} .

c) Determine the optimal time path of k_t and y_t , respectively. *Hint:* you may take as given that the standard transversality condition, $\lim_{t \rightarrow \infty} k_t \eta_t e^{-\rho t} = 0$, where η_t is the co-state variable, is necessary for optimality; as an implication, $c_0 = (F(1, L) - \delta - \gamma_{SP})k_0$ for all $t \geq 0$; why? (a brief verbal account is enough).

- d) Sign the difference between γ_{SP} and γ from Problem 1. What is the economic intuition behind the difference?

We now introduce a benevolent government that is capable of calculating the social planner's solution and is contemplating to implement it by a production subsidy. The idea is to subsidize production at a constant rate $s > 0$ so that if firm i produces and sells Y_i , its revenue is $(1 + s)Y_i$.

- e) Examine whether the government can in this way implement the social planner's solution in the market economy. *Hint:* one approach is to check whether there exists a value of the subsidy rate such that the net rate of return to capital investment implied by the aggregate production technology can be made equal to the net rate of return on saving faced by the consumer.
- f) Suppose that whatever the answer to e), the government launches the subsidy at a certain rate \bar{s} and that it finances the subsidy by a consumption tax. Moreover, assume that the government has no other expenditures and always balances the budget. Find the consumption tax rate needed in this case. *Hint:* at a certain stage in the argument you will need knowledge about what value is taken by c_t/k_t ; here you may use the value implied by the social planner's solution, cf. c).

Problem 3 *Short questions.*

- a) Germany and Japan had a very high per-capita growth rate after the second world war (and up to the mid 1970s). "As predicted by neoclassical growth theory (Solow or Ramsey style), sooner or later the very fast growth came to an end." Do you think this statement makes sense? Briefly explain.
- b) "In Romer's learning-by-investing model the government can just as well finance the optimal subsidy by a tax on labor income as by a consumption tax." True or false? Why?
- c) "In models where technical knowledge is endogenous there is a built-in tendency for either weak or strong scale effects (i.e., scale effects on either levels or growth) to arise." True or false? Why?

d) In the cross-country growth regression in B & S, Chapter 12, the estimated coefficient to country size (as measured by the log of population) is very close to zero and definitely insignificant (see p. 536-537). Some people might interpret this as an indication that the hypothesis about scale effects on growth is falsified. There is, however, at least one serious problem associated with using cross-country regression analysis as a test of this hypothesis. What can this problem be?

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