Economic Growth. Exercises

03.05.2010. Christian Groth

Problem Set V

V.1 N. G. Mankiw, D. Romer, and D. N. Weil (1992) considered the aggregate production function

$$Y_t = AK_t^{\alpha}H_t^{\beta}(T_tL_t)^{1-\alpha-\beta}, \qquad A > 0,\ 0 < \alpha < 1,\ 0 < \beta < 1, \tag{*}$$

where K is aggregate capital input, H aggregate human capital input, T the technology level (broadly defined), and L input of man hours. Assuming $T_t = T_0e^{gt}$, $g > 0$, is the same for all countries in the sample (apart from a noise term affecting T_0), their cross-country regression analysis (98 countries, 1960-1985) found that $\alpha = \beta = 1/3$ fitted the data well.

Let h denote average human capital, i.e., $h \equiv H/L$, and suppose all workers at any time t have the same amount of human capital, equal to h_t.

a) Show that (*) can be rewritten on the form $Y_t = F(K_t, X_tL_t)$.

b) At least when we study individual firms' decisions, this alternative way of writing the production function is more natural and convenient than the form (*). Why?

c) Whether one uses a Solow-style one-sector approach (with given, constant investment rates in physical and human capital) or a Ramsey-style one-sector approach (cf. Exercise IV.1), it can be shown that the economy tends to converge to a steady state with $\tilde{y} = A(\tilde{k}^*)^{\alpha}(\tilde{h}^*)^{\beta}$, where $\tilde{k}^*$ and $\tilde{h}^*$ are the constant steady state values of $\tilde{k} \equiv K/(TL)$ and $\tilde{h} \equiv h/T$. What is the steady state growth rate of $y \equiv Y/L$? Comment in relation to the question: does human capital accumulation drive growth?

d) Section 5.1 in B&S also takes a Ramsey-style one-sector approach to human and physical capital accumulation. Their production function is

$$Y_t = AK_t^{\alpha}(h_tL_t)^{1-\alpha}.$$

Letting $g = 0$ in your result under a), is there still a difference? If so, what is it and what implication does it have in relation to the question about what forces are capable of driving economic growth?

V.2 *Artificial parameter links in B & S's simple increasing variety model* First we consider the simple increasing variety model with permanent monopolies in the B & S text. Firm i ($i = 1, 2, ..., M$) in the manufacturing sector has the production function

$$Y_i = A \left(\sum_{j=1}^{N} x_{ij}{}^{\alpha} \right) L_i^{1-\alpha}, \qquad A > 0,\ 0 < \alpha < 1. \tag{1}$$

Here Y_i, L_i, and x_{ij} denote output of the firm, labor input and input of intermediate good j, respectively ($j = 1, 2, ..., N$; N "large").

a) The symmetry in (1) and the fact that the prices of intermediate goods are all set by monopoly firms at the same level $p = 1/\alpha$, induce firm i to choose $x_{ij} = x_i$ for all j. Explain by a few well-chosen sentences why this is so. Next derive the implied result:

$$Y_i = ANx_i{}^{\alpha} L_i^{1-\alpha}. \tag{2}$$

b) A general feature of increasing variety models is the hypothesis that "variety is productive" or, with a broader formulation, "there are gains by specialization". Is this hypothesis consistent with the equation (2)? Yes or no? Explain.

Let the aggregate input of intermediate goods in the manufacturing sector and the aggregate output in the sector be denoted X and Y, respectively. Thus, $X = NX^m$, where $X^m \equiv (\alpha^2 A)^{1/(1-\alpha)} L$ (the aggregate input of each of the intermediate goods), and $Y = \sum_i Y_i$.

c) Write down an expression for the value added in the manufacturing sector. Comment.

The model leads to the following expression for the aggregate manufacturing production function in equilibrium at time t :

$$Y_t = A(X^m)^{\alpha} L^{1-\alpha} N_t = AX_t^{\alpha}(N_t L)^{1-\alpha}, \qquad A > 0, 0 < \alpha < 1, \tag{3}$$

where L is the constant labor force, $L > 0$. This aggregate manufacturing output is used partly for consumption, C, partly for investment in R&D, R, and partly for replacing the intermediate goods used up in the production of Y,

$$Y_t = C_t + R_t + X_t, \tag{4}$$

and the invention production function is

$$\dot{N} \equiv \frac{dN}{dt} = \frac{R}{\eta}, \qquad \eta > 0,\ \eta \text{ constant.}$$

d) Given this specification of the "growth engine", already the formula (3) gives a hint that the model is (technologically) capable of generating fully endogenous growth. Briefly explain.

In more general increasing variety models[1] (1) is replaced by

$$Y_i = AN^{\beta}(CES_i)^{\alpha}L_i^{1-\alpha}, \qquad A > 0,\ \beta > 0, 0 < \alpha < 1, \tag{5}$$

where the parameter β reflects "gains to specialization" and CES_i is a CES aggregate[2] of the quantities $x_{i1}, ..., x_{iN}$:

$$CES_i \equiv N\left(N^{-1}\sum_{j=1}^{N} {x_{ij}}^{\varepsilon}\right)^{\frac{1}{\varepsilon}}, \qquad 0 < \varepsilon < 1, \tag{6}$$

(This is the standard "CRS definition" of a CES aggregate in that the right-hand side of (6) has CRS with respect to the inputs $x_{i1}, ..., x_{iN}$; in B & S, p. 286, footnote 2, also appears a CES aggregate, but without this convenient CRS property which opens up for "gains to specialization" to appear explicitly *outside* the CES index as in (5).) Again, in equilibrium, because of symmetry and the fact that the prices of intermediate goods will all be set at the same level $P = 1/\varepsilon$, firm i chooses $x_{ij} = x_i$, for all j.

e) "The B & S specification (1)-(2) is a special case of (5)-(6), namely the case $\varepsilon = \alpha$ and $\beta = 1 - \alpha$." True or false? Comment.

f) Why may the parameter link $\varepsilon = \alpha$ be considered problematic?

By the method described in Lecture Note 15 it can be shown that the aggregate production function in manufacturing in the general case (presupposing static efficiency) is

$$Y_t = AN_t^{\beta}X_t^{\alpha}L^{1-\alpha}.$$

g) Suppose $\beta < 1 - \alpha$. Comment on the likely technological capability of this model to generate fully endogenous growth.

[1] For example Jones (AER, 2002) and Alvarez-Pelaez and Groth (2005).
[2] CES = Constant Elasticity of Substitution.

h) Do you consider $\beta > 1 - \alpha$ to be plausible? Why or why not?

V.3 Consider the Jones (1995) R&D-based growth model for a closed economy. For simplicity we ignore the duplication externality. We ignore the Romer-style microeconomic story of the production side in the model and go directly to the aggregate level. With standard notation the aggregate model is:

$$
\begin{aligned}
Y_t &= K_t^{\alpha}(A_t L_{Yt})^{1-\alpha}, \qquad 0 < \alpha < 1, \\
\dot{K}_t &= Y_t - c_t L_t - \delta K_t, \qquad \delta \geq 0, && (*) \\
\dot{A}_t &= \mu A_t^{\varphi} L_{At}, \qquad \mu > 0, \varphi < 1, && (**) \\
L_{Yt} + L_{At} &= L_t, \\
L_t &= L_0 e^{nt}, \ n > 0, \text{ constant.}
\end{aligned}
$$

The household sector is given by a Ramsey household with infinite horizon, pure rate of time preference ρ, and a CRRA instantaneous utility function with parameter $\theta > 0$. To ensure boundedness of the utility integral we assume $\rho - n > (1-\theta)n/(1-\varphi)$.

a) Find the growth rate of "knowledge", A, under the assumption that it is positive and constant. *Hint: S*tart from an expression for g_A derived from (**) and use the growth accounting principle on this expression.

b) Find the growth rate of manufacturing output per capita, $y \equiv Y/L$, under balanced growth. *Hint:* Since the model is not a fully endogenous growth model, the approach to the study of balanced growth is different and more simple than that needed for AK-style models. A good starting point is the growth accounting relation $g_Y = \alpha g_K + (1-\alpha)(g_A + g_{L_Y})$, where one can use the fact that under balanced growth with the standard capital accumulation equation (*) for a closed economy we have $g_Y = g_K$.

c) Find the growth rate of c under balanced growth.

Given the microeconomic increasing-variety set-up with monopolists as in Lecture Note 15 (apart from specialized intermediate goods being replaced by specialized capital goods), it can be shown that the equilibrium real interest rate at time t equals $\alpha^2 Y_t/K_t$. This information is useful for the next questions.

d) Suppose $s_A \equiv L_A/L$ in balanced growth can be increased by an R&D subsidy.

1. Will this affect the long-run per capita growth rate? Comment. *Hint:* It can be shown that the model is saddle-point stable.
2. Will it affect levels under balanced growth? Comment. *Hint:* Find an expression for y in terms of $\tilde{k} \equiv K/(AL_Y)$, s_A and A under balanced growth. Then find an expression for A in terms of L_A under balanced growth. Check that $\tilde{k}$ is independent of s_A; use here that the output-capital ratio in balanced growth can be found from the Keynes-Ramsey rule of the representative household.

e) Is there a scale effect on levels in the model? Comment. *Hint:* From Jones (1995, p. 769) we have that s_A under balanced growth is independent of L. Show by use of the Keynes-Ramsey rule that also $\hat{k}$ under balanced growth is independent of L. Then the stated question can be answered.