Economic Growth Exercise solutions

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Suggested solution to Problem VI.4, g) - k)

For convenience we repeat the equations of the model:

$$\dot{K}_t = T_t I_t - \delta K_t, \qquad \delta > 0, \tag{1}$$

$$T_t = \tilde{\xi} \left(\int_{-\infty}^{t} I_\tau d\tau \right)^{\lambda}, \qquad \tilde{\xi} > 0,\ 0 < \lambda < \frac{1-\alpha}{\alpha}. \tag{2}$$

$$Y_t = A_t {K_t}^{\alpha} {L_t}^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{3}$$

where L_t is labor input, growing at a given constant rate $n \geq 0$, and A_t is TFP, growing at a given constant rate $\gamma \geq 0$. Finally,

$$Y_t = I_t + C_t. \tag{4}$$

g) Since g_T is constant, our result in d) implies

$$g_T = \lambda g_I. \tag{5}$$

And since in e) we have shown that TI/K is constant, we have

$$g_T + g_I = g_K.$$

Substituting (5) gives

$$g_K = (\lambda + 1) g_I. \tag{6}$$

h) By constancy of s, taking growth rates in (3) gives

$$g_I = g_Y = \gamma + \alpha g_K + (1-\alpha)n,$$

which combined with (6) yields

$$g_Y = g_I = \frac{\gamma + (1-\alpha)n}{1 - \alpha(1+\lambda)} > 0. \tag{7}$$

The inequality is due to $\alpha(1+\lambda) < 1$ which is implied by with the parameter restriction $\lambda < (1-\alpha)/\alpha$ in (2).

i) Using (7), we get

$$\begin{aligned} g_y &= g_Y - n = \frac{\gamma + \alpha\lambda n}{1-\alpha(1+\lambda)} \\ &= \frac{\gamma}{1-\alpha(1+\lambda)} + \frac{\alpha\lambda n}{1-\alpha(1+\lambda)}. \end{aligned} \tag{8}$$

The first term in the decomposition in the second line represents the contribution to g_y from disembodied (and exogenous) technical progress. The reason that the learning parameter, λ, appears in the denominator is that exogenous technical progress enlarges investment in the economy and thereby more learning is generated. The second term in (8) represents the contribution to g_y from embodied (and learning-based) technical progress.

j) From (3), where the capital stock is measured in constant efficiency units, we get

$$g_Y = \gamma + \alpha g_K + (1-\alpha)n. \tag{9}$$

Hence, the TFP growth rate is

$$g_{TFP} = g_Y - \alpha g_K - (1-\alpha)n = \gamma.$$

k) Thus, if $\gamma = 0$, growth accounting gives $g_{TFP} = 0$, as if there is no contribution from technical progress. In contrast, the theoretical model implies that, with $\gamma = 0$,

$$g_y = \frac{\alpha\lambda n}{1-\alpha(1+\lambda)},$$

"saying" that all of g_y derives from *embodied* technical progress generated via learning by investing. The conclusion is that when the capital stock is measured in constant efficiency units, growth accounting (at least in the present simple form) does not capture embodied technical progress.

The point can also be articulated in the following way. Simple growth accounting decomposes growth of labor productivity into the TFP growth rate and a "contribution" from growth in the capital-labor ratio:

$$g_y = g_Y - n = \gamma + \alpha(g_K - n) = \gamma + \alpha g_k = g_{TFP} + \alpha g_k.$$

A theoretical model decomposes labor productivity growth into "ultimate sources", as in (8).

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