Economic Growth. Exercise solutions

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Solutions to selected exercises in Problem Set I and II

I.2

a) 7.702 years. If discrete time is used, 8.04 years.

b) 0.13.

c) 50.38 years. If discrete time is used, 52.31 years.

d) A possible answer: No, I do not. So high growth rates have never in history been maintained in more than, say 40 years. It is easier to grow fast, when you start at a low per capita income level. The currently high growth in China is due to:

1. A technology gap and imitation of the better technology of the most advanced countries. When the gap is reduced, this source of high growth disappears. Catching-up is limited.
2. Institutional shift to the use of market mechanisms, whereby a structural transformation takes places and resources are moved from low-productivity activities (say agriculture) to high-productivity activities (say manufacturing, IT industry, services).
3. In the last decades China has had a very high investment-income ratio, but due to diminishing returns to capital the resulting very high growth rate is not likely to be maintainable.

I.3

a) We say that a given collection of countries show σ *convergence* with respect to a given measure of dispersion if this measure of dispersion, applied to income per capita or output per worker across the countries, declines systematically over time. On the other hand, the countries show σ *divergence* if the dispersion increases systematically over time.

A relevant measure of dispersion is for example the standard deviation of *relative* income per capita (or of $\log y$ where $y \equiv Y/L$). The standard deviation of income per capita as such is not a very relevant measure. This is because income per capita is trending upwards and the standard deviation is not a scale-free measure.

The reported data shows a tendency for world income per capita relative to income per capita in the US 1952-96 to increase over time, i.e., the ratio

$$\frac{GDP_{world}/L_{world}}{GDP_{US}/L_{US}}$$

tends to increase. Although this tendency is not an unambiguous indication of σ convergence (because of the coarse partition into only the U.S. and the world), it is at least consistent with σ convergence. More detailed data across the countries in the world (Sala-i-Martin 2006) do in fact show a (weak) tendency to σ convergence, at least in the last 20 years, when countries' per-capita income is *weighted by population size* as a fraction of world population. Without such weighting, data for countries in the world show slight σ divergence (this is because the developments in small slow-growing countries in, e.g., Africa and Latin America then count equally much as the development in fast-growing large countries in Asia like India and China).

b) We say that a large heterogeneous group of countries (say the countries in the world) show *unconditional* (or absolute) income convergence if income convergence occurs for the whole group without conditioning on specific structural characteristics of the countries. If income convergence occurs only among a subgroup of the countries, namely such countries that in advance share the same "structural characteristics", then we say there is *conditional* income convergence.

c) Let country i have a per capita production function $y_i = f(k_i, T_i)$, where k_i is the capital intensity and T_i is the technology level of country i. As explanations of the observed σ convergence (for countries weighted by population size) one could imagine the following:

   (a) *Solow-type transition dynamics.* If countries have similar structural characteristics (i.e., they have access to the same technology and they share the same parameters s, δ and n), but different initial conditions, then they would be converging towards the same steady state ($y_i \to \tilde{y}^* T$ for $t \to \infty$, where $\tilde{y} \equiv Y/(TL)$) and therefore show σ convergence. There are two kinds of problems with such an explanation: a) The countries of the world are generally

not closed economies, but parts of an international economic system. b) The countries of the world are far from having similar structural characteristics. (An analogue argument goes through if we think of Ramsey-type transition dynamics and replace the Solow parameter s by the two Ramsey parameters ρ and θ).

(b) *Factor movements across countries and regions.* Factors tend to move to regions where they get the highest remuneration.

(c) *Technological catching up.* In general countries do not have access to the same technology. It takes time for technology to diffuse across countries; adoption of new technologies is costly and requires infrastructure, human capital etc. After the second world war and especially since the 1980s, the economies of the world have generally become more and more open economies (less restrictions on trade and capital movements). This promotes technological catching-up. Still, as shown in Problem 3 of Homework 1, even countries that participate in a fully integrated world market for goods and financial capital need not display complete σ convergence.

II.1

a) They should all be interpreted as flows: dim(output) = goods per time unit, say per year; dim(capital input) = machine-hours per year; dim(labour input) = man-hours per year.

b) In (2) K is simply "the number of machines", a stock, and therefore $\dim(K) =$ machine units.

c) Yes, the same symbol, K, is used for a flow in (1) and for a stock in (2).

d) We introduce rates of utilization, μ and η, so that $Y = F(\mu K, \eta L, T)$, where $\dim(\mu)$ = machine-hours per machine per year, and $\dim(\eta)$ = man-hours per worker per year. Then the interpretation of (1) is that μ and η are assumed constant and both normalized to 1.

e) $Y = F(\mu_i K, L, T)$, where $i = A, B$, and $\mu_A = 1$, $\mu_B = 2$.

f) $Y_A < Y_B < 2Y_A$. Doubling capital input without changing labour input gives higher output, but not a doubled output.

II.4

a) True.

b) The statement is misleading since "Ramsey model" without further predicate must refer to the standard Ramsey model for a closed economy. The stated prediction requires that the countries are also similar with respect to the other key parameters of the model since these influence the steady state of a closed economy. In the Ramsey model these other key parameters are ρ, θ, and n.

If we consider a Ramsey-style model for a small open economy acting in a fully integrated world market for goods and financial capital, the statement is essentially true, however. We see this immediately when we consider how the competitive firm adjusts its capital-labor ratio to a constant real interest rate, given from the world market.

c) False. A counter example is the following. Suppose country A has $\tilde{k} = \tilde{k}^*$ and therefore $g_y = x$, whereas country B has $\tilde{k} > \tilde{k}^*$ and therefore $g_y = \frac{f'(\tilde{k})\dot{\tilde{k}}}{f(\tilde{k})} + x < x$, where the inequality follows from the fact that $\dot{\tilde{k}} < 0$ when $\tilde{k}_t > \tilde{k}^*$. Nevertheless, country B is definitely "further away from its steady state".

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