Economic Growth. Lecture Note 4.

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Partial catching up and persistent technology differences

1 Introduction

This note adds further perspectives to the article by Bernard and Jones (EJ, 1996). The authors raise the question:

> should we expect TFP and labor productivity convergence within the group of more developed countries, like for example the OECD countries?

By a simple Solow-style model Bernard and Jones show that even if the countries have more or less the same saving rate and population growth rate, we should not expect the answer to be necessarily a yes. In general, at a given point in time, countries may not have access to the same technology in a given industry. It takes time for technology to diffuse between countries. In addition to adjustment lags other factors may make technology differences persistent, including the fact that the product and industry composition varies across countries and such variation is likely to persist.

Bernard and Jones' data for 14 OECD countries 1970-1987 indicate that although at the aggregate level labor productivity and Total Technology Productivity feature some convergence, in the manufacturing sector this is not so. This is remarkable in view of the fact that most R&D and international trade occurs in this sector.

The purpose of this lecture note is theoretical. I want to underline:

- alternative hypotheses concerning the exact form of the technological catching-up process are possible;
- the possibility of overtaking in the countries' productivity race should not be ruled out;

- the degree of integration of the world market for financial capital has increased considerably since the late 1980s and so the Solow-style setup used by Bernard and Jones may not be the msot natural one for a discussion of what kind of convergence or absence of convergence we should expect in the future.

2 Catching-up hypotheses

After the second world war, the economies of the world have generally become more and more open economies (less restrictions on trade and capital movements). This promotes technological catching-up which can be modelled in alternative ways. In this and the next section we consider two simple ways.

Let T_i be the technology level of country i, $i = 1, 2,..., N$. Let $T_w(t)$ be the frontier technology level in the world at time t. In the Bernard and Jones article the frontier technology level seems identified with the technology level of the U.S.; indeed, the U.S. is often considered as the "technological leader" after the first world war. Yet, an alternative interpretation is that the frontier technology level refers to something global and is a frontier pushed forward as a result of R&D and learning by doing in the industrialized world as a whole. If all the N countries that we consider are relatively small compared with the industrialized world as a whole, we may regard the evolution of the frontier technology level as largely exogenous from the point of view of each single country.

2.1 Complete catching up without overtaking

Whatever the interpretation, we assume the frontier technology level grows at a given constant rate, $g > 0$, i.e., $T_w(t) = T_w(0)e^{gt}$. By definition, we have initially $T_i(0) \leq T_w(0)$ for all i.

One possible hypothesis concerning technological catching-up is the following:

$$\frac{\dot{T}_i(t)}{T_i(t)} = g + \xi_i(\frac{T_w(t)}{T_i(t)} - 1), \qquad \xi_i > 0. \tag{1}$$

The size of $T_w(t)/T_i(t) - 1$ indicates the *technology gap* at time t. The parameter ξ_i is sometimes called the *catching up ability* or the *learning ability* of country i and is generally assumed to depend on such factors as the level of human capital, the degree of openness, and the "quality" of the institutions of the country. According to this formulation, if country i is initially behind the frontier, then $\dot{T}_i(t)/T_i(t) > g$. Since $T_i(t)$ remains growing at a higher rate than $T_w(t)$ as long as there is a technology gap, $T_w(t)/T_i(t)$ will be falling and approach one from above. Let us be precise about this:

CLAIM 1 Let the catching-up process be given by (1) and let $T_i(0) < T_w(0)$. Then $T_w(t)/T_i(t) > 1$ for all $t \geq 0$, but $\lim_{t\to\infty} T_w(t)/T_i(t) = 1$.

Proof. Mathematically it is convenient to consider the inverse ratio, $x(t) \equiv T_i(t)/T_w(t)$, a measure of country i's *lag* relative to the frontier. The growth rate of $x(t)$ is

$$\frac{\dot{x}}{x} = \frac{\dot{T}_i}{T_i} - \frac{\dot{T}_w}{T_w} = g + \xi_i(\frac{T_w(t)}{T_i(t)} - 1) - g = \xi_i(\frac{1}{x} - 1).$$

Multiplying through by x we thus have $\dot{x} = \xi_i(1-x)$. This is a linear differential equation with constant coefficients. Writing it on the standard form,

$$\dot{x} + \xi_i x = \xi_i, \tag{2}$$

we find, from formula 1 of the appendix, the solution

$$x(t) = (x(0) - x^*)e^{-\xi_i t} + x^* \rightarrow x^* = 1,$$

for $t \rightarrow \infty$, since $\xi_i > 0$ and $x^* = \xi_i/\xi_i = 1$, by setting $\dot{x} = 0$ in (2). Moreover, $T_i(0) < T_w(0)$ implies $x(0) < 1$, and so $x(t) < 1$ for all $t \geq 0$, which shows that $T_i(t)/T_w(t)$ approaches 1 from below, i.e., $T_w(t)/T_i(t)$ does so from above, as was to be shown. □

Here we have "complete technology catching-up", but "no overtaking".

Note, however, that even though the technology gap in this case ultimately disappears, labor productivities need not converge. This is because there may remain technology differences between the countries emanating from differences in product and industry composition. I will here illustrate that this is so not only in a Solow-style setup, but also in a setup where the countries trade in an integrated world market for goods and financial capital.

Let country i, $i = 1, 2, ..., n$, face a *constant* real interest rate $r > 0$, given from the fully integrated world market for goods and financial capital. Then, given country i's production function on intensive form,

$$\tilde{y}_i(t) \equiv \frac{y_i(t)}{T_i(t)} \equiv \frac{Y_i(t)}{T_i(t)L_i(t)} = f_i(\tilde{k}_i(t)),$$

where $\tilde{k}_i(t) \equiv K_i(t)/(T_i(t)L_i(t))$, profit maximizing firms will under perfect competition choose a time-independent effective capital intensity, $\tilde{k}_i^*$, satisfying

$$f_i'(\tilde{k}_i^*) = r + \delta_i,$$

where δ_i is the country-specific capital depreciation rate, here assumed constant over time. Consequently, $y_i(t) \equiv \tilde{y}_i(t)T_i(t) = f_i(\tilde{k}_i^*)T_i(t)$.

Considering the frontier technology as associated with a specific country, w, we get the relative labor productivity

$$\frac{y_i(t)}{y_w(t)} = \frac{f_i(\tilde{k}_i^*)T_i(t)}{f_w(\tilde{k}_w^*)T_w(t)} \rightarrow \frac{f_i(\tilde{k}_i^*)}{f_w(\tilde{k}_w^*)}$$

for $t \rightarrow \infty$, since $T_i(t)/T_w(t) \rightarrow 1$ for $t \rightarrow \infty$. Only if $f_i(\cdot) = f_w(\cdot)$ and $\delta_i = \delta_w$, will $y_i(t)/y_w(t) \rightarrow 1$ for $t \rightarrow \infty$.[1]

2.2 The Bernard and Jones formulation

Bernard and Jones (1996) propose another hypothesis about technological catching up, implying that both *incomplete* technology catching-up and *overtaking* are possible. The latter possibility is not mentioned by the authors, probably because it only arises if $\xi_i > \xi_w$, a condition implicitly ruled out by Bernard and Jones.

Their hypothesis is that

$$\frac{\dot{T}_i(t)}{T_i(t)} = \xi_i \frac{T_w(t)}{T_i(t)}, \tag{3}$$

where, again, $T_w(t) = T_w(0)e^{gt}$ and $T_w(0) > T_i(0)$. As before, the size of $T_w(t)/T_i(t) - 1$ indicates the technology gap at time t.

If $T_w(t)$ refers to the technology level of a single leader country, for instance the U.S., the learning ability of this country is $\xi_w = g$, in view of $\dot{T}_w(t)/T_w(t) = \xi_w$, by (3). If instead $T_w(t)$ refers to a frontier technology level in the world and is not identical to the technology level of any single country, we may still, if convenient, identify the frontier growth rate g with a "frontier learning ability" ξ_w.

CLAIM 2 Let the catching-up process be given by (3) and let $T_i(0) < T_w(0)$. Then

$$\lim_{t\rightarrow\infty} T_w(t)/T_i(t) = \frac{g}{\xi_i} \gtreqless 1 \text{ for } \frac{\xi_w}{\xi_i} \gtreqless 1, \text{ respectively.}$$

[1] Yet, adding stochastic elements, we would even in this case rather expect the standard deviation of $y_i(t)/y_w(t)$ across the countries to converge over time to some *positive* number.

Proof. Mathematically it is again convenient to consider the inverse ratio, $x(t) \equiv T_i(t)/T_w(t)$, a measure of country i's *lag* relative to the frontier. The growth rate of $x(t)$ is

$$\frac{\dot{x}}{x} = \frac{\dot{T}_i}{T_i} - \frac{\dot{T}_w}{T_w} = \xi_i \frac{T_w(t)}{T_i(t)} - g = \xi_i \frac{1}{x} - g.$$

Multiplying through by x we thus have $\dot{x} = \xi_i - gx$. This is a linear differential equation with constant coefficients. Writing it on the standard form,

$$\dot{x} + gx = \xi_i,$$

we find, from formula 1 of the appendix, the solution

$$x(t) = (x(0) - x^*)e^{-\xi_i t} + x^* \to x^* = \frac{\xi_i}{g},$$

for $t \to \infty$, since $\xi_i > 0$. Since $g = \xi_w$, we have hereby proved the claim. □

Interpreting ξ_w as referring to a specific country, the technology leader, Claim 2 thus shows:

(i) if $\xi_i < \xi_w$, although the technology gap is declining over time, it never disappears: country i never catches up fully with the technology leader, cf. Fig. 1;

(ii) if $\xi_i = \xi_w$, country i tends to catch up fully (but no more) with the technology leader;

(iii) if $\xi_i > \xi_w$, $T_w(t)/T_i(t)$ declines and reaches 1 in finite time, say at time t_0. But $T_w(t)/T_i(t)$ continues to decline until ultimately $T_w(t)/T_i(t) = \xi_w/\xi_i < 1$.

In case (iii) country i not only catches up in finite time, but *overtakes* the current leader and becomes a potential new leader in the sense that $T_i(t) > T_w(t)$ for all $t > t_0$. Fig. 2 illustrates. At time t_0 country i reaches the frontier and after time t_0 country i's superior ability to manage new technology shows up as $T_i(t) > T_w(t)$.

Admittedly, however, for $t > t_0$ the model no more seems entirely adequate, if we stick to the interpretation of ξ_w as referring to a specific country. Indeed, for $t > t_0$ $T_w(t) < T_i(t)$ and so it is not clear why the catching-up process in (3) should continue to hold, since there is nothing for country i any more to imitate from country w.

Personally, I therefore prefer the alternative interpretation which does not identify $T_w(t)$ with a specific country, but simply with the frontier technology level in the world.

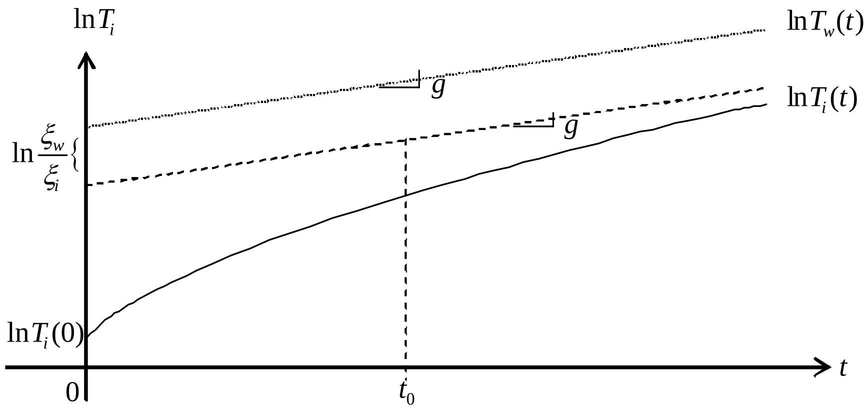

Figure 1: The technology gap is never eliminated because $\xi_i < \xi_w$.

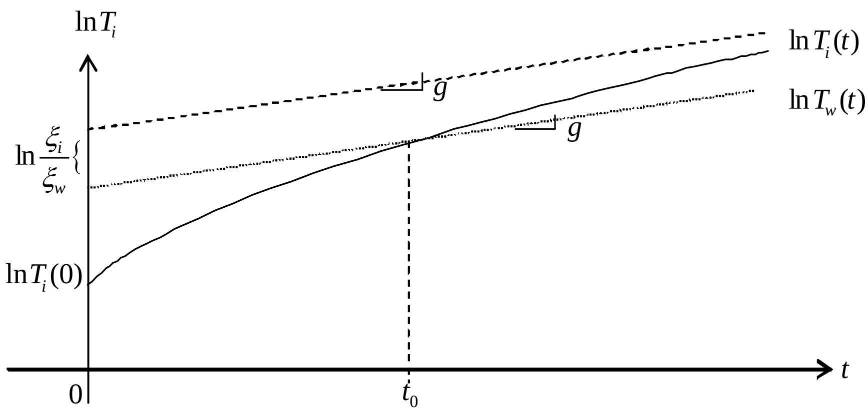

Figure 2: Overtaking takes place because $\xi_i > \xi_w$.

This frontier is pushed forward as a result of R&D and learning by doing by the most innovative firms in the industrialized world as a whole. Then it is natural to assume that the learning ability ξ_i of the specific countries is never higher than $g(= \xi_w)$. A picture like that in Fig. 2 will never arise, but two countries, i and j, may still overtake *each other*, depending on the size relationship between ξ_i and ξ_j.

Of course, both the catching-up hypotheses, (1) and (3), considered here are ad hoc. A more micro-founded approach would better be able to tell when imitation ceases and independent innovation takes over.

3 Appendix: Solution formulas for linear differential equations of first order

In the B & S textbook and other places we sometimes consider explicit solutions of linear differential equations. Apart from my general recommendation that you provide yourself with a handy mathematics manual (e.g., Berck and Sydsæter, Economists' Mathematical Manual, Springer-Verlag), I list here the most important formulas.[2]

1. $\dot{x}(t) + ax(t) = b$, with $a \neq 0$ and initial condition $x(0) = x_0$. Solution:

$$x(t) = (x_0 - x^*)e^{-at} + x^*, \text{ where } x^* = \frac{b}{a}.$$

2. $\dot{x}(t) + ax(t) = b(t)$, with initial condition $x(0) = x_0$. Solution:

$$x(t) = x_0 e^{-at} + e^{-at} \int_0^t b(s)e^{as} ds.$$

Special case: $b(t) = ce^{ht}$, with $h \neq -a$ and initial condition $x(0) = x_0$. Solution:

$$x(t) = x_0 e^{-at} + e^{-at} c \int_0^t e^{(a+h)s} ds = (x_0 - \frac{c}{a+h})e^{-at} + \frac{c}{a+h}e^{ht}.$$

3. $\dot{x}(t) + a(t)x(t) = b(t)$, with initial condition $x(0) = x_0$. Solution:

$$x(t) = x_0 e^{-\int_0^t a(\tau)d\tau} + e^{-\int_0^t a(\tau)d\tau} \int_0^t b(s)e^{\int_0^s a(\tau)d\tau} ds.$$

In the proofs of the claims 1 and 2 above we applied formula 1.

[2] The Math appendix in B & S may also be useful.