Written Exam for the M.Sc. in Economics 2009-II

Economic Growth

Master's Course

June 12, 2009

(4-hour closed book exam)

NOTICE:

- The Appendix reported at page 5 contains solution formulas to first order differential equations which might turn out to be useful in the course of the exam.

Problem 1: Exercise

Consider a closed market economy with N profit maximizing firms, operating under perfect competition (N "large"). There is a representative household (family dynasty) with L members at time t. Assume $L = L_0 e^{nt}$, where n is constant, $n \geq 0$. Each household member supplies one unit of labour per time unit. Aggregate output is Y per time unit, and output is used for consumption, $C \equiv cL$, and investment in physical capital K, i.e., $Y = C + \dot{K} + \delta K$, where $\delta \geq 0$ is the rate of physical decay of capital. Variables are dated implicitly. The initial value $K_0 > 0$ is given. There is a perfect market for loans at the real rate of interest r. There is perfect foresight.

The production function for firm i ($i = 1, 2, ..., N$) is

$$Y_i = F(K_i, TL_i), \tag{1}$$

where F is neoclassical and has CRS. The variable T evolves according to

$$T = T_t = e^{xt} K_t^{\lambda}, \quad x \geq 0, 0 < \lambda \leq 1, \tag{2}$$

where x and λ are constants and $K_t = \sum_i K_{it}$. Each firm is small and takes K_t as not affected by its own behavior.

a. Briefly interpret (1) and (2).

b. In general equilibrium, determine r and the aggregate production function at time t.

c. Assume $x > 0$ and $\lambda < 1$. Determine the rate of growth of Y and $y \equiv Y/L$ under balanced growth. *Hint:* use the proposition about equivalence of balanced growth and constancy of certain key ratios.

d. Comment on the model in relation to different types of endogenous growth.

From now on, let $\lambda = 1$ and $x = n = 0$.

e. Assume that the representative household has infinite horizon, an instantaneous utility function with absolute elasticity of marginal utility equal to a constant $\theta > 0$ and a constant rate of time preference w.r.t. utility, $\rho > 0$. Let $F_1(1, L) > \delta + \rho$. Determine the equilibrium rate of growth of c, k ($\equiv K/L$) and y, respectively. In case you need to introduce a restriction on some parameters, do it.

f. Now, introduce a government that pursues two activities:(i) it pays a subsidy, s, to the firms so that their capital costs reduce to

$$(1-s)(r+\delta)$$

per unit of capital per time unit; (ii) it finances this subsidy by a constant consumption tax τ. The government budget is always balanced. In particular, the subsidy is financed by a consumption tax and no other expenditures take place, that is

$$\tau c L = s(r+\delta)K$$

g. Could there be good economic reasons for such a subsidy? Comment.

h. Provide an analysis of whether there is a level of the subsidy rate such that the social planner's allocation can in principle be implemented.

Problem 2: Exercise

Consider the following growth model for a closed economy with a government sector. Firm i employs the following technology:

$$Y_{it} = AK_{it}^{\alpha}L_{it}^{1-\alpha}\hat{G}_t^{\pi}, \quad 0<\alpha<1, \quad \pi>0 \tag{1}$$

where A is a constant, K_{it} denotes the firm-specific capital stock, L_{it} total labor input in firm i, while

$$\hat{G}_t = \frac{G_t}{K_t^{\phi}L_t^{\delta}}, \quad 0<\phi<\alpha<1, \quad 0<\delta<1 \tag{2}$$

where G_t represents government investments in infrastructure, while K_t and L_t are the aggregate stock of private capital and the total labor force in the economy, respectively. Let r_t denote the real interest rate, and w_t the real wage. All markets are competitive, and the price of output is normalized to 1. For simplicity it is assumed that capital does not depreciate. G_t is financed by a wealth tax, levied on the households. The government balances the budget at all points in time, i.e. $G_t = \tau K_t$, where τ is the (time constant) wealth tax rate. Finally, the total size of the labor force is constant at all points in time, hence $L_t = L$.

a. Provide an interpretation of equation (2).

b. Solve the profit maximization problem for firm i, and proceed to show that the aggregate production function can be written as

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha}\hat{G}_t^{\pi}$$

c. What would π need to fulfill so that the model can exhibit fully endogenous growth? Assume the restriction just derived holds. The representative agent maximizes discounted utility from consumption. More specifically, the problem is

$$\max_{\{c_t\}_{t=0}^{\infty}} \int_0^{\infty} \frac{c_t^{1-\theta}-1}{1-\theta} e^{-\rho t} dt, \quad \theta > 0, \quad \rho > 0$$

$$\begin{aligned} &\text{s.t.} \\ c_t &\geq 0 \\ \dot{k}_t &= (r-\tau) k_t + w_t - c_t, \quad k_0 \text{ given} \\ \lim_{t\to\infty} k_t e^{-\int_{s=0}^{t} r_s ds} &\geq 0 \end{aligned}$$

where the wealth of the representative agent equals the capital/labor ratio, $k_t \equiv K_t/L$.

d. (i) Solve the consumer's problem and derive the growth rate of GDP per capita. (ii) Explain why the tax rate, τ, is related to the growth rate in the manner suggested by the formula.

e. The growth rate depends on the size of the labor force: is it possible to impose a restriction on certain parameters so as to eliminate scale effects in the present model, while preserving endogenous growth?

Problem 3: Short Essay Questions

a. *In brief, virtually all the R&D-based models in the literature share a prediction of "scale effects": if the level of resources devoted to R&D- measured, say, by the number of scientists engaged in R&D- is doubled, then the per capita growth rate of output should also double, at least in the steady state. Empirically, of course, such a prediction receives little support.* Discuss this statement from Jones (Journal of Political Economy, 1995), making clear whether he refers to strong or weak scale effects, and suggest possible mechanisms to overcome the rise of these counterfactual effects in R&D-based models of economic growth.

b. Comment on the models of endogenous growth proposed by Arrow (Review of Economic Studies, 1962) and Romer (Journal of Political Economy, 1986) in relation to the concepts of fully endogenous growth and semi-endogenous growth. In addition, discuss in which case taxes and subsidies may have long-run growth effects, as compared to the case in which they can only exert level effects.

Appendix: solution formulas to first order differential equations

1. $\dot{x}(t) + ax(t) = b$, with $a \neq 0$ and initial condition $x(0) = x_0$. Solution:

$$x(t) = (x_0 - x^*)e^{-at} + x^*, \text{ where } x^* = \frac{b}{a}.$$

2. $\dot{x}(t) + ax(t) = b(t)$, and initial condition $x(0) = x_0$. Solution:

$$x(t) = x_0 e^{-at} + e^{-at} \int_0^t b(s)e^{as} ds.$$

Special case: $b(t) = ce^{ht}$, with $h \neq -a$ and initial condition $x(0) = x_0$. Solution:

$$x(t) = x_0 e^{-at} + e^{-at} c \int_0^t e^{(a+h)s} ds = (x_0 - \frac{c}{a+h})e^{-at} + \frac{c}{a+h} e^{ht}.$$

3. $\dot{x}(t) + a(t)x(t) = b(t)$, with initial condition $x(0) = x_0$. Solution:

$$x(t) = x_0 e^{-\int_0^t a(\tau)d\tau} + e^{-\int_0^t a(\tau)d\tau} \int_0^t b(s) e^{\int_0^s a(\tau)d\tau} ds.$$