e) How, if at all, does a subsidy $\sigma \in (0,1)$ to R&D, financed by a lump-sum tax, such that the cost per unit of R&D labor is $(1-\sigma)w$, affect s_R and g_c, respectively, along the BGP? *Hint:* a look at f) of Problem VII.11 might be useful. Comment.

f) Does the subsidy affect the BGP? Why or why not?

VII.13 *Comparative analysis in the Romer-Jones model with Ramsey households.* We consider the same model and use the same notation as in Exercise VII.12, including $\varphi < 1$, $n > 0$, $0 \leq \xi < 1$, and the R&D subsidy $\sigma \in (0,1)$ (it is an advantage if you have already solved this problem).

a) Is the problem of the representative household as stated just above question a) in Exercise VII.12 still stated correctly after the introduction of the R&D subsidy financed by lump-sum taxes? Why or why not?

A convenient way of writing the model solution for s_R under balanced growth is:

$$s_R = \frac{1}{1 + \frac{1-\sigma}{\alpha}\left(\frac{\rho - n}{g_A} + \theta\right)}, \text{ with } g_A = \frac{1-\xi}{1-\varphi} n. \qquad (*)$$

b) On the basis of the general formula $s_R = (1 + \frac{(1-\sigma)(r-n)}{\alpha g_A})^{-1}$, show that (*) holds in the present case with Ramsey households.

c) Indicate the sign of the effect on s_R from a rise in ρ, θ, n,and σ, respectively. In each case, give some intuition. *Hint:* recalling intermediate results such as $P_A = \pi/(r-n)$ and $P_A \bar{\eta} = (1-\sigma)w = (1-\sigma)\partial Y/\partial L_Y = (1-\sigma)(1-\alpha)Y/L_Y$ may be of help.

d) Indicate the sign of the effect on s_R from a rise in φ and ξ, respectively. Because of mutual dependencies at several levels (balanced growth in general equilibrium) are involved, here it is not easy to briefly give an intuition. But an intermediate result from f) of Exercise VII.12 may at least, in combination with (*), provide a preliminary *rationalization* of the signs. That intermediate result is that equilibrium in the labor market with active R&D under balanced growth requires that

$$\frac{g_A}{r-n} = \frac{1-\sigma}{\alpha} \frac{L_A}{L_Y}. \qquad (**)$$

Explain the link between this result and the sign of the effect on s_R from a rise in φ and ξ, respectively.