Economic Growth Note on Exercise III.6a)

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We read:

> *Hint 2.* Consider the ratio $x(t) \equiv A_i(t)/A_w(t)$, a measure of (better to say: a reverse measure of) country i's *lag* relative to the frontier; express the growth rate of x in terms of x, ξ_i, and g; this should give you a linear first-order differential equation with constant coefficients; then apply the brief math manual in the appendix.

Question received from a student: How can we derive the growth rate of x in terms of x, ξ_i, and g? Since this first step is important for the remainder of III. 6 and even for III.7, here is my answer.

Recall the rule that the growth rate (in continuous time) of a ratio is the growth rate of the numerator minus the growth rate of the denominator, a rule which we get by "take logs and time derivatives" (Jones, App. A.1.4, or LN, Ch. 3, App. A). Indeed,

$x(t) \equiv A_i(t)/A_w(t)$ implies $\ln x(t) = \ln A_i(t) - \ln A_w(t)$. Now take the derivative w.r.t. time on both sides to get:

$$
\begin{aligned}
\frac{\dot{x}}{x} &= \frac{\dot{A}_i}{A_i} - \frac{\dot{A}_w}{A_w} \quad \text{(this is a manifestation of the ratio rule)} \\
&= \xi_i \frac{A_w}{A_i} - g = \xi_i x^{-1} - g \Rightarrow \\
\dot{x} &= \xi_i - gx,
\end{aligned}
$$

which is a linear differential equation in x.