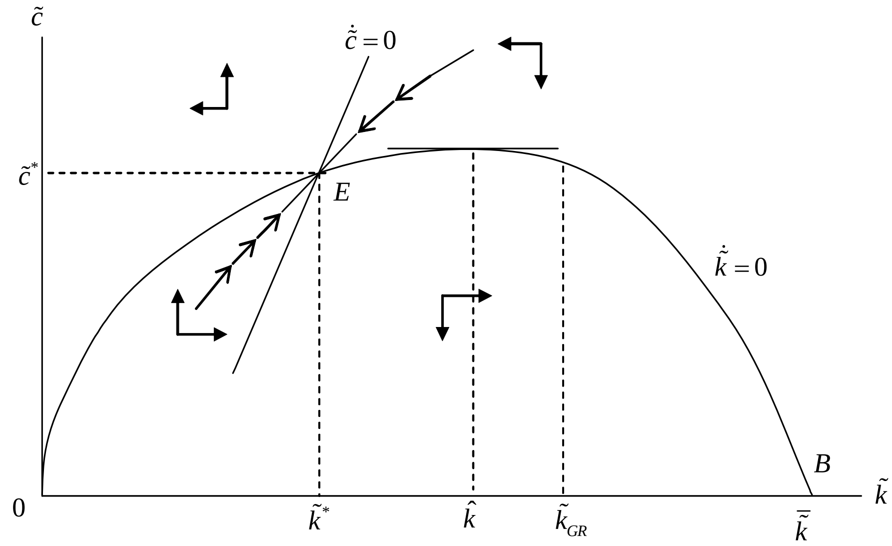

Figure 12.1: Phase diagram for the Arrow model.

Before determining the slope of the $\dot{\tilde{c}} = 0$ locus, it is convenient to consider the steady state, $(\tilde{k}^*, \tilde{c}^*)$.

Steady state

In a steady state $\tilde{c}$ and $\tilde{k}$ are constant so that the growth rate of C as well as K equals $\dot{A}/A + n$, i.e.,

$$\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} + n = \lambda\frac{\dot{K}}{K} + n.$$

Solving gives

$$\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{n}{1-\lambda}.$$

Thence, in a steady state

$$g_c = \frac{\dot{C}}{C} - n = \frac{n}{1-\lambda} - n = \frac{\lambda n}{1-\lambda} \equiv g_c^*, \quad \text{and} \tag{12.15}$$

$$\frac{\dot{A}}{A} = \lambda\frac{\dot{K}}{K} = \frac{\lambda n}{1-\lambda} = g_c^*. \tag{12.16}$$

The steady-state values of r and $\tilde{k}$, respectively, will therefore satisfy, by (12.11),

$$r^* = f'(\tilde{k}^*) - \delta = \rho + \theta g_c^* = \rho + \theta\frac{\lambda n}{1-\lambda}. \tag{12.17}$$

To ensure existence of a steady state we assume that the private marginal product of capital is sufficiently sensitive to capital per unit of effective labor, from now called the "capital intensity":

$$\lim_{\tilde{k}\to 0} f'(\tilde{k}) > \delta + \rho + \theta\frac{\lambda n}{1-\lambda} > \lim_{\tilde{k}\to\infty} f'(\tilde{k}). \tag{A1}$$

The transversality condition of the representative household is that $\lim_{t\to\infty} a_t e^{-\int_0^t (r_s - n)ds} = 0$, where a_t is per capita financial wealth. In general equilibrium $a_t = k_t \equiv \tilde{k}_t A_t$, where A_t in steady state grows according to (12.16). Thus, in steady state the transversality condition can be written

$$\lim_{t\to\infty} \tilde{k}^* e^{(g_c^* - r^* + n)t} = 0. \tag{TVC}$$

For this to hold, we need

$$r^* > g_c^* + n = \frac{n}{1-\lambda}, \tag{12.18}$$

by (12.15). In view of (12.17), this is equivalent to

$$\rho - n > (1-\theta)\frac{\lambda n}{1-\lambda}, \tag{A2}$$

which we assume satisfied.

As to the slope of the $\dot{\tilde{c}} = 0$ locus we have, from (12.14),

$$c'(\tilde{k}) = f'(\tilde{k}) - \delta - \frac{1}{\lambda}(\tilde{k}\frac{f''(\tilde{k})}{\theta} + g_c) > f'(\tilde{k}) - \delta - \frac{1}{\lambda}g_c, \tag{12.19}$$

since $f'' < 0$. At least in a small neighborhood of the steady state we can sign the right-hand side of this expression. Indeed,

$$f'(\tilde{k}^*) - \delta - \frac{1}{\lambda}g_c^* = \rho + \theta g_c^* - \frac{1}{\lambda}g_c^* = \rho + \theta\frac{\lambda n}{1-\lambda} - \frac{n}{1-\lambda} = \rho - n - (1-\theta)\frac{\lambda n}{1-\lambda} > 0, \tag{12.20}$$

by (12.15) and (A2). So, combining with (12.19), we conclude that $c'(\tilde{k}^*) > 0$. By continuity, in a small neighborhood of the steady state, $c'(\tilde{k}) \approx c'(\tilde{k}^*) > 0$. Therefore, close to the steady state, the $\dot{\tilde{c}} = 0$ locus is positively sloped, as indicated in Figure 12.1.

Still, we have to check the following question: In a neighborhood of the steady state, which is steeper, the $\dot{\tilde{c}} = 0$ locus or the $\dot{\tilde{k}} = 0$ locus? The slope of the latter is $f'(\tilde{k}) - \delta - n/(1-\lambda)$, from (12.13). At the steady state this slope is

$$f'(\tilde{k}^*) - \delta - \frac{1}{\lambda}g_c^* \in (0, c'(\tilde{k}^*)),$$