Chapter 13

Perspectives on learning by doing and learning by investing

By adding some theoretical and empirical perspectives to learning-by-doing and learning-by-investing models of endogenous growth, this chapter is a follow-up on Chapter 12. The contents are:

1. Learning by doing, learning by using, learning by watching
2. Empirics on learning by investing
3. Disembodied vs. embodied technical change*
4. Static comparative advantage vs. dynamics of learning by doing*

The growth rate of any time-dependent variable $z > 0$ is written $g_z \equiv \dot{z}/z$. In this chapter the economy-wide technology level at time t is denoted T_t rather than A_t.

13.1 Learning by doing, learning by using, learning by watching

The term *learning by doing* refers to the hypothesis that accumulated work experience, including repetition of the same type of action, improves workers' productivity and adds to technical knowledge. In connection with training in applying new production equipment, sometimes the related term *learning by using* is appropriate. In a broader context, the literature sometimes refers to spillover effects as *learning by watching.*

A specific form of learning by doing is called learning by investing and is treated separately in Section 13. 2 and 13.3.

A learning-by-doing model typically combines an aggregate CRS production function,

$$Y_t = F(K_t, T_t L_t), \tag{13.1}$$

with a learning function, for example,

$$\dot{T}_t = BY_t^{\lambda}, \qquad B > 0, 0 < \lambda \leq 1, \tag{13.2}$$

where λ is a learning parameter and B is a constant that, depending on the value of λ and the complete model in which (13.2) is embedded, is either an unimportant constant that depends only on measuring units or a parameter of importance for the productivity level or even the productivity growth rate. In Section 13.4 below, on the resource curse problem, we consider a two-sector model where each sector's productivity growth is governed by such a relationship.[1]

Another learning hypothesis is of the form

$$\dot{T}_t = BT_t^{\lambda} L_t^{\mu}, \qquad T_0 > 0 \text{ given}, B > 0, \lambda \leq 1, \mu > 0. \tag{13.3}$$

Here both λ and μ are learning parameters, reflecting the elasticities of learning w.r.t. the technology level and labor hours, respectively. The higher the number of human beings involved in production and the more time they spend in production, the more experience is accumulated. Sub-optimal ingredients in the production processes are identified and eliminated. The experience and knowledge arising in one firm or one sector is speedily diffused to other firms and other sectors in the economy (knowledge spillovers or *learning by watching*), and as a result the aggregate productivity level is increased.[2]

Since hours spent, L_t, is perhaps a better indicator for "new experience" than output, Y_t, specification (13.3) may seem more appealing than specification (13.2). So this section concentrates on (13.3).

If the labor force is growing, λ should be assumed strictly less than one, because with $\lambda = 1$ there would be a built-in tendency to forever faster growth, which does not seem plausible. In fact, $\lambda < 0$ can not be ruled out; that would reflect that learning becomes more and more difficult ("the easiest ideas are found first"). On the other hand, the case of "standing on

[1] In his Chapter 20, Section 20.4, on industrialization and structural change Acemoglu considers a model with two sectors, an agrarian and a manufacturing sector, where in the latter learning by doing in the form (13.2) with $\lambda = 1$ plays an important role.

[2] Diffusion of proficiency also occurs via apprentice-master relationships.

the shoulders" is also possible, that is, the case $0 < \lambda \leq 1$, which is the case where new learning becomes easier, the more is learnt already.

In "very-long-run" growth theory concerned with human development in an economic history perspective, the L in (13.3) has been replaced simply by the size of population in the relevant region (which may be considerably larger than a single country). This is the "population breeds ideas" view, cf. Kremer (1993). Anyway, many simple models consider the labor force to be proportional to population size, and then it does not matter whether we use the learning-by-doing interpretation or the population-breeds-ideas interpretation.

The so-called Horndal effect (reported by Lundberg, 1961) was one of the empirical observations motivating the learning-by-doing idea in growth theory:

> "The Horndal-iron works in Sweden had no new investment (and therefore presumably no significant change in its methods of production) for a period of 15 years, yet productivity (output per man-hour) rose on the average close to 2 % per annum. We find again steadily increasing performance which can only be imputed to learning from experience" (here cited after Arrow, 1962).

Similar patterns of on-the-job productivity improvements have been observed in ship-building, airframe construction, and chemical industries. On the other hand, within a single production line there seems to be a tendency for this kind of productivity increases to gradually peter out, which suggests $\lambda < 0$ in (13.3). We may call this phenomenon "diminishing returns in the learning process": the potential for new learning gradually evens out as more and more learning has already taken place. But new products are continuously invented and the accumulated knowledge is transmitted, more or less, to the production of these new products that start on a "new learning curve", along which there is initially "a large amount to be learned".[3] This combination of qualitative innovation and continuous productivity improvement through learning *may* at the aggregate level end up in a $\lambda \geq 0$ in (13.3).

In any case, whatever the sign of λ at the aggregate level, with $\lambda < 1$, this model is capable of generating sustained endogenous per capita growth (without "growth explosion") if the labor force is growing at a rate $n > 0$. Indeed, as in Chapter 12, there are two cases that are consistent with a balanced growth path (BGP for short) with positive per capita growth,

[3] A *learning curve* is a graph of estimated productivity (or its inverse, cf. Fig. 13.1 or Fig. 13.2 below) as a function of cumulative output or of time passed since production of the new product began at some plant.

namely the case $\lambda < 1$ combined with $n > 0$, and the case $\lambda = 1$ combined with $n = 0$.

We will show this for a closed economy with $L_t = L_0 e^{nt}$, $n \geq 0$, and with capital accumulation according to

$$\dot{K}_t = I_t - \delta K_t = Y_t - C_t - \delta K_t, \qquad K_0 > 0 \text{ given.} \tag{13.4}$$

13.1.1 The case: $\lambda < 1$ in (13.3)

Let us first consider the growth rate of $y \equiv Y/L$ along a BGP. There are two steps in the calculation of this growth rate.

Step 1. Given (13.4), from basic balanced growth theory (Chapter 4) we know that along a BGP with positive gross saving, not only are, by definition, g_Y and g_K constant, but they are also the same, so that Y_t/K_t is constant over time. Owing to the CRS assumption, (13.1) implies that

$$1 = F(\frac{K_t}{Y_t}, \frac{T_t L_t}{Y_t}). \tag{13.5}$$

When Y_t/K_t is constant, $T_t L_t/Y_t \equiv T_t/y_t$ must be constant, whereby

$$g_T = g_y = g_Y - n, \tag{13.6}$$

a constant.

Step 2. Dividing through by T_t in (13.3), we get

$$g_T \equiv \frac{\dot{T}_t}{T_t} = A T_t^{\lambda-1} L_t^{\mu}.$$

Taking logs gives $\log g_T = \log A + (\lambda - 1)\log T + \mu \log L$. And taking the time derivative on both sides of this equation leads to

$$\frac{\dot{g}_T}{g_T} = (\lambda - 1) g_T + \mu n. \tag{13.7}$$

In view of g_T being constant along a BGP, we have $\dot{g}_T = 0$, and so (13.7) gives

$$g_T = \frac{\mu n}{1 - \lambda},$$

presupposing $\lambda < 1$. Hence, by (13.6),

$$g_y = \frac{\mu n}{1 - \lambda}.$$

Under the assumption that $n > 0$, this per capita growth rate is positive, whatever the sign of λ. Given n, the growth rate is an increasing function of *both* learning parameters. Since a positive per capita growth rate can in the long run be maintained only if supported by $n > 0$, this is an example of *semi-endogenous exponential growth* (as long as n is exogenous).

This model thus gives growth results somewhat similar to the results in Arrow's learning-by-investing model, cf. Chapter 12. In both models the learning is an unintended by-product of the work process and construction of investment goods, respectively. And both models assume that knowledge is non-appropriable (non-exclusive) and that knowledge spillovers across firms are fast (in the time perspective of growth theory). So there are positive externalities which may motivate government intervention.

Methodological remark: Different approaches to the calculation of long-run growth rates Within this semi-endogenous growth case, depending on the situation, different approaches to the calculation of long-run growth rates may be available. In Chapter 12, in the analysis of the Arrow case $\lambda < 1$, the point of departure in the calculation was the steady state property of Arrow's model that $\tilde{k} \equiv K/(TL)$ is a constant. But this point of departure presupposes that we have established a well-defined steady state in the sense of a stationary point of a complete dynamic system (which in the Arrow model consists of two first-order differential equations in $\tilde{k}$ and $\tilde{c}$, respectively), usually involving also a description of the household sector.

In the present case we are not in this situation because we have not specified how the saving in (13.4) is determined. This explains why above (as well as in Chapter 10) we have taken another approach to the calculation of the long-run growth rate. We simply assume balanced growth and ask what the growth rate must then be. If the technologies in the economy are such that per capita growth in the long run can only be due to either exogenous productivity growth or semi-endogenous productivity growth, this approach is usually sufficient to determine a unique growth rate.

Note also, however, that this latter feature is in itself an interesting and useful result (as exemplified in Chapter 10). It tells us what the growth rate *must* be in the long run provided that the system converges to balanced growth. The growth rate will be the same, independently of the market structure and the specification of the household sector, that is, it will be the same whether, for example, there is a Ramsey-style household sector or an overlapping generations set-up.[4] And at least in the first case the growth

[4] Specification of these things is needed if we want to study the transitional dynamics: the adjustment processes outside balanced growth/steady state, including the question of

rate will be the same whatever the size of the preference parameters (the rate of time preference and the elasticity of marginal utility of consumption). Moreover, only if economic policy affects the learning parameters or the population growth rate (two things that are often ruled out inherently by the setup), will the long-run growth rate be affected. Still, economic policy can *temporarily* affect economic growth and in this way affect the *level* of the long-run growth path.

13.1.2 The case $\lambda = 1$ in (13.3)

With $\lambda = 1$ in (13.3), the above growth rate formulas are no longer valid. But returning to (13.3), we have $g_T = BL_t^{\mu}$. Then, unless $n = 0$, the growth rate of y will tend to rise forever, since we have $g_T = BL_0^{\mu}e^{\mu nt} \to \infty$ for $n > 0$.

So we will assume $n = 0$. Then $L_t = L_0$ for all t, implying $g_T = BL_0^{\mu}$ for all t. Since both B and L_0 are exogenous, it is *as if* the rate of technical progress, g_T, were exogenous. Yet, technical progress is generated by an internal mechanism. If the government by economic policy could affect B or L_0, also g_T would be affected. In any case, under balanced growth, (13.5) holds again and so $T_tL_t/Y_t = T_t/y_t$ must be constant. This implies $g_y = g_T = BL_0^{\mu} > 0$. Consequently, positive per capita growth can be maintained forever without support of growth in any exogenous factor. So we consider *fully endogenous exponential growth*.

As in the semi-endogenous growth case we can here determine the growth rate along a BGP independently of how the household sector is described. And preference parameters do *not* affect the growth rate. The fact that this is so even in the fully-endogenous growth case is due to the "law of motion" of technology making up a subsystem that is independent of the remainder of the economic system. This is a special feature of the "growth engine" (13.3). Although it is not a typical ingredient of endogenous growth models, this growth engine can not be ruled out *apriori*. The simple alternative, (13.2), is very different in that the endogenous aggregate output, Y_t, is involved. We return to (13.2) in Section 13.4 below.

Before proceeding, a brief remark on the explosive case $\lambda > 1$ in (13.2) or (13.3) is in place. If we imagine $\lambda > 1$, growth becomes explosive in the extreme sense that output as well as productivity, hence also per capita consumption, will tend to *infinity in finite time*. This is so even if $n = 0$. The argument is based on the mathematical fact that, given a differential equation $\dot{x} = x^a$, where $a > 1$ and $x_0 > 0$, the solution x_t has the property

convergence to balanced growth/steady state.

that there exists a $t_1 > 0$ such that $x_t \to \infty$ for $t \to t_1$. For details, see Appendix B.

13.2 Disembodied learning by investing

In the above framework the work process is a source of learning whether it takes place in the consumption or capital goods sector. This is *learning by doing* in a broad sense. If the source of learning is specifically associated with the construction of capital goods, the learning by doing is often said to be of the form of *learning by investing.* Why in the headline of this section we have added the qualification "disembodied", will be made clear in Section 13.3. Another name for learning by investing is *investment-specific learning by doing.*

The prevalent view in the empirical literature seems to be that learning by investing is the most important form of learning by doing; ship-building and airframe construction are prominent examples. To the extent that the construction of capital equipment is based on more complex and involved technologies than is the production of consumer goods, we are also, intuitively, inclined to expect that the greatest potential for productivity increases through learning is in the investment goods sector.[5]

In the simplest version of the learning-by-investment hypothesis, (13.3) above is replaced by

$$T_t = \left(\int_{-\infty}^{t} I_s^n ds \right)^{\lambda} = K_t^{\lambda}, \quad 0 < \lambda \leq 1, \tag{13.8}$$

where I_s^n is aggregate *net* investment. This is the hypothesis that the economy-wide technology level T_t is an increasing function of society's previous experience, proxied by cumulative aggregate net investment.[6] The Arrow and Romer models, as described in Chapter 12, correspond to the cases $0 < \lambda < 1$ and $\lambda = 1$, respectively.

In this framework, where the "growth engine" depends on capital accumulation, it is only in the Arrow case that we can calculate the per-capita growth rate along a BGP without specifying anything about the household sector.

[5] After the information-and-communication technology (ICT) revolution, where a lot of technically advanced consumer goods have entered the scene, this traditional presumption may be less compelling.

[6] Contrary to the dynamic learning-by-doing specification (13.3), there is here no good reason for allowing $\lambda < 0$.

13.2.1 The Arrow case: $\lambda < 1$ and $n \geq 0$

We may apply the same two steps as in Section 13.1.1. Step 1 is then an exact replication of step 1 above. Step 2 turns out to be even simpler than above, because (13.8) immediately gives $\log T = \lambda \log K$ so that $g_T = \lambda g_K$, which substituted into (13.6) yields

$$g_T = \lambda g_K = g_y = g_Y - n = g_K - n.$$

From this follows, first,

$$g_K = \frac{n}{1-\lambda}, \tag{13.9}$$

and, second,

$$g_y = \frac{\lambda n}{1-\lambda}.$$

Alternatively, we may in this case condense the two steps into one by rewriting (13.5) in the form

$$\frac{Y_t}{K_t} = F(1, \frac{T_t L_t}{K_t}) = F(1, K_t^{\lambda-1} L_t),$$

by (13.8). Along the BGP, since Y/K is constant, so must the second argument, $K_t^{\lambda-1} L_t$, be. It follows that

$$(\lambda - 1) g_K + n = 0,$$

thus confirming (13.9).

Whatever the approach to the calculation, the per capita growth rate is here tied down by the size of the learning parameter and the growth rate of the labor force.

13.2.2 The Romer case: $\lambda = 1$ and $n = 0$

In the Romer case, however, the growth rate along a BGP cannot be determined until the saving behavior in the economy is modeled. Indeed, the knife-edge case $\lambda = 1$ opens up for many different per capita growth rates under balanced growth. Which one is "selected" by the economy depends on how the household sector is described.

For a Ramsey setup with $n = 0$ the last part of Chapter 12 showed how the growth rate generated by the economy depends on the rate of time preference and the elasticity of marginal utility of consumption of the representative household. Growth is here *fully-endogenous* in the sense that a positive per capita growth rate can be maintained forever without the support by growth in any exogenous factor. Moreover, according to this model, economic policy that internalizes the positive externality in the system can raise not only the productivity level, but also the long-run productivity growth rate.

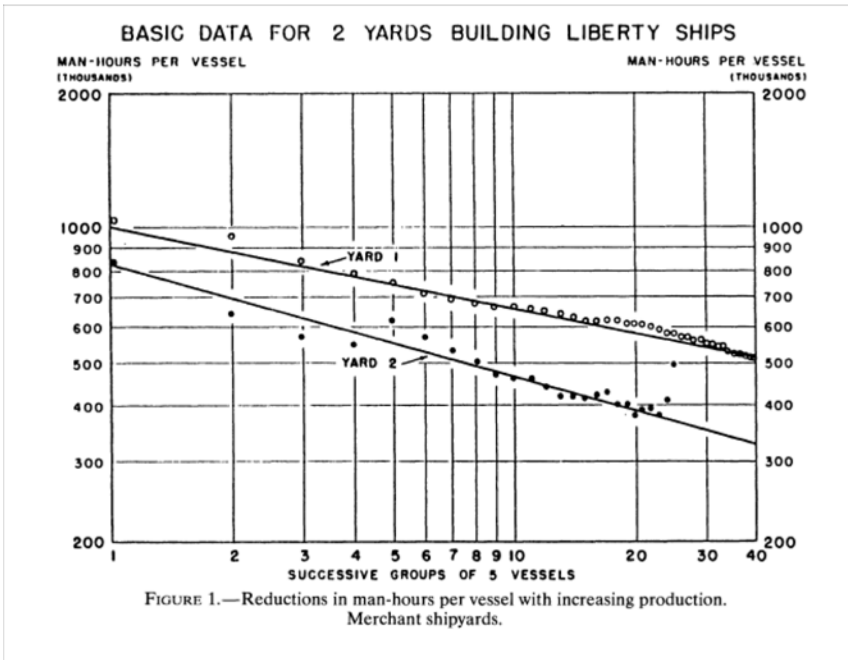

Figure 13.1: Man-hours per vessel against cumulative number of vessels completed to date in shipyard 1 and shipyard 2, respectively. Log-log paper. Source: Searle (1945).

13.2.3 The size of the learning parameter

What is from an empirical point of view a plausible value for the learning parameter, λ? This question is important because quite different results emerge depending on whether λ is close to 1 or considerably lower (fully-endogenous growth versus semi-endogenous growth). At the same time the question is not easy to answer because λ in the models is a parameter that is meant to reflect the aggregate effect of the learning going on in single firms and spreading across firms and industries.

Like Lucas (1993), we will consider the empirical studies of on-the-job productivity increases in ship-building by Searle (1945) and Rapping (1965). Both studies used data on the production of different types of cargo vessels during the second world war. Figures 1 and 2 are taken from Lucas' review article, Lucas (1993), but the original source is Searle (1945). For the vessel type called "Liberty Ships" Lucas cites the observation by Searle (1945):

> "the reduction in man-hours per ship with each doubling of cumulative output ranged from 12 to 24 percent."

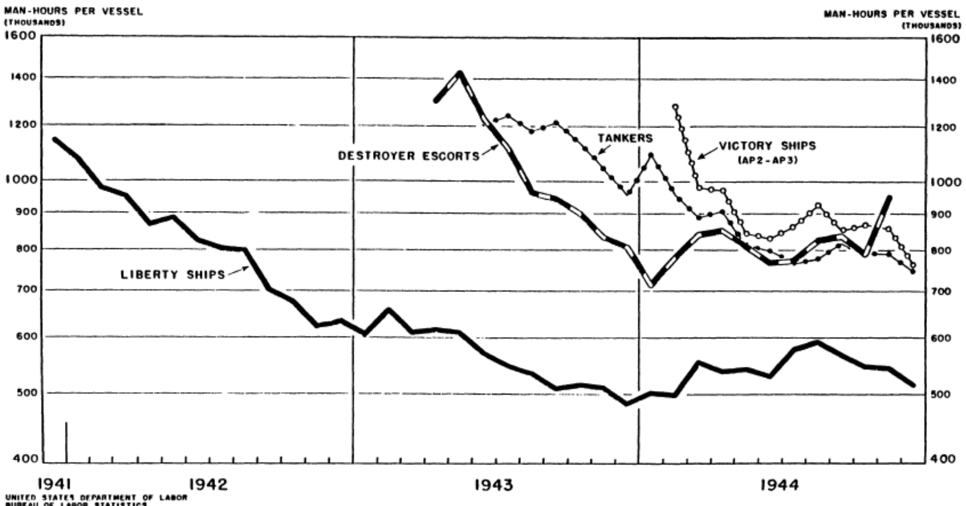

FIGURE 2.—Unit man-hour requirements for selected shipbuilding programs. Vessels delivered December 1941–December 1944.

Figure 13.2: Average man-hours (over ten shipyards) per vessel against calendar time. Four different vessel types. Source: Searle (1945).

Let us try to connect this observation to the learning parameter λ in Arrow's and Romer's framework. We begin by considering firm i which operates in the investment goods sector. We imagine that firm i's equipment is unchanged during the observation period (as is understood in the above citation as well as the citation from Arrow (1962) in Section 13.1). Let firm i's current output and employment be Y_{it} and L_{it}, respectively. The current labor productivity is then $a_{it} = Y_{it}/L_{it}$. Let the firm's *cumulative* output be denoted Q_{it}. This cumulative output is a part of cumulative investment in society. At the micro-level the learning-by-investing hypothesis is the hypothesis that labor productivity is an increasing function of the firm's cumulative output, Q_{it}.

In figures 1 and 2 the dependent variable is not directly labor productivity, but its inverse, namely the required man-hours per unit of output, $m_{it} = L_{it}/Y_{it} = 1/a_{it}$. Figure 13.1 suggests a log-linear relationship between this variable and the cumulative output:

$$\log m_{it} = \alpha - \beta \log Q_{it}. \tag{13.10}$$

That is, as cumulative output rises, the required man-hours per unit of output declines over time in this way:

$$m_{it} = \frac{e^{\alpha}}{Q_{it}^{\beta}}.$$

Equivalently, labor productivity rises over time in this way:

$$a_{it} = \frac{1}{m_{it}} = e^{-\alpha} Q_{it}^{\beta}.$$

So, specifying the relationship by a power function, as in (13.8), makes sense.

Now, let $t = t_1$ be a fixed point in time. Then, (13.10) becomes

$$\log m_{it_1} = \alpha - \beta \log Q_{it_1}.$$

Let t_2 be the later point in time where cumulative output has been doubled. Then at time t_2 the required man-hours per unit of output has declined to

$$\log m_{it_2} = \alpha - \beta \log Q_{it_2} = \alpha - \beta \log(2Q_{it_1}).$$

Hence,

$$\log m_{it_1} - \log m_{it_2} = -\beta \log Q_{it_1} + \beta \log(2Q_{it_1}) = \beta \log 2. \tag{13.11}$$

Lucas' citation above from Searle amounts to a claim that

$$0.12 < \frac{m_{it_1} - m_{it_2}}{m_{it_1}} < 0.24. \tag{13.12}$$

By a first-order Taylor approximation we have $\log m_{it_2} \approx \log m_{it_1} + (m_{it_2} - m_{it_1})/m_{it_1}$. Hence, $(m_{it_1} - m_{it_2})/m_{it_1} \approx \log m_{it_1} - \log m_{it_2}$. Substituting this into (13.12) gives, approximately,

$$0.12 < \log m_{it_1} - \log m_{it_2} < 0.24.$$

Combining this with (13.11) gives $0.12 < \beta \log 2 < 0.24$ so that

$$0.17 = \frac{0.12}{\log 2} < \beta < \frac{0.24}{\log 2} = 0.35.$$

Rapping (1965) finds by a more rigorous econometric approach β to be in the vicinity of 0.26 (still ship building). Arrow (1962) and Solow (1997) refer to data on airframe building. This data roughly suggests $\beta = 1/3$.

How can this be translated into a guess about the size of the "aggregate" learning parameter λ in (13.8)? This is a complicated question and the subsequent remarks are very tentative. First of all, the potential for both internal and external learning seems to vary a lot across different industries. Second, the amount of spillovers can not simply be added to the β above, since they are already partly included in the estimate of β. Even theoretically, the role of experience in different industries cannot simply be added up because to some extent there is redundancy due to *overlapping* experience and sometimes the learning in other industries is of limited relevance. Given that we are interested in an upper bound for λ, a "guestimate" is that the spillovers matter for the final λ at most the same as β from ship building so that $\lambda \leq 2\beta$.[7]

On the basis of these casual considerations we claim that a λ higher than about 2/3 may be considered fairly implausible. This speaks for the Arrow case of semi-endogenous exponential growth rather than the Romer case of fully-endogenous exponential growth, at least as long as we think of learning by investing as the sole source of productivity growth. Another point is that to the extent learning is internal and at least temporarily appropriable, we should expect at least some firms to internalize the phenomenon in its optimizing behavior (Thornton and Thompson, 2001). Although the learning is far from fully excludable, it takes time for others to discover and imitate technical and organizational improvements. Many simple growth models ignore this and treat all learning by doing and learning by investing as a 100 percent externality, which seems an exaggeration.

A further issue is to what extent learning by investing takes the form of *disembodied* versus *embodied* technical change. This is the topic of the next section.

13.3 Disembodied vs. embodied technical change

Arrow's and Romer's models build on the idea that the *source* of learning is primarily experience in the investment goods sector. Both models assume that the learning, via knowledge spillovers across firms, provides an engine of productivity growth in essentially *all* sectors of the economy. And both models (Arrow's, however, only in its simplified version, which we considered

[7] For more elaborate studies of empirical aspects of learning by doing and learning by investing, see Irwin and Klenow (1994), Jovanovic and Nyarko (1995), and Greenwood and Jovanovic (2001). Caballero and Lyons (1992) find clear evidence of positive externalities across US manufacturing industries. Studies finding that the quantitative importance of spillovers is significantly smaller than required by the Romer case include Englander and Mittelstadt (1988) and Benhabib and Jovanovic (1991). See also the surveys by Syverson (2011) and Thompson (2012).

Although in this lecture note we focus on learning as an externality, there exists studies focusing on *internal* learning by doing, see, e.g., Gunn and Johri, 2011.

in Chapter 12, not in its original version) assume that a firm can benefit from recent technical advances irrespective of whether it buys new equipment or just uses old equipment. That is, the models assume that technical change is *disembodied*.

13.3.1 Disembodied technical change

Disembodied technical change occurs when new technical knowledge advances the combined productivity of capital and labor independently of whether the workers operate old or new machines. Consider again (13.1) and (13.3). When the K_t appearing in (13.1) refers to the total, historically accumulated capital stock, then the interpretation is that the higher technology level generated in (13.3) or (13.8) results in higher productivity of *all* labor, independently of the vintage of the capital equipment with which this labor is combined. Thus also firms with old capital equipment benefit from recent advances in technical knowledge. No new investment is needed to take advantage of the recent technological and organizational developments.

Examples of this kind of productivity increases include improvement in management and work practices/organization and improvement in accounting.

13.3.2 Embodied technical change

In contrast, we say that technical change is *embodied*, if taking advantage of new technical knowledge requires construction of new investment goods. The newest technology is incorporated in the design of newly produced equipment; and this equipment will not participate in subsequent technical progress. An example: only the most recent vintage of a computer series incorporates the most recent advance in information technology. Then investment goods produced later (investment goods of a later "vintage") have higher productivity than investment goods produced earlier at the same resource cost. Whatever the source of new technical knowledge, investment becomes an important bearer of the productivity increases which this new knowledge makes possible. Without new investment, the potential productivity increases remain potential instead of being realized.[8]

One way to formally represent embodied technical progress is to write

[8] The concept of embodied technical change was introduced by Johansen (1959) and Solow (1960). The notion of Solow-neutral technical change is related to embodied technical change and capital of different vintages.

capital accumulation in the following way,

$$\dot{K}_t = q_t I_t - \delta K_t, \tag{13.13}$$

where I_t is gross investment at time t and q_t measures the "quality" (productivity) of newly produced investment goods. The rising level of technology implies rising q_t so that a given level of investment gives rise to a greater and greater addition to the capital stock, K, measured in efficiency units. Even if technical change does not directly appear in the production function, that is, even if for instance (13.1) is replaced by $Y_t = F(K_t, L_t)$, the economy may in this manner still experience a rising standard of living.

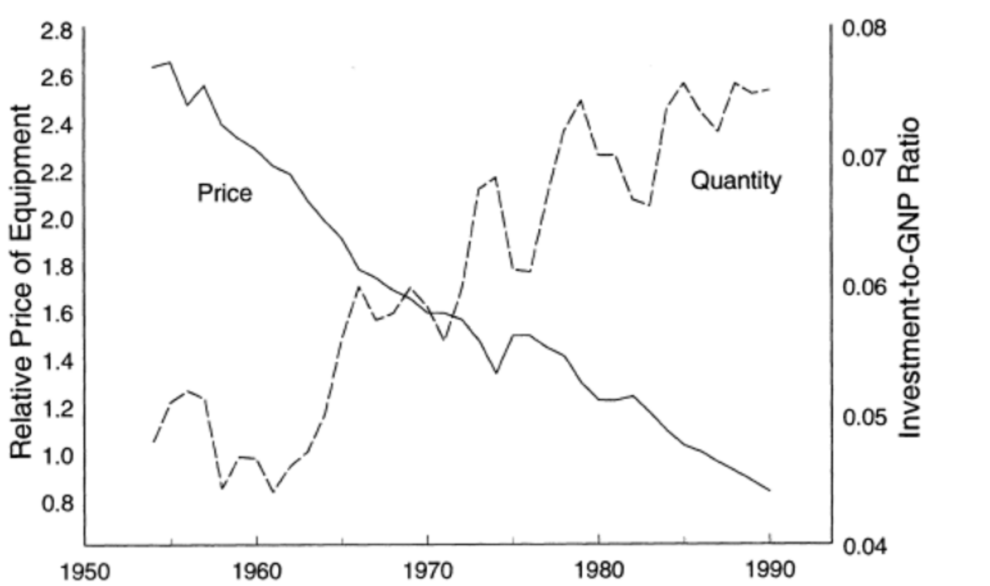

Figure 13.3: Relative price of equipment and quality-adjusted equipment investment-to-GNP ratio. Source: Greenwood, Hercowitz, and Krusell (1997).

Embodied technical progress is likely to result in a steady decline in the price of capital equipment relative to the price of consumption goods. This prediction is confirmed by the data. Greenwood et al. (1997) find for the U.S. that the relative price, p, of capital equipment has been declining at an average rate of 0.03 per year in the period 1950-1990, cf. the "Price" curve in Figure 13.3.[9] As the "Quantity" curve in Figure 13.3 shows, over

[9] The relative price index in Fig. 13.3 is based on the book by R. Gordon (1990), which is an attempt to correct previous price indices for equipment by better taking into account quality improvements in new equipment.

the same period there has been a secular rise in the ratio of new equipment investment (in efficiency units) to GNP; note that what in the figure is called the "investment-to-GNP Ratio" is really "quality-adjusted investment-to-GNP Ratio", qI/GNP, not the usual investment-income ratio, I/GNP.

Moreover, the correlation between de-trended p and de-trended qI/GNP is -0.46. Greenwood et al. interpret this as evidence that technical advances have made equipment less expensive, triggering increases in the accumulation of equipment both in the short and the long run. The authors also estimate that embodied technical change explains 60% of the growth in output per man hour.

13.3.3 Embodied technical change and learning by investing

Whether technological progress is disembodied or embodied says nothing about whether its *source* is exogenous or endogenous. Indeed, the increases of q in (13.13) may be modeled as exogenous or endogenous. In the latter case, a popular hypothesis is that the source is learning by investing. This learning may take the form (13.8) above. In that case the experience that matter for learning is cumulative *net* investment.

An alternative hypothesis is:

$$q_t = \left(\int_{-\infty}^{t} I_s ds \right)^{\lambda}, \qquad 0 < \lambda \leq \bar{\lambda}, \tag{13.14}$$

where I_s is *gross* investment at time s. Here the experience that matter has its basis in cumulative *gross* investment. An upper bound, $\bar{\lambda}$, for the learning parameter is introduced to avoid explosive growth. The hypothesis (13.14) seems closer to both intuition and the original ideas of Arrow:

> "Each new machine produced and put into use is capable of changing the environment in which production takes place, so that learning is taking place with continually new stimuli" (Arrow, 1962).

Contrary to the integral based on net investment in (13.8), the integral in the learning hypothesis (13.14) does not allow an immediate translation into an expression in terms of the accumulated capital stock. Instead a new state variable, cumulative gross investment, enter the system and opens up for richer dynamics.

We may combine (13.14) with an aggregate Cobb-Douglas production function,

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}. \tag{13.15}$$

Then the upper bound for the learning parameter in (13.14) is $\bar{\lambda} = (1-\alpha)/\alpha$.[10]

The case $\lambda < (1-\alpha)/\alpha$

Suppose $\lambda < (1-\alpha)/\alpha$. Using (13.14) together with (13.13), (13.15), and $I = Y - C$, one finds, under balanced growth with $s = I/Y$ constant and $0 < s < 1$,

$$g_K = \frac{(1-\alpha)(1+\lambda)n}{1-\alpha(1+\lambda)}, \tag{13.16}$$

$$g_q = \frac{\lambda}{1+\lambda} g_K, \tag{13.17}$$

$$g_Y = \frac{1}{1+\lambda} g_K, \tag{13.18}$$

$$g_c = g_y = g_Y - n = \frac{\alpha\lambda n}{1-\alpha(1+\lambda)}, \tag{13.19}$$

cf. Appendix A. We see that $g_y > 0$ if and only if $n > 0$. So exponential growth is here semi-endogenous.

Let us assume there is perfect competition in all markets. Since q capital goods can be produced at the same minimum cost as one consumption good, the equilibrium price, p, of capital goods in terms of the consumption good must equal the inverse of q, that is, $p = 1/q$. With the consumption good being the numeraire, let the rental rate in the market for capital services be denoted R and the real interest rate in the market for loans be denoted r. Ignoring uncertainty, we have the no-arbitrage condition

$$\frac{R_t - (\delta p_t - \dot{p}_t)}{p_t} = r_t, \tag{13.20}$$

[10] An alternative to the specification of embodied learning by gross investment in (13.14) is

$$q_t = \left(\int_{-\infty}^{t} q_s I_s ds \right)^{\tilde{\lambda}}, \qquad 0 < \tilde{\lambda} \leq \bar{\tilde{\lambda}},$$

implying that it is cumulative quality-adjusted gross investment that matters, cf. Greenwood and Jovanovic (2001). If combined with the production function (13.15) the appropriate upper bound on the learning parameter, $\tilde{\lambda}$, is $\bar{\tilde{\lambda}} = 1 - a$.

where $\delta p_t - \dot{p}_t$ is the true economic depreciation of the capital good per time unit. Since $p = 1/q$, (13.17) and (13.16) indicate that along a BGP the relative price of capital goods will be declining according to

$$g_p = -\frac{(1-\alpha)\lambda n}{1-\alpha(1+\lambda)} < 0.$$

Note that $g_K > g_Y$ along the BGP. Is this a violation of Proposition 1 of Chapter 4? No, that proposition presupposes that capital accumulation occurs according to the standard equation (13.4), not (13.13). And although g_K differs from g_Y, the output-capital ratio in *value* terms, $Y/(pK)$, is constant along the BGP. In fact, the BGP complies entirely with Kaldor's stylized facts if we interpret "capital" as the value of capital, pK.

The formulas (13.16) and (13.19) display that $\alpha(1+\lambda) < 1$ is needed to avoid a forever rising growth rate if $n > 0$. This inequality is equivalent to $\lambda < (1-\alpha)/\alpha$ and confirms that the upper bound, $\bar{\lambda}$, in (13.14) equals $(1-\alpha)/\alpha$. With $\alpha = 1/3$, this upper bound is 2. The bound is thus no longer 1 as in the simple learning-by-investing model of Section 13.2. The reason is twofold, namely partly that now q is formed via cumulative gross investment instead of net investment, partly that the role of q is to strengthen capital formation rather than the efficiency of production factors in aggregate final goods produce.

When $n = 0$, the system can no longer generate a constant positive per capita growth rate (exponential growth). Groth et al. (2010) show, however, that the system is capable of generating *quasi-arithmetic growth.* This class of growth processes, which fill the whole range between exponential growth and complete stagnation, was briefly commented on in Section 10.5 of Chapter 10.

The case $\lambda = (1-\alpha)/\alpha$ and $n = 0^*$

When $\lambda = (1-\alpha)/\alpha$, we have $\alpha(1+\lambda) = 1$ and so the growth formulas (13.16) and (13.19) no longer hold. But the way that (13.17) and (13.18) are derived (see Appendix A) ensures that these two equations remain valid along a BGP. Given $\lambda = (1-\alpha)/\alpha$, (13.17) can be written $g_q = (1-\alpha)g_K$, which is equivalent to

$$q_t = BK_t^{1-\alpha}$$

along a BGP (B is some positive constant to be determined).

To see whether a BGP exists, note that (13.14) implies

$$g_q = \frac{\dot{q}_t}{q_t} = \lambda q_t^{-1/\lambda} I_t = \lambda q_t^{-\alpha/(1-\alpha)} I_t = \lambda B^{-\alpha/(1-\alpha)} K_t^{-\alpha} I_t = \lambda B^{-\alpha/(1-\alpha)} K_t^{-\alpha} sY_t,$$

considering a BGP with $s = I/Y$ constant. Substituting (13.15) into this, we get

$$g_q = \lambda B^{-\alpha/(1-\alpha)} K_t^{-\alpha} s K_t^{\alpha} L^{1-\alpha} = \lambda B^{-\alpha/(1-\alpha)} s L^{1-\alpha}. \tag{13.21}$$

If $n = 0$, the right-hand side of (13.21) is constant and so is $g_K = g_q/(1-\alpha)$, by (13.17), and $g_Y = \alpha g_K = \alpha g_q/(1-\alpha)$, by (13.19).

If $n > 0$ at the same time as $\lambda = (1-\alpha)/\alpha$, however, there is a tendency to a forever rising growth rate in q, hence also in K and Y. No BGP exists in this case.

Returning to the case where a BGP exists, a striking feature revealed by (13.21) is that the saving rate, s, matters for the growth rate of q, hence also for the growth rate of K and Y, respectively, along a BGP. As in the Romer case of the disembodied learning-by-investing model, the growth rates along a BGP cannot be determined until the saving behavior in the economy is modeled.

So the considered knife-edge case, $\lambda = (1-\alpha)/\alpha$ combined with $n = 0$, opens up for many different per capita growth rates under balanced growth. Which one is "selected" by the economy depends on how the household sector is described. In a Ramsey setup with $n = 0$ one can show that the growth rate under balanced growth depends negatively on the rate of time preference and the elasticity of marginal utility of consumption of the representative household. And not only is growth in this case *fully endogenous* in the sense that a positive per capita growth rate can be maintained forever without the support by growth in any exogenous factor. An economic policy that subsidizes investment can generate not only a transitory rise in the productivity growth rate, but also a permanently higher productivity growth rate.

In contrast to the Romer (1986) model, cf. Section 13.2.2 above, we do not here end up with a reduced-form AK model. Indeed, we end up with a model with transitional dynamics, as a consequence of the presence of *two* state variables, K and q.

If instead $\alpha > 1/(1+\lambda)$, we get a tendency to explosive growth – infinite output in finite time – a not plausible scenario, cf. Appendix B.

13.4 Static comparative advantage vs. dynamics of learning by doing*

In this section we will briefly discuss a development economics perspective of the above learning-based growth models.

More specifically we will take a look at the possible "conflict" between static comparative advantage and economic growth. The background to this possible "conflict" is the dynamic externalities inherent in learning by doing and learning by investing.[11]

13.4.1 A simple two-sector learning-by-doing model

We consider an isolated economy with two production sectors, *sector 1* and *sector 2*, each producing its specific consumption good. Labor is the only input and aggregate labor supply L is constant. There are many small firms in the two sectors. Aggregate output in the sectors are:

$$Y_{1t} = T_{1t}L_{1t}, \tag{13.22}$$
$$Y_{2t} = T_{2t}L_{2t}, \tag{13.23}$$

where

$$L_{1t} + L_{2t} = L.$$

There are *sector-specific* learning-by-doing externalities in the following form:

$$\dot{T}_{1t} = B_1 Y_{1t}, \qquad B_1 \geq 0, \tag{13.24}$$
$$\dot{T}_{2t} = B_2 Y_{2t}, \qquad B_2 \geq 0. \tag{13.25}$$

Although not visible in our aggregate formulation, there are substantial knowledge spillovers across firms within the sectors. Across sectors, spillovers are assumed negligible.

Assume firms maximize profits and that there is perfect competition in the goods and labor markets. Then, prices are equal to the (constant) marginal costs. Let the relative price of sector 2-goods in terms of sector-1 goods be called p_t (i.e., we use sector-1 goods as numeraire). Let the hourly wage in terms of sector-1 goods be w_t. In general equilibrium with production in both sectors we then have

$$T_{1t} = p_t T_{2t} = w_t,$$

saying that the value of the (constant) marginal productivity of labor in each sector equals the wage. Hence,

$$p_t \frac{T_{2t}}{T_{1t}} = 1 \quad \text{or} \quad p_t = \frac{T_{1t}}{T_{2t}}, \tag{13.26}$$

saying that the relative price of the two goods is inversely proportional to the relative labor productivities in the two sectors. The demand side, which

[11] Krugman (1987), Lucas (1988, Section 5).

is not modelled here, will of course play a role for the final allocation of labor to the two sectors.

Taking logs in (13.26) and differentiating w.r.t. t gives

$$\frac{\dot{p}_t}{p_t} = \frac{\dot{T}_{1t}}{T_{1t}} - \frac{\dot{T}_{2t}}{T_{2t}} = \frac{B_1 Y_{1t}}{T_{1t}} - \frac{B_2 Y_{2t}}{T_{2t}} = B_1 L_{1t} - B_2 L_{2t},$$

using (13.24) and (13.25). Thus,

$$\dot{p}_t = (B_1 L_{1t} - B_2 L_{2t}) p_t.$$

Assume sector 2 (say some industrial activity) is more disposed to learning-by-doing than sector 1 (say mining) so that $B_2 > B_1$. Consider for simplicity the case where at time 0 there is symmetry in the sense that $L_{10} = L_{20}$. Then, the relative price p_t of sector-2 goods in terms of sector-1 goods will, at least initially, tend to diminish over time. The resulting substitution effect is likely to stimulate demand for sector-2 goods. Suppose this effect is large enough to ensure that $L_2 = Y_2/T_2$ never becomes lower than $B_1 L_1/B_2$, that is, $B_2 L_2 \geq B_1 L_1$ for all t. Then the scenario with $\dot{p} \leq 0$ is sustained over time and the sector with highest growth potential remains a substantial constituent of the economy. This implies sustained economic growth in the aggregate economy.

Now, suppose the country considered is a rather backward, developing country which until time t_0 has been a closed economy (very high tariffs etc.). Then the country decides to open up for free foreign trade. Let the relative world market price of sector 2-goods be $\bar{p}$, which we for simplicity assume is constant. At time t_0 there are two alternative possibilities to consider:

Case 1: $\bar{p} > \frac{T_{1t_0}}{T_{2t_0}}$ (world-market price of good 2 higher than the opportunity cost of producing good 2). Then the country specializes fully in sector-2 goods. Since this is the sector with a high growth potential, economic growth is stimulated. The relative productivity level T_{1t}/T_{2t} decreases so that the scenario with $\bar{p} > T_{1t}/T_{2t}$ remains. A virtuous circle of dynamics of learning by doing is unfolded and high economic growth is sustained.

Case 2: $\bar{p} < \frac{T_{1t_0}}{T_{2t_0}}$ (world-market price of good 2 lower than the opportunity cost of producing good 2). Then the country specializes fully in sector-1 goods. Since this is the sector with a low growth potential, economic growth is impeded or completely halted. The relative productivity level T_{1t}/T_{2t} does not decrease. Hence, the scenario with $\bar{p} < T_{1t}/T_{2t}$ sustains itself and persists. Low or zero economic growth is sustained. The static comparative advantage in sector-1 goods remains and the country is locked in low growth.

If instead $\bar{p}$ is time-dependent, suppose $\dot{\bar{p}}_t < 0$ (by similar arguments as for the closed economy). Then the case 2 scenario is again self-sustaining.

The point is that there may be circumstances (like in case 2), where temporary protection for a backward country is growth promoting (this is a specific kind of "infant industry" argument).

13.4.2 A more robust specification

The way (13.24) and (13.25) are formulated, we have

$$\frac{\dot{T}_{1t}}{T_{1t}} = B_1 L_{1t}, \tag{13.27}$$

$$\frac{\dot{T}_{2t}}{T_{2t}} = B_2 L_{2t}, \tag{13.28}$$

by (13.22) and (13.23). Thus, the model implies scale effects on growth, that is, *strong* scale effects.

An alternative specification introduces limits to learning-by-doing in the following way:

$$\dot{T}_{1t} = B_1 Y_{1t}^{\lambda_1}, \qquad \lambda_1 < 1,$$

$$\dot{T}_{2t} = B_2 Y_{2t}^{\lambda_2}, \qquad \lambda_2 < 1.$$

Then (13.27) and (13.28) are replaced by

$$\frac{\dot{T}_{1t}}{T_{1t}} = B_1 T_{1t}^{\lambda_1 - 1} L_{1t}^{\lambda_1}, \tag{13.29}$$

$$\frac{\dot{T}_{2t}}{T_{2t}} = B_2 T_{2t}^{\lambda_2 - 1} L_{2t}^{\lambda_2}. \tag{13.30}$$

Now the problematic strong scale effect has disappeared. At the same time, since $\lambda_1 - 1 < 0$ and $\lambda_2 - 1 < 0$, (13.29) and (13.30) show that growth peters out as long as the "diminishing returns" to learning-by-doing are not offset by an increasing labor force or an additional source (outside the model) of technical progress. If $n > 0$, we get sustained growth of the semi-endogenous type as in the Arrow model of learning-by-investing.

Yet the analysis may still be a basis for an "infant industry" argument. If the circumstances are like in case 2, temporary protection may help a backward country to enter a higher long-run path of evolution. Stiglitz underlines South Korea as an example:

> What matters is *dynamic* comparative advantage, or comparative advantage in the long run, which can be shaped. Forty years ago, South Korea had a comparative advantage in growing rice. Had

> it stuck to that strength, it would not be the industrial giant that it is today. It might be the world's most efficient rice grower, but it would still be poor (Stiglitz, 2012, p. 2).

This point is related to two different aspects of technical knowledge. On the one hand, technical knowledge is a nonrival good and this non-rivalness speaks for *openness,* thereby improving conditions for knowledge spillovers and learning from other countries. On the other hand, the potential for knowledge accumulation and internal learning by doing is different in different production sectors. And some sectors with a lot of internal learning potential and economies of scale never gets started unless to begin with they are protected from foreign competition.

13.4.3 Resource curse?

The analysis also suggests a mechanism that, along with others, may help explaining what is known as the *resource curse* problem. This problem refers to the paradox that being abundant in natural resources may sometimes seem a curse for a country rather than a blessing. At least quite many empirical studies have shown a negative correlation between resource abundance and economic growth (see, e.g., Sachs and Warner 1995, Gylfason et al., 1999).

The mechanism behind this phenomenon could be the following. Consider a mining country with an abundance of natural resources in the ground. Empirically, growth in total factor productivity in mining activity is relatively low. Interpreting this as reflecting a relatively low learning potential, the mining sector may be represented by sector 1 above. Given the abundance of natural resources, T_{1t_0} is likely to be high relative to the productivity in the manufacturing sector, T_{2t_0}. So the country is likely to be in the situation described as case 2. As a result, economic growth may never get started.

The basic problem here is, however, not of an economic nature in a narrow sense, but rather of an institutional character. Taxation on the natural resource and use of the tax revenue for public investment in growth promoting factors (infrastructure, health care, education, R&D) or directly in the sector with high learning potential can from an economic point of view circumvent the curse to a blessing. It is not the natural resources as such, but rather barriers of a political character, conflicts of interest among groups and social classes, even civil war over the right to exploit the resources, or dominance by foreign superpowers, that may be the obstacles to a sound economic development (Mehlum et al., 2006). An additional potential obstacle is related to the possible response of a country's real exchange rate, and therefore its competitiveness, to a new discovery of natural resources in a country.[12]

Summing up: Discovery of a valuable mineral in the ground in a country with weak institutions may, through corruption etc. have adverse effects on resource allocation and economic growth in the country. But: "Resources should be a blessing, not a curse. They can be, but it will not happen on its own. And it will not happen easily" (Stiglitz, 2012, p. 2).

13.5 Appendix

A. Balanced growth in the embodied technical change model with investment-specific learning

In this appendix the results (13.16), (13.17), (13.18), and (13.19) are derived. The model is:

$$Y = K^{\alpha}L^{1-\alpha}, \qquad 0<\alpha<1, \tag{13.31}$$

$$I = Y - C, \tag{13.32}$$

$$\dot{K} = qI - \delta K, \tag{13.33}$$

$$q_t = \left(\int_{-\infty}^{t} I_s ds\right)^{\lambda}, \qquad 0<\lambda\leq\bar{\lambda}, \tag{13.34}$$

$$L = L_0 e^{nt}, \qquad n \geq 0. \tag{13.35}$$

Consider a BGP. By definition, Y, K, and C then grow at constant rates, not necessarily positive. With $s = I/Y$ constant and $0 < s < 1$, (13.31) gives

$$g_I = g_Y = \alpha g_K + (1-\alpha)n, \tag{13.36}$$

a constant. By (13.33), $g_K = q\frac{I}{K} - \delta$, showing that qI/K is constant along a BGP. Hence,

$$g_q + g_I = g_K, \tag{13.37}$$

and so also g_q must be constant. From (13.34) follows that $g_q = \lambda q^{-1/\lambda}I$. Taking logs in this equation and differentiating w.r.t. t gives

$$\frac{\dot{g}_q}{g_q} = -\frac{1}{\lambda}g_q + g_I = 0,$$

in view of constancy of g_q. Substituting into (13.37) yields $(1+\lambda)g_I = g_K$, which combined with (13.36) gives

$$g_K = \frac{(1-\alpha)(1+\lambda)n}{1-\alpha(1+\lambda)},$$

[12] Ploeg (2011) provides a survey over different theories related to the resource curse problem. See also Ploeg and Venables (2012) and Stiglitz (2012).

which is (13.16). In view of $g_q = \lambda g_I = \lambda g_Y = \lambda(g_y + n) = \lambda g_K/(1+\lambda)$, the results (13.17), (13.18), and (13.19) immediately follow.

B. Big bang a hair's breadth from the AK

Here we shall prove the statement in Section 13.5.2: a hair's breadth from the AK assumption the technology is so productive as to generate infinite output in finite time.

The simple AK model as well as reduced-form AK models end up in an aggregate production function

$$Y = AK.$$

We ask the question: what happens if the exponent on K is not exactly 1, but slightly above. For simplicity, let $A = 1$ and consider

$$Y = K^{\alpha}, \qquad \alpha = 1 + \varepsilon, \ \varepsilon \gtrapprox 0.$$

Our claim is that *if* $\alpha > 1$, a constant saving rate, s, will generate infinite Y and C in finite time.

We embed the technology in a Solow-style model with $\delta = n = 0$ and get:

$$\dot{K} \equiv \frac{dK}{dt} = sK^{\alpha}, \qquad 0 < s < 1, \qquad K(0) = K_0 > 0 \text{ given.} \tag{13.38}$$

We see that not only is $\dot{K} > 0$ for all $t \geq 0$, but $\dot{K}$ is increasing over time since K is increasing. So, for sure, $K \to \infty$, but how fast?

One way of answering this question exploits the fact that $\dot{x} = x^a$ is a Bernouilli equation and can be solved by considering the transformation $z = x^{1-\alpha}$ as we do in Chapter 7 and Exercise III.3. Closely related to that method is the approach below, which may have the advantage of being somewhat more transparent and intuitive.

To find out, note that (13.38) is a separable differential equation which implies

$$K^{-\alpha} dK = s dt.$$

By integration,

$$\begin{aligned} \int K^{-\alpha} dK &= \int s dt + \mathcal{C} \ \Rightarrow \\ \frac{K^{-\alpha+1}}{1-\alpha} &= st + \mathcal{C}, \end{aligned} \tag{13.39}$$

where $\mathcal{C}$ is some constant, determined by the initial condition $K(0) = K_0$. For $t = 0$ (13.39) gives $\mathcal{C} = K_0^{-\alpha+1}/(1-\alpha)$. Consequently, the solution $K = K(t)$ satisfies

$$\frac{K_0{}^{1-\alpha}}{\alpha - 1} - \frac{K(t)^{1-\alpha}}{\alpha - 1} = st. \tag{13.40}$$

As t increases, the left-hand side of this equation follows suit since $K(t)$ increases and $\alpha > 1$. There is a $\bar{t} < \infty$ such that when $t \to \bar{t}$ from below, $K(t) \to \infty$. Indeed, by (13.40) we see that such a $\bar{t}$ must be the solution to the equation

$$\lim_{K(t)\to\infty} \left(\frac{K_0{}^{1-\alpha}}{\alpha - 1} - \frac{K(t)^{1-\alpha}}{\alpha - 1} \right) = s\bar{t}.$$

Since

$$\lim_{K(t)\to\infty} \left(\frac{K_0{}^{1-\alpha}}{\alpha - 1} - \frac{K(t)^{1-\alpha}}{\alpha - 1} \right) = \frac{K_0{}^{1-\alpha}}{\alpha - 1},$$

we find

$$\bar{t} = \frac{1}{s}\frac{K_0{}^{1-\alpha}}{\alpha - 1}.$$

To get an idea about the implied order of magnitude, let the time unit be one year and $s = 0.1$, $K_0/Y_0 = K_0^{1-\alpha} = 2$, and $\alpha = 1.05$. Then $\bar{t} = 400$ years. So the Big Bang ($Y = \infty$) would occur in 400 years from now if $\alpha = 1.05$.

As Solow remarks (Solow 1994), this arrival to the Land of Cockaigne would imply the "end of scarcity", a very optimistic perspective.

In a discrete time setup we get an analogue conclusion. With airframe construction in mind let us imagine that the learning parameter λ is slightly above 1. Then we must accept the implication that it takes only a finite number of labor hours to produce an infinite number of airframes. This is because, given the (direct) labor input required to produce the q'th in a sequence of identical airframes is proportional to $q^{-\lambda}$, the total labor input required to produce the first q airframes is proportional to $1/1 + 1/2^{\lambda} + 1/3^{\lambda} + ... + 1/k^{\lambda}$. Now, the infinite series $\sum_{k=1}^{\infty} 1/q^{\lambda}$ converges if $\lambda > 1$. As a consequence only a finite amount of labor is needed to produce an infinite number of airframes. "This seems to contradict the whole idea of scarcity", Solow observes (Solow 1997, p. 8).

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