Short Note 1. Economic Growth.

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Robustness issues and scale effects

This note adds some conceptual and empirical perspectives to the discussion in Chapter 5 and 9 in Jones and Vollrath (2013).

1 Different growth patterns

Notation: $Y = GDP$, $y \equiv Y/L$, and $g_y \equiv \dot{y}/y$; time is continuous.

Economic growth can take different forms. It can be *exponential*:

$$y_t = y_0 e^{gt}, \qquad g > 0. \tag{1}$$

Ignoring business cycle fluctuations this describes quite well what we have seen in industrialized economies since the industrial revolution (with $g \in (0.01, 0.02)$ on annual basis). When growth is exponential, the *growth rate,* $\dot{y}/y$, is a positive constant, here equal to g.

Growth can alternatively take the form of *arithmetic growth:*

$$y_t = y_0 + \alpha t, \qquad \alpha > 0. \tag{2}$$

Here $\dot{y} = \alpha$, the *momentum,* is a positive constant. So, in spite of the growth rate, $\dot{y}/y$, approaching zero for t going to infinity, we have $y_t \to \infty$ for $t \to \infty$.

More generally, growth can take the form of *quasi-arithmetic growth*:

$$y_t = y_0(1 + \alpha\beta t)^{1/\beta}, \qquad \alpha > 0,\ \beta > 0. \tag{3}$$

In the special case $\beta = 1$ and $y_0 = 1$, this is arithmetic growth. The parameter β is the *damping coefficient.* The case of *strict stagnation*, $y_t = y_0$ for all $t \geq 0$, can be interpreted as the limiting case $\beta \to \infty$.[1] On the other hand, in the limit, when $\beta \to 0$ (no damping), the growth path (3) becomes exponential growth, $y_t = y_0 e^{\alpha t}$.[2]

[1] To see this, use L'Hôpital's rule for "∞/∞" on $\ln y_t = \ln y_0 + \frac{1}{\beta}\ln(1 + \alpha\beta t)$. The term *stagnation* also covers the case of *asymptotic stagnation* where in spite of $\dot{y} > 0$ for all $t \geq 0$, $\dot{y}$ goes towards zero fast enough so that there is an upper bound, $\bar{y}$, for y_t, i.e., $y_t < \bar{y}$ for all $t \geq 0$. For instance *logistic growth* has this property. (Logistic growth is the growth path generated by the differential equation $\dot{y}_t = \alpha y_t(1 - y_t/\bar{y})$, $\alpha > 0$, $0 < y_0 < \bar{y}$.)

[2] To see this, use L'Hôpital's rule for "0/0" on $\ln y_t = \ln y_0 + \frac{1}{\beta}\ln(1 + \alpha\beta t)$.

These alternative growth patterns can be generated for alternative parameter values of essentially the same model, namely a model that leads to the differential equation

$$\dot{y}_t = \alpha y_0^{\beta} {y_t}^{1-\beta}, \qquad \alpha > 0, \beta \geq 0. \tag{4}$$

In case $\beta = 0$, (4) is a linear differential equation that has the solution (1) with $g = \alpha$, which is exponential growth. In case $\beta > 0$, (4) is an autonomous Bernoulli equation that has the solution (3), which is quasi-arithmetic growth.[3] For alternative values of β between 0 and infinity, quasi-arithmetic growth covers the whole range between exponential growth and strict stagnation. We rule out the case of $\beta < 0$ which would imply that the model could only temporarily describe reality, because $\beta < 0$ leads to *explosive* growth: y_t approaching infinity in *finite* time (the "end of scarcity").

Several prominent macroeconomists, e.g., Lawrence Summers, Robert Gordon, and our own Charles Jones, predict that economic growth in the future will be lower than what we have seen in the 20th century. One of the reasons emphasized by Jones and others is the slowdown of population growth and thereby, everything else equal, dampening of growth of the source of new ideas. Along this line, in a coming exercise you will be asked to show what long-run growth pattern the horizontal innovations model with $\varphi < 1$ and $n = 0$ implies.

2 The term "endogenous growth" and all that

How terms like "endogenous growth" and "semi-endogenous growth" are defined varies in the literature. In this course we use the following definitions. A model features:

endogenous growth if $y_t \to \infty$ for $\to \infty$, and the source of this evolution is some *internal* mechanism in the model (rather than exogenous technology growth);

fully-endogenous growth if growth is endogenous in such way that $y_t \to \infty$ for $\to \infty$ occurs even if there is no support by growth in any exogenous factor;[4]

[3] It is clear that with $0 < \beta < \infty$, the solution formula (3) can not be extended, without bound, *backward* in time. For $t = -(\alpha\beta)^{-1} \equiv \bar{t}$, we get $y_t = 0$, and thus, according to (4), $y_t = 0$ for all $t \leq \bar{t}$. This should not, however, be considered a necessarily problematic feature. A certain growth regularity need not be applicable to all periods in history. It may apply only to specific historical epochs characterized by a particular institutional environment.

[4] An alternative name for this case is *strictly endogenous growth.*

semi-endogenous growth if growth is endogenous in such way that $y_t \to \infty$ for $\to \infty$ occurs only if the growth path is supported by growth in *some* exogenous factor (for example exogenous growth in the labor force).

If in the above three cases, the weak growth criterion "$y_t \to \infty$ for $\to \infty$" is replaced by exponential growth, then we speak of *endogenous, fully-endogenous,* and *semi-endogenous exponential growth,* respectively. If instead the weak growth criterion is replaced by, for instance, arithmetic growth, we speak of *endogenous, fully-endogenous,* and *semi-endogenous arithmetic growth,* respectively,

An example of fully endogenous exponential growth is the endogenous growth generated in the Romer case ($\varphi = 1, n = 0$) of the horizontal innovations model. An example of semi-endogenous exponential growth is the Jones case ($\varphi < 1, n > 0$) of the horizontal innovations model.

When Romer's case is combined with Ramsey households, we get steady-state results of the following kind: $\partial g_y^*/\partial \rho < 0$ and $\partial g_y^*/\partial \theta < 0$ (standard notation). That is, preference parameters matter for long-run growth. This suggests, at least at the theoretical level, that taxes and subsidies, by affecting incentives, may have effects on long-run growth.

In any case, fully-endogenous exponential growth is technologically possible if and only if there are *non-diminishing returns* (at least asymptotically) *to the producible inputs* in the growth-generating sector(s), also called the *growth engine.* The growth engine in an endogenous growth model is defined as the set of input-producing sectors or activities using their own output as input. This set may consist of only one element, for instance the R&D sector in the horizontal innovations model, the manufacturing sector in the simple AK model, and the educational sector in the Lucas (1988) model. A model is capable of generating fully-endogenous exponential growth if the growth engine has *CRS w.r.t. producible inputs.*

No argument, however, like the replication argument for CRS w.r.t. the *rival* inputs exists regarding CRS w.r.t. the *producible inputs.* This theoretical limitation, combined with strong empirically founded skepticism, motivated Jones to introduce his *semi-endogenous* version of the horizontal innovations model (Jones1995a, 1995b), where $\varphi < 1, n > 0$. In that version, in the long run

$$g_y = g_k = g_c = \frac{n}{1 - \varphi} \equiv g_y^*. \tag{5}$$

If a certain degree, ξ, of R&D overlap is added, $0 \leq \xi < 1$, we instead get $g_y^* = (1 - \xi)n/(1-\varphi)$.[5]

So, in this case, if and only if $n > 0$, can a positive constant per capita growth rate be maintained forever. Only when the R&D outcome is *assisted* by growth in the exogenous *source* of ideas, population, is the growth engine strong enough to maintain exponential growth. The key role of population growth derives from the fact that at the aggregate level there are increasing returns to scale w.r.t. capital, labor, *and* knowledge. For the increasing returns to be sufficiently exploited to generate exponential growth, population growth is needed. Note that if the Jones case is combined with Ramsey households, we get $\partial g_y^*/\partial\rho = 0 = \partial g_y^*/\partial\theta$, that is, preference parameters do not matter for *long-run* growth (only for the *level* of the growth path, see Section 4 below). This suggests that taxes and subsidies do not have *long-run* growth effects. Yet, in the Jones model and similar semi-endogenous growth models, economic policy can have important permanent *level* effects. Moreover, the only temporary growth effects can be quite durable because the speed of convergence is low (see Jones, 1995a).

Strangely enough, some textbooks (for example Barro and Sala-i-Martin, 2004) do not call much attention to the distinction between fully-endogenous growth and semi-endogenous growth (and even less attention to the distinction between exponential growth and weaker forms of growth). Rather, they tend to use the term "endogenous growth" as synonymous with what we here call "fully-endogenous exponential growth". But there is certainly no reason to rule out *apriori* the parameter cases corresponding to semi-endogenous growth.

In the Acemoglu textbook (Acemoglu, 2009, p. 448), "semi-endogenous growth" is defined or characterized as endogenous growth where the long-run per capita growth rate of the economy "does not respond to taxes or other policies". As an implication, endogenous growth which is not semi-endogenous is in Acemoglu's text implicitly defined as endogenous growth where the long-run per capita growth rate of the economy *does* respond to taxes or other policies.

We have defined the distinction between "semi-endogenous growth" and "fully-endogenous growth" in a different way. In our terminology, this distinction does not coincide with the distinction between policy-dependent and policy-invariant growth. Indeed, in our ter-

[5] Of course the model shifts from featuring "semi-" to featuring "fully-endogenous" exponential growth if the model is extended with an internal mechanism *determining* the population growth rate. Jones (2003) takes steps towards such a model.

minology positive per capita growth may rest on an "exogenous source" in the sense of deriving from exogenous technical progress and yet the long-run per capita growth rate may be policy-dependent. In Chapter 16 of the lecture notes we will see an example in connection with the Dasgupta-Heal-Solow-Stiglitz model, also known as the DHSS model.

There also exist models that according to our definition feature *semi*-endogenous growth and yet the long-run per capita growth rate is policy-*dependent* (Cozzi, 1997; Sorger, 2010). Similarly, there exist models that according to our definition feature fully-endogenous exponential growth and yet the long-run per capita growth rate is policy-*invariant* (some learning-by-doing models have this property).

Before proceeding, a word of warning. The distinction between "exogenous" and "endogenous" growth is only meaningful within a *given meta-theoretical framework*. It is always possible to make the meta-theoretical framework so broad that the per capita growth rate *must* be considered endogenous within that framework. From the perspective of society as a whole we can imagine many different political and institutional structures − as witnessed by long-run historical evolution − some of which clearly are less conducive to economic growth than others. From this broad point of view, growth is always endogenous.

3 Robustness of endogenous growth models

The horizontal innovations model illustrates the fact that endogenous growth models with exogenous population typically exist in two varieties or cases. One is the fully-endogenous growth case where a particular value is imposed on a key parameter in the growth engine. This value is such that there are constant returns (at least asymptotically) to *producible* inputs in the growth engine of the economy. In the "corresponding" semi-endogenous growth case, the key parameter is allowed to take any value in an open interval. The endpoint of this interval appears as the "knife-edge" value assumed in the fully-endogenous growth case.

Although the two varieties build on qualitatively the same mathematical model of a certain growth mechanism (say, research and development or learning by doing, to be considered later in the course), the long-run results turn out to be very sensitive to which of the two cases is assumed. In the fully-endogenous growth case a positive per-capita growth rate is maintained forever without support of growth in any exogenous factor. In the semi-endogenous growth case, the growth process needs "support" by some growing exogenous factor in order for sustained growth to be possible. The established terminology is somewhat seductive here. "Fully endogenous" sounds as something going much deeper than "semi-endogenous". But nothing of that sort should be implied. It is just a matter of different parameter values.

As Solow (1997, pp. 7-8) emphasizes in connection with learning-by-investing models (with constant population), the knife-edge case assumed in the fully-endogenous growth versions is a very special case, indeed an "extreme case, not something intermediate". A value slightly above the knife-edge value leads to explosive growth: infinite output in finite time even when $n = 0$. And a value slightly below the knife-edge value leads to growth petering out in the long run when $n = 0$.

Whereas the strength of the semi-endogenous growth case is its theoretical and empirical robustness, the convenience of the fully-endogenous growth case is that it has much simpler dynamics. Then the question arises to what extent a fully-endogenous growth model can be seen as a useful approximation to its semi-endogenous growth "counterpart". Imagine that we contemplate applying the fully-endogenous growth case as a basis for making forecasts or for policy evaluation in a situation where the "true" case is the semi-endogenous growth case. Then we would like to know: Are the impulse-response functions generated by a shock in the fully-endogenous growth case an *acceptable approximation* to those generated by the same shock in the corresponding semi-endogenous growth case for *a sufficiently long time horizon to be of interest*?[6] The answer is "yes" if the critical parameter has a value "close" to the knife edge value and "no" otherwise. How close it need be, depends on circumstances. My own tentative impression is that usually it is "closer" than what the empirical evidence warrants.

Even if a single growth-generating mechanism, like learning by doing, does not in itself seem strong enough to generate a reduced-form AK model (the fully-endogenous growth case), there might exist complementary factors and mechanisms that in total could generate something close to a reduced-form AK model. The time-series test by, for instance, Jones (1995b) and Romero-Avila (2006), however, reject this.[7]

Comment on "growth petering out" when $n = 0$ The above-mentioned "petering out" of long-run growth in the semi-endogenous case when $n = 0$ takes different forms in

[6] Obviously, the ultimate effects of the shock tend to be very different in the two models.

[7] For an opposite view, see Kocherlakota and Yi (1997). There is a longstanding discussion about these time-series econometric issues. See the course website under Supplementary Material.

different models. When exponential growth cannot be sustained in a model, sometimes it remains true that nevertheless $y \to \infty$ for $t \to \infty$, for instance in the form of quasi-arithmetic growth, and sometimes instead asymptotic stagnation results.[8]

Another issue is whether there exist factors that in spite of $n = 0$ (or, to be more precise, in spite of n decreasing, possibly to zero as projected by the United Nations (2013) to happen within a century from now) may *replace* the growth-supporting role of population growth under semi-endogenous parameter conditions like $\varphi < 1$. Both urbanization and the evolution of digital information and communication technologies seem likely for a long time to at least help in that direction.

4 Weak and strong scale effects

The distinction between weak and strong scale effects is important. In the Romer case ($\varphi = 1, n = 0$) of the horizontal innovations model there a *strong scale effect:*

$$\frac{\partial g_y^*}{\partial L} > 0. \tag{6}$$

Interpreting the size ("scale") of the economy as measured by the size, L, of the labor force, we call such an effect a *strong scale effect,* that is, "scale" has an effect on the long-run *growth rate.* This kind of scale effect has clearly been rejected by the empirics, cf. Jones and Vollrath (2013, p. 106).

Scale effects can be of a less dramatic form. In this case we speak of a *weak scale effect* or a *scale effect on levels.* This form arises when φ is less than 1. We see from (5) that in the Jones case ($\varphi < 1, n > 0$) of the horizontal innovations model, the steady state growth rate is independent of the *size* of the economy. Consequently, in Jones' version there is no strong scale effect. Yet there is a scale effect on *levels* unless $\varphi = 0$. If $\varphi > 0$, the scale effect is positive in the sense that along a steady state growth path, $(y_t^*)_{t=0}^{\infty}$,

$$\frac{\partial y_t^*}{\partial L_0} > 0, \tag{7}$$

cf. Exercise VII.7.

The result (7) says the following. Suppose we consider two closed economies characterized by the same parameters, including the same $n > 0$ and the same $\varphi \in (0, 1)$. The economies differ only w.r.t. initial size of the labor force. Suppose both economies are

[8] See Groth et al., 2010.

in steady state. Then, according to (7), the economy with the larger labor force has, for all t, larger output per unit of labor. The background is the increasing returns to scale w.r.t. capital, labor, *and* technical knowledge, which in turn is due to *technical knowledge being a non-rival good* – its use by one firm does not (in itself) limit the amount of knowledge available to other firms.[9] In a large economic system, say an integrated set of open economies, *more* people benefit from a given increase in knowledge than in a small economic system. At the same time the per capita cost of creating the increase in knowledge is less in the large system than in the small system.

The scale effect on levels displayed by (7) can be shown to be increasing in the parameter φ, which measures the elasticity of the economy-wide R&D productivity w.r.t. the stock of knowledge. When $\varphi \to 1$, the scale effect becomes more and more powerful. In the limit it ends up as a scale effect on the growth rate, as in the Romer case.

5 Discussion

Are there good theoretical and/or empirical reasons to believe in the existence of (positive) scale effects on levels or perhaps even on growth in the long run?

Let us start with some theoretical considerations.

5.1 Theoretical aspects

From the point of view of theory, we should recognize the likelihood that offsetting forces are in play. On the one hand, there is the problem of *limited natural resources*. For a given level of technology, if there are CRS w.r.t. capital, labor, *and* land (or other natural resources), there are diminishing returns to capital and labor taken together. In this *Malthusian* perspective, an increased scale (increased population) results, everything else equal, in lower rather than higher per capita output, that is, a negative scale effect should be expected.

On the other hand, there is the *anti-Mathusian* view that repeated improvements in technology tend to overcome, or rather *more* than overcome, this Malthusian force, if appropriate socio-economic conditions are present. Here the theory of endogenous technical change comes in by telling us that a large population may be good for technical

[9] By patent protection, secrecy, and copyright some aspects of technical knowledge are sometimes *partially* and *temporarily excludable,* but that is another matter.

progress if the institutions in society are growth-friendly. A larger population breeds more ideas, the more so the better its education is; a larger population also promotes division of labor and larger markets. This helps the creation of new technologies or, from the perspective of an open economy, it helps the local adoption of already existing technologies outside the country. In a less spectacular way it helps by furthering day-by-day productivity increases due to learning by doing and learning by watching. The non-rival character of technical knowledge is an important feature behind all this. It implies that output per capita depends on the *total* stock of ideas, not on the stock per person. This implies − everything else equal − an advantage of scale.

In the models considered so far in this course, natural resources and the environment have been more or less ignored. Here only a few remarks about this limitation. The approach we have followed is intended to clarify certain *mechanisms* − in abstraction from numerous things. The models in focus have primarily been about aspects of an industrialized economy. Yet the natural environment is always a precondition. A tendency to positive scale effects on levels *may* be more or less counteracted by *congestion* and aggravated *environmental problems* ultimately caused by increased population and a population density above some threshold.

What can we say from an *empirical* point of view?

5.2 Empirical aspects

First of all we should remember that in view of cross-border diffusion of ideas and technology, a positive scale effect (whether weak or strong) should not be seen as a prediction about individual countries, but rather as pertaining to larger regions, nowadays probably the total industrialized part of the world. So cross-country regression analysis is not the right framework for testing for scale effects, whether on levels or the growth rate. The relevant scale variable is not the size of the country, but the size of a larger region to which the country belongs, perhaps the whole world; and multivariate time series analysis seems the most relevant approach.

Since in the last century there has been no clear upward trend in per capita growth rates in spite of a growing world population (and also a growing population in the industrialized part of the world separately), most economists do not believe in *strong* scale effects. But on the issue of *weak* scale effects the opinion is definitely more divided.

Considering the *very*-long run *history* of population and per capita income of different regions of the world, there clearly exists evidence in favour of scale effects (Kremer, 1993). Whether advantages of scale are present also in a contemporary context is more debated. Recent econometric studies supporting the hypothesis of positive scale effects on levels include Antweiler and Trefler (2002) and Alcalá and Ciccone (2004). Finally, considering the economic growth in China and India since the 1980s, we must acknowledge that this impressive performance at least does *not* speak *against* the existence of positive scale effects on levels.

Acemoglu seems to find positive scale effects on levels plausible at the theoretical level (pp. 113-114). At the same time, however, later in his book he seems somewhat skeptical as to the existence of empirical support for this. Indeed, with regard to the fact that R&D-based theoretical growth models tend to generate at least weak scale effects, Acemoglu claims: "It is not clear whether data support these types of scale effects" (Acemoglu, 2009, p. 448).

My personal view on the matter is that we should, of course, recognize that offsetting forces, coming from our finite natural environment, are in play and that a lot of uncertainty is involved. Nevertheless it seems likely that at least up to a certain point there are positive scale effects on levels.

5.3 Policy implications

If this holds true, it supports the view that international economic integration is generally a good idea. The concern about congestion and environmental problems, in particular global warming, should probably, however, preclude recommending governments and the United Nations to try to *promote* population growth.

Moreover, it is important to remember the distinction between the global and the local level. The n in the formula (5) refers to a much larger region than a single country; we may refer to this region as "the set of knowledge-producing countries in the world". No recommendation of higher population growth in a single country is implied by this theoretical formula. When discussing economic policy from the perspective of a single country, all aspects of relevance in the given local context should be incorporated. For a developing country with limited infrastructure and weak educational system, family-planning programs and similar may in many cases make sense from both a social and a productivity point of view (cf. Dasgupta, 1995).

6 References

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