Written exam for the M. Sc. in Economics. Summer 2012

Economic Growth

Master's Course

June 15, 2012

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The weighting of the problems is: Problem 1: 20%, Problem 2: 50%, and Problem 3: 30%.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 Consider a pre-industrial economy described by:

$$\begin{aligned} Y_t &= A_t^{\sigma} L_t^{\alpha} Z^{1-\alpha}, \qquad \sigma > 0, 0 < \alpha < 1, && (1)\\ \dot{A}_t &= \lambda A_t^{\varepsilon} L_t, \qquad \lambda > 0, 0 < \varepsilon < 1, \qquad A_0 > 0 \text{ given}, && (2)\\ L_t &= \frac{Y_t}{\bar{y}} \equiv \varphi Y_t, \qquad \bar{y} > 0, && (3) \end{aligned}$$

where Y is aggregate output, A the level of technical knowledge, L the labor force (= population), and Z the amount of land (fixed). Time is continuous and it is understood that a kind of Malthusian population mechanism is operative behind the scene; that is, (3) should be seen as a short-cut.

a) Interpret the equations, including $\bar{y}$.

From now, let $Z = 1$.

b) Show that the dynamics of the model reduce to the "law of motion":

$$\dot{A}_t = \hat{\lambda} A_t^{\varepsilon + \frac{\sigma}{1-\alpha}}, \qquad \text{where } \hat{\lambda} \equiv \lambda \varphi^{\frac{1}{1-\alpha}}.$$

c) Define $\mu \equiv \varepsilon + \frac{\sigma}{1-\alpha}$. Suppose $\mu > 1$. Find the evolution of A_t. *Hint:* let $x \equiv A^{1-\mu}$ and consider the implied differential equation for x, applying that a differential equation $\dot{x}_t + a x_t = b$, with $a \neq 0$ and x_0 given, has the solution:

$$x_t = (x_0 - x^*)e^{-at} + x^*, \text{ where } x^* = \frac{b}{a}.$$

d) Can we from this analysis infer anything about the possibility of persistence of the Malthusian regime as described by the model? Why or why not?

Problem 2 Consider a closed market economy with N profit maximizing firms, operating under perfect competition (N "large"). The size of the labor force (= employment = population) is a constant, L. Aggregate output at time t is Y_t per time unit. Output is used for consumption, $C_t \equiv c_t L$, and investment in physical capital K_t so that $\dot{K}_t = Y_t - C_t - \delta K_t$, where $K_0 > 0$ is given and $\delta \geq 0$ is the rate of physical decay of capital. The initial value $K_0 > 0$ is given. There is a perfect market for loans with a short-term real interest rate r_t. There is no uncertainty (perfect foresight).

The production function for firm i ($i = 1, 2, ..., N$) is

$$Y_{it} = F(K_{it}, A_t L_{it}),$$

where F is neoclassical and has CRS. The variable A_t evolves according to

$$A_t = K_t,$$

where $K_t = \sum_i K_{it}$.

a) Briefly interpret the assumption that $A_t = K_t$ as a special case of some general hypothesis.

Suppose each firm is small relative to the economy as a whole and perceives it has no influence on aggregate variables.

b) In general equilibrium, determine r and the aggregate production function at time t.

Let the household sector be Ramsey-style with inelastic labor supply, instantaneous CRRA utility of per capita consumption with parameter $\theta > 0$, and a constant utility discount rate $\rho > 0$. Finally, we assume that $\rho > (1 - \theta)g$, where g is the equilibrium growth rate of c.

c) Determine g. What parameter restriction is needed to ensure $g > 0$? From now, assume this restriction satisfied.

d) Determine the equilibrium growth rate for k ($\equiv K/L$) and y, respectively. *Hint:* In answering you may refer to general knowledge about a certain class of models provided you have shown that the present model belongs to this class.

e) What would happen if the population were growing? Why?

We introduce a government which contemplates (i) to pay an investment subsidy $\sigma \in (0, 1)$ to the firms so that their capital costs are reduced to $(1 - \sigma)(r + \delta)$ per unit of capital per time unit; (ii) to finance this subsidy by a consumption tax rate, τ.

Suppose you, as an economic advisor, are asked by the government to suggest an optimal size of σ, given that the social welfare function coincides with the criterion function of the representative household.

f) Derive a formula for the recommendable size of σ. *Hint:* Set up the social planner's problem, derive the first-order conditions and the transversality condition. Determine the implied growth rate of c_t. Next, use your general knowledge about a certain class of models to determine the growth rates of k_t and y_t (a brief verbal account is enough). Finally, use that for σ to be optimal, σ should ensure that the net rate of return on saving faced by the consumer equals the net rate of return to capital investment implied by the aggregate production technology.

g) Assume that the government always balances the budget and has no other expenditures than the investment subsidy. Find the consumption tax rate, τ, needed to balance the budget. *Hint:* At a certain stage in the argument you will need knowledge about what value is taken by c/k in the social planner's solution. You do not have to derive this value; it is given here: $c/k = F(1, L) - \delta - g_{SP}$.

h) Given the model, *is* the suggested policy (σ, τ) optimal or might there for example be distortionary effects associated with the financing? Discuss.

i) Whatever the answer to h), briefly suggest other subsidy policies which might do the job.

j) Briefly evaluate the model.

Problem 3 *Short questions*

a) In the theory of countries' technology catching-up, one suggested hypothesis is that

$$\frac{\dot{A}_t}{A_t} = \xi \frac{\tilde{A}_t}{A_t}, \qquad \xi > 0,$$

where A_t is the technology level of the considered country and $\tilde{A}_t$ is the world frontier technology level. A further suggested hypothesis is that the ability to catch-up is an increasing function of average human capital, h, in the country, i.e., $\xi = \varphi(h)$, $\varphi' > 0$. Briefly discuss what effect in this context a general health improvement in the country may have in the longer run.

b) In poor countries the capital intensity measured as K/L tends to be much lower than in rich countries. Can we, within a neoclassical framework of diminishing marginal productivity of capital, explain why capital flows from rich to poor countries are not much larger than they are? Why or why not?

c) In the Acemoglu textbook a model where long-run productivity growth is driven by a combination of physical and human capital accumulation is presented. Set up the aggregate production function of the model. Briefly comment on what you think are strengths and/or weaknesses of the model.

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