Protection for Sale to Oligopolists*

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Abstract

In Grossman and Helpman’s (1994) canonical "Protection for Sale" (PFS) model political competition between industry lobbies is purely driven by their interests as consumers. This paper introduces demand linkages and oligopolistic competition into PFS framework to address the rivalry among lobbies due to product substitutability. It shows that increased substitutability weakens the interest groups’ incentives to lobby and reduces tariff distortions. This may explain why empirical tests of PFS find surprisingly little impact of lobbies on the government trade policy decision. The paper also analyzes endogenous lobby formation, suggesting that demand linkages may adversely affect industry decision to get organized.

Keywords: Lobbying, Endogenous Trade Policy, Substitutability

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1 Introduction

How do organized lobby groups affect trade policy? While there are a number of theoretical explanations suggested in the literature (see e.g. Rodrik (1995) for a review), Grossman and Helpman’s (1994) "Protection for Sale" model (henceforth PFS) is the most influential by far. PFS explicitly describes a mechanism through which interest groups’ contributions influence the policy-maker’s decision for trade protection, providing micro-foundations for the previous approaches. In addition, the model’s prediction for the equilibrium protection pattern relates the industry’s trade tariff to a number of observable variables, thereby providing a coherent framework for empirical testing.

As is well known, PFS neglects some important issues. In particular, it abstract from strategic market interactions and production linkages. Indeed, the demand structure in PFS implies that lobby groups do not behave strategically in the product markets. Also, factor-specific production eliminates any competition between different lobbies in the factor market. Hence, their interests only diverge to the extent that each industry wants to increase its profits by raising the price of its own good, while all other organized industries aim at reducing the same price in order to increase their members’ consumer surplus. That is, political rivalry among the lobbies arises purely from the desire of the members of different industry lobbies to defend their interests as consumers, as is recognized by Grossman and Helpman (p.849). ¹

This paper adds a more realistic justification for the political competition among the organized groups by introducing demand linkages and oligopolistic market structure into the PFS framework. More precisely, it addresses the rivalry between the lobbies that arises due to the substitutability between goods. The paper studies the impact of these demand-side interactions on the determination of trade policy, the intensity of inter-industry lobbying competition and lobby formation.

In the original PFS model the consumers’ utility function is separable and quasi-linear, implying that the demand for each good is independent of the prices of the other goods. Besides, the small open economy assumption implies that the producers face perfectly elastic demand. Thus, the prices in other industries do not affect the producers’ incentives to lobby. To address the inter-industry rivalry in lobbying we relax these assumptions and allow for both demand linkages and imperfectly competitive industries. More precisely, we consider a utility function that admits cross-price effects on demand, and assume that each good is produced by an international oligopoly and sold in internationally segmented markets. The first part of the paper shows that the presence of substitutes may reduce interest group incentives to lobby due to competition in the goods market. If demands are interdependent, an increase in the price

¹See e.g. Baldwin and Robert-Nicoud (2006) for further discussion of concerns caused by this feature of PFS.
of a good causes its demand to shift towards the substitutes. A decrease in the price of the substitute has a similar effect. The interest group takes this into account when lobbying the government to increase the price of its own good (as a producer receiving profit from selling this good) and reduce the price of all other goods (as a group of consumers). Therefore, the lobbying strategy of an organized industry becomes less aggressive. As a result, with an increase in the degree of substitutability, the protection of the organized industries falls, and the protection of the non-organized industries increases, relative to the first-best benchmark. That is, other things equal, the trade tariffs in economies producing more substitutable products (or having a more competitive industry structure) should be closer to the socially optimal levels.

This result suggests a new explanation for the known puzzle of the empirical studies of the PFS model. These studies commonly find that the government puts unexpectedly low weight on the lobby contributions relative to the welfare loss. That is, the interest groups are found to have surprisingly little impact on the government trade policy decision, which causes a concern about the empirical significance of the PFS model. E.g., Gawande and Krishna (2003) write that "it is enough to cast doubt on the value of viewing trade policy determination through this political economy lens".

Existing explanations for this finding can be roughly classified in two groups. The first group argues that direct empirical testing of PFS model neglects the presence of other interest groups whose objectives are opposed to the objectives of organized groups in PFS. The conflicting objectives imply that the lobbying efforts of competing interest groups would offset each other, leading to less protection. For example, Gawande and Bandyopadhyay (2000) introduce political competition between the upstream and downstream producers, and Gawande, Krishna and Robbins (2006) introduce counter-lobbying by the foreign organized groups. The second view attributes the puzzling finding to overestimated substitutability between domestic and foreign goods within the same industry. Facchini et al. (2010) argue that the original PFS model considers domestic and foreign goods to be homogenous. However, if the domestic and foreign goods are imperfect substitutes, domestic organized groups would be less interested in protection, which leads to lower trade barriers. This paper provides an alternative explanation, suggesting that smaller deviations from the first-best protection rates may result from product substitutability between the industries. We argue that the empirical tests of the original PFS setting underestimate the cross-product substitutability. The threat of losing demand to a substitute-producing industry, not accounted for in PFS, weakens organized groups’ incentives to lobby and leads to less protection.

The second part of the paper addresses the impact of product substitutability on the incentives to form

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2 E.g. Goldberg and Maggi (1999); Gawande and Bandyopadhyay (2000).
a lobby. The original PFS model assumes an exogenous lobby group structure. A natural extension is to endogenize the lobby formation. However, in the PFS model, the interests of different industry groups are opposed to each other. So if an additional lobby formation stage is introduced in their model, all industries would likely get organized (in the absence of the lobby formation costs).

However, in the presence of demand linkages, at a sufficiently high degree of substitution a non-organized industry becomes protected without paying for it. The reason is that the organized industry cannot lobby to increase its own price without losing a substantial part of its consumers switching to the cheaper (but similar) good. For the same reason, it cannot lobby for lowering the price of the substitute. Thus, the non-lobbying industry gets a free ride on the lobbying industry efforts. If instead both these substitute-producing industries are organized, they both contribute to the government for (potentially higher) protection. Comparing these two outcomes, we demonstrate that the industry may better off not being organized. As a result, with endogenous lobby formation, fewer industries get organized and lobbying becomes less intense.

There are several studies that extend the PFS framework to address the political rivalry among the organized groups. Gawande and Bandyopadhyay (2000) introduce supply-side interactions through a single importable intermediate input. Cadot, de Melo and Olarreaga (2001) strengthen the degree of inter-industry interactions even further, assuming that the industries compete for a common scarce resource and each good is produced using other goods as intermediate inputs. These extensions help obtain predictions consistent with the observed stylized facts, e.g., escalation of protection rates with the degree of processing, or higher average protection for poor countries. However, to our knowledge, this paper is the first one to study the rivalry arising from the demand-side interactions. A related paper, Bombardini and Trebbi (2009), considers oligopolistic markets with differentiated products and discusses the lobbying incentives arising from product substitutability. Yet, their setting differs from the PFS approach. In particular, in their paper the organized groups can only lobby either for the own tariff or for economy-wide one. This restriction limits the possibility of addressing the political competition among the lobbies. In addition, their model relies on Bertrand competition, implying prohibitive tariffs and zero imports of differentiated goods. While the logic of our modelling approach works also in the case of price competition, most of our results are based on Cournot competition, thereby avoiding such an extreme prediction.

This paper is not the first to endogenize lobby formation in the PFS model or discuss the possibility

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3 Similarly to PFS, we neglect any intra-industry organizational conflict.
4 Chang (2005) incorporates monopolistic competition into the PFS framework, but in her setup there is substitutability only across varieties within the same sector, and no demand linkages across sectors.
of free-riding. Mitra (1999) adds an initial stage to the PFS model, letting the owners of each specific factor decide in Nash equilibrium whether it is profitable to incur a fixed cost of forming a lobby. Magee (2002) employs a two-stage game, where in the first stage, industry representatives and the policy maker determine the tariff schedule and, in the second stage, every firm in the industry decides whether to contribute to the lobbying effort (defecting is infinitely punished). Bombardini (2008) looks at the lobbying incentives of individual firms, again assuming a fixed cost of lobby participation. These papers address the collective action problem at the intra-industry level. We, instead, relate lobby formation to the inter-industry demand links, thereby avoiding the issue of exogenous fixed costs. As mentioned above, Bombardini and Trebbi (2009) also consider the impact of inter-industry links on lobbying organization. However, they study the incentives for joint vs. individual lobbying rather then endogenous lobby formation.

The remainder of the paper is organized as follows: Section 2 describes the model setup, Section 3 discusses the equilibrium structure of protection in the presence of demand linkages, Section 4 analyzes the effect of the degree of substitution on the extent of protection, Section 5 studies endogenous lobby formation and Section 6 concludes. All derivations and proofs are relegated to the appendix.

2 The model

Consider an economy populated by individuals of total mass 1. The individuals have identical preferences represented by the quasi-linear utility function

\[ U(x_0, x_1, ..., x_m) = x_0 + \hat{U}(x_1, ..., x_m), \tag{1} \]

where \( x_i \) denotes consumption of good \( i \), and there are \( m + 1 \) goods in the economy. This specification allows for the cross-price effects between goods \( 1, ..., m \), which constitutes the main difference with the original PFS model.\(^5\) More precisely, we assume the quadratic sub-utility function:

\[ \hat{U}(x_1, ..., x_m) = \sum_{k=1}^{m} x_k - \frac{1}{2} \sum_{k=1}^{m} x_k^2 - \sigma \sum_{k=1}^{m} \sum_{j=k+1}^{m} x_k x_j, \tag{2} \]

where \( \sigma \in [0, 1] \) reflects the substitutability between goods \( x_1, ..., x_m \).

This functional form (2) yields a familiar linear inverse demand function for each good \( i = 1, ..., m \)

\[ p_i = 1 - x_i - \sigma \sum_{j \neq i} x_j, \]

where \( p_i \) denotes the domestic price of good \( i \). For \( \sigma \in (0, 1) \), considering only interior solutions, the

\(^5\)The original PFS model uses the separable utility function \( U(x_0, x_1, ..., x_m) = x_0 + \sum_{i=1}^{m} \hat{u}_i(x_1, ..., x_m) \), which implies that demand functions are independent of the prices of the other goods.
resulting domestic demand function for good \( i \) is linear as well

\[
d_i(p_1, ..., p_m) = \frac{1}{((m-1)\sigma + 1)(1-\sigma)} \left( (1-\sigma) - ((m-2)\sigma + 1) p_i + \sigma \sum_{j \neq i} p_j \right).
\] (3)

The demand for good \( i \) decreases in its own price and increases in prices of goods \( 1, 2, ..., i-1, i+1, ..., m \), so good \( i \) and goods \( 1, 2, ..., i-1, i+1, ..., m \) are (imperfect) substitutes.

Due to the quasi-linearity of the utility function, any income effect is totally captured by the consumption of good 0, which is chosen to be a numeraire. As a result, the associated indirect utility function for each individual is

\[
V(p) = E - \sum_{k=1}^{m} p_k d_k(p) + \hat{U}(d_1(p), ..., d_m(p)),
\]

where \( p \) is the vector of domestic prices \( (p_1, ..., p_m) \) and \( E \) is total expenditure.

All goods are produced using a single factor, labor. The numeraire good 0 is produced with an input-output coefficient of 1 and freely traded in a perfectly competitive international market. As a result, the wage rate in this economy is equal to 1. The other \( m \) goods are produced by a CRS technology as well, but are sold at internationally segmented oligopolistic markets. More precisely, good \( k \) is supplied by \( n_k > 0 \) identical domestic firms and \( n_k^* > 0 \) identical foreign firms, which compete in quantities. The total number of firms operating in sector \( k \) is given by \( N_k = n_k + n_k^* \).

It takes \( c_k \) units of labor to produce one unit of good \( k \), both at home and abroad.\(^6\) We assume that each firm is a pure profit maximizer and that the number of firms is fixed, so that no entry or exit decisions are taken. The government in the economy sets trade taxes and/or subsidies on the goods \( i = 1, ..., m \). Resulting net revenue is redistributed equally among the voting population.

For each non-numeraire good, the domestic and the foreign market are segmented and production is characterized by the constant marginal costs. As a result, firms’ production decisions are made separately for the domestic and the international market. Thus, the government can set import and export trade policies separately. In this paper, we concentrate on the government decision about import tariffs, which determines the production and consumption in the domestic market. We also assume that the foreign government does not impose any export tariffs on its firm, so that in the absence of trade intervention of the domestic government, firms at home and abroad are equally efficient. As we do not study the strategic interaction between the domestic and foreign governments in setting trade policy, this assumption does not affect the model’s qualitative results.

Denote the domestic import tariff in sector \( k \) by \( \tau_k \), and the vector of the import tariffs \( (\tau_1, ..., \tau_m) \) by

\(^6\)The assumption about symmetric costs at home and abroad is made for computational convenience. It does not change the predictions of the model.
Then, in Cournot-Nash equilibrium, each of the \( n_k \) identical domestic firms in sector \( k \) chooses the same output level \( q_k(\tau) \). Similarly, each of the \( n^*_k \) foreign firms in sector \( k \) chooses the same production level \( q^*_k(\tau) \). The resulting profit levels are given by \( \pi_k(\tau) \) for each domestic firm in industry \( k \), and \( \pi^*_k(\tau) \) for each foreign firm, respectively. Note also that the market clearing condition implies that

\[
d_k(p(\tau)) = n_k q_k(\tau) + n^*_k q^*_k(\tau).
\]

Each individual is endowed with some labor and may also own some claims to the profit of a firm in at most one industry. These claims are indivisible and non-tradable. Thus, individual income is the sum of wages, government transfers and possibly claims to a domestic firm’s profit.

The owners of firms in the same industry may choose to organize and form a lobby group trying to influence the government in its decision about trade policies. The joint welfare of the members of such a group \( i \) comprising share \( a_i \) of the total population is

\[
W_i(\tau) = l_i + n_i \pi_i(\tau) + a_i \left[ \sum_{k=1}^{m} \tau_k n^*_k q^*_k(\tau) - \sum_{k=1}^{m} p_k d_k(p(\tau)) + \tilde{U}(d_1(p(\tau)), ..., d_m(p(\tau))) \right],
\]

where \( l_i \) denotes the total labor endowment of group \( i \) members, and the first sum in the brackets represents the tax revenue collected by the government. Each lobby \( i \) may contribute to the government an amount \( C_i(\tau) \) conditional on the trade policy vector, or, equivalently, the domestic price vector.

The objective function of the government is

\[
G(\tau) = \sum_{i \in L} C_i(\tau) + a W(\tau),
\]

where \( L \) is the exogenously given set of organized sectors, \( a > 0 \) is the weight the government attaches to aggregate welfare \( W(\tau) \) in the economy, and the latter is given by

\[
W(\tau) = l + \sum_{k=1}^{m} n_k \pi_k(\tau) + \left[ \sum_{k=1}^{m} \tau_k n^*_k q^*_k(\tau) - \sum_{k=1}^{m} p_k d_k(p(\tau)) + \tilde{U}(d_1(p(\tau)), ..., d_m(p(\tau))) \right].
\]

The game timing is as follows: first, the trade policy is set and then firms make their production decisions and consumption takes place. The trade policy is determined in two substages. In the first substage, lobbies simultaneously announce their contribution schedules, i.e., the amount contributed as a function of the import tariffs vector. In the second substage, the government chooses policy by maximizing its objective function over the suggested contribution schedules. The resulting equilibrium is (a) a set of contribution functions, each maximizing the welfare of the respective lobby members given the schedules of all other lobbies, the tariff decision of the government and the production decisions of the firms, (b) the tariffs vector maximizing the government’s objective function under these contribution schedules, and (c) for any firm, a set of output levels as functions of the trade policy.
3 Protection and substitutability

In this section we characterize an equilibrium of the game and compare the resulting trade policy with and without substitutability between the goods.

We solve the game backwards. In the last stage the firms compete in quantities given the trade policy. As introduced above, the equilibrium levels of output and profit for a domestic (foreign) firm in sector $k$ are given by $q_k(\tau)$ and $\pi_k(\tau)$ ($q^*_k(\tau)$ and $\pi^*_k(\tau)$, respectively). The following lemma summarizes the results concerning the responsiveness of domestic and foreign supply and profit to the trade tariffs.

**Lemma 1** *In the Cournot-Nash equilibrium*

- a) An increase in the trade tariff in sector $k$ raises the output of a domestic firm and lowers the output of a foreign firm in sector $k$, and weakly raises output in both types of firms in all other sectors $j \neq k$,

  $$\frac{\partial q_k(\tau)}{\partial \tau_k} > 0, \quad \frac{\partial q^*_k(\tau)}{\partial \tau_k} < 0,$$
  $$\frac{\partial q_k(\tau)}{\partial \tau_j} \geq 0, \quad \frac{\partial q^*_k(\tau)}{\partial \tau_j} \geq 0 \text{ if } j \neq k.$$

- b) The profit of a domestic firm in sector $k$ increases in the tariff on each good $i = 1, \ldots, m$,

  $$\frac{\partial \pi_k(\tau)}{\partial \tau_j} > 0.$$

Lemma 1 follows from several observations. First, consider the own-tariff effect. Due to Cournot competition and linear demand functions, the home and foreign outputs in sector $k$ are strategic substitutes. A domestic import tariff in sector $k$ shifts the foreign firm’s response curve in the south-west direction. Therefore, an increase in $\tau_k$ reduces the output of the foreign firm and increases the output of the domestic firm in sector $k$. Now, turn to the cross-tariff effect. The output produced by either a domestic or a foreign firm in sector $k$ and the output produced by a foreign firm in sector $j$ are strategic substitutes. As a result, an increase in $\tau_j$ induces higher output of both domestic and foreign firms in sector $k$. Finally, the domestic price of good $k$ increases in response to an increase in the import tariff on any good, which yields the result for the profit.

Before proceeding, we establish another useful auxiliary result, characterizing the sensitivity of imports to the trade tariffs. Denote by $m_k(\tau)$ total imports in sector $k$,

$$m_k(\tau) = n^*_k q^*_k(\tau).$$

Consider the matrix $M$ of the derivatives of the import with respect to the trade tariffs

$$M = \left( \frac{\partial m_k}{\partial \tau_j} \right), \quad 1 \leq k, j \leq N. \quad (6)$$
Lemma 2. The matrix $M$ is invertible and all entries of the negative of its inverse $H = -(M)^{-1}$ are greater or equal to zero

$$H = (h_{ij})_{i,j=1,...,n} \geq 0.$$  

Now we are ready to proceed backward in the game to the determination of the trade policy. The problem has the structure of a menu-auction, as characterized by Bernheim and Whinston (1986). Similarly to PFS approach, we concentrate on locally truthful equilibria. That is, we assume that the contribution schedules are differentiable, which will imply that they are locally truthful around equilibrium, i.e., a marginal change in each lobby’s contribution to the government resulting from a small policy change is exactly equal to the respective marginal change of this lobby’s welfare.

Local truthfulness combined with the optimality of the equilibrium price vector for the government results in the condition (equation (12) in PFS)

$$\sum_{i \in L} \nabla W_i(\tau) + a \nabla W(\tau) = 0.$$  

(7)

With the use of expressions for the lobbies’ and social welfare (4) and (5) system (7) can be rewritten as

$$\tau_i = \sum_{j=1}^{m} h_{ij} \left( \sum_{k=1}^{m} \frac{(I_k + a)}{(a + \alpha_L)} n_k \frac{\partial \pi_k(\tau)}{\partial \tau_j} - \sum_{k=1}^{m} \frac{\partial p_k}{\partial \tau_j} d_k(p(\tau)) + n^*_i q^*_j(\tau) \right), \quad i = 1, ..., m,$$

(8)

where $I_k$ is an indicator function taking the value of 1 if industry $k$ is organized, and 0 otherwise, $\alpha_L = \sum_{i \in L} \alpha_i$ is the total share of population in the organized industries and $h_{ij}$ are the elements of the matrix $H = -(M)^{-1}$. The system of equations (8) characterizes the equilibrium trade policy and in what follows we concentrate on analyzing this system.

First, consider the case of a separable utility function, $\sigma = 0$. This treatment parallels the original PFS setting in an imperfectly competitive environment. Zero substitutability implies that all cross-derivatives are zero: $\partial m_k(\tau)/\partial \tau_j = \partial \pi_k(\tau)/\partial \tau_j = \partial p_k/\partial \tau_j = 0$ for $k \neq j$, the matrix of import derivatives $M$ is diagonal, and so is matrix $H$. The diagonal elements of $H$ are given by

$$h_{ii} = \frac{1}{(-\partial m_i(\tau)/\partial \tau_i)} > 0, \quad i = 1, ..., m$$

Therefore, the equation (8) becomes

$$\tau_i = \frac{1}{(-\partial m_i(\tau)/\partial \tau_i)} \left[ (I_i + a) n_i \frac{\partial \pi_i(\tau)}{\partial \tau_i} - \left( \frac{\partial p_i}{\partial \tau_i} d_i(p(\tau)) - n^{*_i} q^{*_i}(\tau) \right) \right],$$

(9)

and in the absence of lobbying, the first-best trade tariffs are determined by

$$\tau^0_i = \frac{1}{(-\partial m_i(\tau)/\partial \tau_i)} \left[ n_i \frac{\partial \pi_i(\tau)}{\partial \tau_i} - \left( \frac{\partial p_i}{\partial \tau_i} d_i(p(\tau)) - n^{*_i} q^{*_i}(\tau) \right) \right].$$

(10)

Due to imperfect competition in the goods markets, free trade is no longer socially optimal. But, as in
the original PFS model, the organized industries experience higher protection than in the first-best equilibrium. In turn, the non-organized industries are underprotected. Indeed, other things equal, equations (9) and (10) differ by the term
\[
\frac{1}{(-\partial m_i(\tau)/\partial \tau_i)} \left[ \left( \frac{I_i + a}{a + a_L} - 1 \right) n_i \frac{\partial \pi_i(\tau)}{\partial \tau_i} = \frac{(I_i - a_L)}{(a + a_L)} \left( \frac{n_i}{(-\partial m_i(\tau)/\partial \tau_i)} \frac{\partial \pi_i(\tau)}{\partial \tau_i} \right) \right],
\]
(11)

By Lemma 1 import falls and profit increases in their own tariff so that the sign of expression (11) is determined by the sign of the difference \((I_i - a_L)\). Assume that the organized industries do not comprise the entire population \((a_L < 1)\). Then, other things equal, for an organized industry \((I_i = 1)\) the tariff is higher than the first-best tariff as \((I_i - a_L) > 0\). Similarly, for an unorganized industry \((I_i = 0)\) the tariff set in the presence of lobby groups is lower than the first-best tariff. Note that if all industries are organized and every voter belongs to some lobby \((a_L = I_i = 1 \ \forall j)\), the protection rates are equal to the first-best ones. That is, like in PFS, the lobbying efforts of different industries exactly offset each other.

Now, let us see how the relaxation of utility separability influences the equilibrium import tariffs. That is, we analyze the system of equations (8) in the presence of cross-price effects. Similarly to above, take the first-best trade tariffs as the reference point. They are given by equation
\[
\tau^0_i = \sum_{j=1}^{m} h_{ij} \left( \sum_{k=1}^{m} n_k \frac{\partial \pi_k(\tau)}{\partial \tau_j} - \sum_{k=1}^{m} \frac{\partial p_k d_k(\mathbf{p}(\tau)) + n_j q_j^*(\tau)}{\partial \tau_j} \right).
\]
(12)

Again, if all industries are organized and every voter belongs to some lobby \((a_L = I_j = 1 \ \forall j)\), the equilibrium protection rates are first-best.

However, unlike the case with a separable utility (system (9)), the comparison of equations (8) and (12) does not imply that the non-organized industries are always underprotected and the organized industries are overprotected. Indeed, in the presence of substitutability between products, the trade tariffs are directly affected by the sensitivity of supply and demand in the other industries and the organizational status of these industries. That is, the terms corresponding to the reaction of the other industries to the increase in \(\tau_i\), directly appear in the tariff equation for industry \(i\).

In particular, the negative effect of industry \(l\) being organized on industry \(i\)’s protection is weaker if industries \(i\) and \(l\) produce substitutes. To see this, first, assume that the demand functions for goods are independent \((\sigma = 0)\), so that the tariff for industry \(i\) is given by equation (9). Other things equal, industry \(l\) getting organized reduces the protection for industry \(i\) only through an increase in \(a_L\) (as the corresponding factor \((I_i + a)/(a + a_L)\) decreases and \(n_i (\partial \pi_i(\tau)/\partial \tau_i) > 0\)). Now, instead, consider the case when the goods are substitutes and the tariff for industry \(i\) is given by equation (8)). As above, other things equal, industry \(l\) getting organized reduces the industry \(i\) tariff through higher \(a_L\) in factors.
\((I_k + a)/(a + a_L)\) for all \(k \in \{1, \ldots, m\}\), as \(\partial \pi_k(\tau)/\partial \tau_j \geq 0\) and \(h_{ij} \geq 0\) by Lemma 2 for all \(i, j\).

However, it also has a positive effect on industry \(i\)’s protection through an increase in \(I_i\) from 0 to 1 in factor \((I_i + a)/(a + a_L)\). It is unclear which effect dominates, but this argument suggests that the overall negative impact on protection of good \(i\) resulting from industry \(l\) being organized is weaker in case of substitutability between the goods.

The intuition behind this effect is straightforward: in the absence of substitution\(^7\) the consumption of each non-numeraire good is fully determined by the price of this good only. As a result, each organized group has two goals. First, it aims at raising the trade tariff in its own sector, which increases its market share, the own good price and, in the end, the profit of the lobby. At the same time, it attempts to reduce the tariffs on all other goods it consumes, as lower tariffs entail lower consumption prices. However, in the presence of substitutability between the goods, such a lobbying strategy may cause consumers to switch consumption from more protected goods to less protected ones. To limit substitution, organized industries tend to apply more ”moderate” lobbying strategies. That is, they try to maintain a balance between decreasing the price of the other goods and increasing its own price.

This effect is best understood in case the ownership of industry \(l\) is very highly concentrated, that is, the share \(\alpha_l\) of the population entitled to its profit is zero. Then, the total share of the population in the organized industries \(\alpha_L\) does not change when industry \(l\) becomes organized. In this case, the effect of other industries’ prices on lobby \((l)’s welfare is negligible, so that they have no consumer welfare gain from the trade interventions in other sectors. Therefore, in the absence of substitutability, if industry \(l\) gets organized, it has no impact on the protection of any other industry \(i \neq l\) (indeed, as \(\alpha_L\) does not change, equations (9) for tariffs in industries \(i \neq k\) are unaffected). However, with substitutability between the goods, industry \(l\) still lobbies for additional protection of all substitute-producing industries because it is concerned with maintaining demand for its own good. That is, it increases industry \(i\) protection, as the term

\[
\sum_{j=1}^{m} h_{ij} \frac{I_i}{(a + \alpha_L)} \frac{\partial \pi_i(\tau)}{\partial \tau_j} \geq \sum_{j=1}^{m} h_{ij} \frac{1}{(a + \alpha_L)} \frac{\partial \pi_i(\tau)}{\partial \tau_j}
\]

entering the equation for the equilibrium tariff \(\tau_i\) is positive.

In PFS, the interests of different industry groups are opposed to each other. Here, we see that non-organized industries may benefit from the contributions of organized ones by ”exploiting” the demand properties. We return to this discussion in the subsequent sections and show that this effect caused by substitutability of products can lead to free-riding in the lobbying behavior.

\(^7\)That is, if the utility function is quasi-linear and separable in goods \(i = 1, \ldots, m\).
The above results are obtained for Cournot competition. However, it is easily seen that they also hold if we keep the assumption of linear demand and constant marginal costs, but allow firms to compete in prices in Bertrand-fashion. First, assume that domestic and foreign goods are homogenous. Notice that the only interesting case to consider is the one with a single producer in (at least some of) the domestic industries. Indeed, if all domestic sectors are oligopolistic, Bertrand competition implies that domestic firms’ profits are zero independently of the tariffs. Thereby, there will be no conflict of interests in the tariff-setting game between the domestic industries, and no lobbying. If some of the domestic sectors are monopolistic, then under positive tariff, the domestic market in these sectors will be served by the domestic firm at the price of \( p_i = c_i + \tau_i \). Under negative tariff all the sales will be done by the foreign producers at the price of either \( c_i \) (if there is a single foreign producer in this sector), or \( p_i = c_i + \tau_i < c_i \) (if there are multiple foreign firms). In the absence of substitutability each organized sector would therefore lobby for a positive tariff in its own sector and a non-positive tariff in all other sectors. However, when different sectors produce (imperfect) substitutes, the prices become strategic complements: a higher tariff (and price) in one sector implies a possibility for a higher price in the substitute-producing sector. Therefore in this situation the organized sectors would lobby for more protection in other sectors and, perhaps, somewhat less protection in its own sector.

If instead domestic and foreign goods are non-homogeneous and all producers still compete in Bertrand-fashion, then higher tariffs on foreign-produced goods attract more demand into domestic sectors. Indeed, prices are strategic complements and therefore, an increase in sector \( k \) tariff would cause an increase in prices in all sectors \( i = 1, \ldots, n \). From the profit maximization problem it follows that, in equilibrium, higher domestic prices entail higher domestic outputs

\[
q_i = (p_i - c_i)\left(\frac{\partial q_i}{\partial p_i}\right),
\]

as \( -\frac{\partial q_i}{\partial p_i} \) is positive. The same is true for foreign output in sector \( i \) for a given tariff \( \tau_i \)

\[
q_i^* = (p_i - c_i - \tau_i)\left(\frac{\partial q_i^*}{\partial p_i}\right).
\]  (13)

Therefore, domestic output increases in all tariffs and foreign output increases in other sectors’ tariffs. Furthermore, the sensitivity of foreign price with respect to the own tariff never exceeds one (an increase in industry \( i \)’s tariff is fully transmitted into the price increase only in case domestic and foreign goods in industry \( i \) are perfect substitutes). Formula (13) then implies that the foreign output is decreasing in its own tariff. It can be shown that under reasonable assumptions, the corresponding matrix \( M \) is invertible, Lemma 2 holds and thus the qualitative results do not change.
4 The level of protection

The analysis in the previous section illustrated the mechanism through which the demand linkages may weaken interest groups’ incentives to lobby, leading to less protection for the organized industries and more for the non-organized industries. However, one potential criticism of this approach can be that the analysis is based on the "other things equal" assumption. To put it differently, the system (8) that defines equilibrium tariffs consists of implicit rather than explicit equations: many of the right-hand-side variables are functions of the trade tariffs. The "other things equal" analysis abstracts from this dependency and bases the comparison on exogenous variation of the right-hand-side variables, thereby ignoring the equilibrium feedback effect from tariffs to the demand/profits etc.\textsuperscript{8}

In this section we address this criticism by providing an explicit solution for the equilibrium trade tariffs. This allows us to directly access the impact of substitutability on the trade protection in equilibrium. In particular, we examine the monotonicity of this relationship, checking whether more substitutability in consumer preferences causes less protection.

To make the analysis tractable, we assume that there is only one domestic and one foreign firm operating in each imperfectly competitive sector, \( n_k = n_k^* = 1 \), so that the respective non-numeraire good \( k \) is supplied by a duopoly. Furthermore, we limit ourselves to the case of two non-numeraire goods, \( m = 2 \), and assume the marginal costs of production to be equal across sectors, \( c_1 = c_2 \). The share of the owners of industry 1 in total population is denoted by \( \alpha \).

Within this context we study the following question: Assume that one industry, say, industry 1, is organized in a lobby group, while the other industry is not. How do the trade tariffs in such an economy change with the change in the degree of substitutability between the industries’ products?

For a given vector of trade tariffs \( \tau = (\tau_1, \tau_2) \) the output of the domestic firm in sector \( k = 1, 2 \) in an interior Nash equilibrium\textsuperscript{9} is given by

\[
q_k(\tau) = \frac{1 - c}{3 + 2\sigma} + \frac{3 - 2\sigma^2}{9 - 4\sigma^2} \tau_k + \frac{\sigma}{9 - 4\sigma^2} \tau_{-k}, \tag{14}
\]

and the output of the foreign firm is

\[
q_k^*(\tau) = \frac{1 - c}{3 + 2\sigma} - \frac{6 - 2\sigma^2}{9 - 4\sigma^2} \tau_k + \frac{\sigma}{9 - 4\sigma^2} \tau_{-k}. \tag{15}
\]

As only industry 1 is organized, the total share of population in the organized industries is \( \alpha_L = \alpha \). Substituting this relation and expressions (14), (15) into the system (8) we solve for the equilibrium

---

\textsuperscript{8}Same criticism applies to the original PFS model.

\textsuperscript{9}Here and thereafter we concentrate on interior solutions. The necessary condition for an interior solution is \( a + 3\alpha \geq 2 \), see the proof of Proposition 3 for more detail.
protection rates as functions of exogenous parameters of the model, such as degree of substitutability between the products, total size of the organized groups, weight of the social welfare in the government payoff function etc. The resulting trade tariff for industry 1 - the organized industry - is given by

\[ \tau_1(\sigma, a, a, c) = (1 - c)F(\sigma, a, a)/D(\sigma, a, a), \]  

(16)

and the tariff for the non-organized industry 2 is determined by

\[ \tau_2(\sigma, a, a, c) = (1 - c)G(\sigma, a, a)/D(\sigma, a, a), \]  

(17)

where the functional forms of \( F(\cdot) \), \( G(\cdot) \) and \( D(\cdot) \) are given by

\[
F(\sigma, a, a) = 4(a + 1)(a + 2a)\sigma^3 + 4(a^2 - 3a - 2a - 3a^2 - 3\alpha)\sigma^2
- (9a^2 + 26a\alpha + 10a + 13a^2 + 14\alpha)\sigma + (3a + a + 2)(9a + 11\alpha),
\]

\[
G(\sigma, a, a) = 4a(2a - 2a - 1)\sigma^3 - 4(a + 2a + 3a\alpha - a^2 + 3a^2)\sigma^2
+ (18a - 26a\alpha - 9a^2 - 13a^2 + 14\alpha)\sigma + (3a + a)(9a + 11\alpha - 2),
\]

\[
D(\sigma, a, a) = 4(a + 2a - 1)(a + 2a)\sigma^4 + (14a - 118a\alpha - 45a^2 - 81a^2 + 22a)\sigma^2
+ (9a + 11\alpha - 2)(9a + 11\alpha).
\]

As a benchmark, consider the first-best government trade policy \( \tau^0 \), which would be implemented in the absence of any organized interests, \( \alpha_L = I_1 = I_2 = 0 \). This optimal policy is characterized by the import taxes

\[ \tau^0_1(\sigma, c) = \tau^0_2(\sigma, c) = (1 - c)/(\sigma + 1), \]  

(19)

which is along the lines of the standard results for the linear demand specification (see, e.g. Brander and Spencer (1984)).

Our aim is to show that the increase in the degree of competition in the product market (represented by the increase in the degree of substitution between the products) results in a more moderate equilibrium trade protection for the organized interest group.

However, in exploring this relationship we would like to abstract from the strategic trade policy effect caused by increase in substitutability. Indeed, equation (19) demonstrated that higher degree of substitution decreases the protection level even in the absence of lobbying – the first-best trade tariffs decline in \( \sigma \). The intuition is as follows: A domestic import tax in sector \( i \) is aimed at increasing the market share of the domestic firm.\(^{10}\) As the degree of substitutability increases, the effect of the trade tax in sector \( i \) also extends to the firms in sector \( \neg i \), thereby increasing their output and improving their market

\(^{10}\) Subject to the respective loss in consumer welfare and the change in the tax revenues.
position. Hence, due to the competition arising from substitutability, the overall effect of a tax in sector $i$ on the firm in sector $i$ becomes weaker, leaving less room for strategic trade policy.

We isolate the impact of substitutability on trade policy through lobbying by analyzing how the trade tariffs in a lobbying equilibrium differ from the first-best trade policy benchmark. More precisely, we consider the ratios of trade tariffs in industry $i$ with and without lobbying

$$T_i (\sigma, a, a) = \tau_i (\sigma, a, a) / \tau_i^0 (\sigma), \quad i = 1, 2,$$

which we henceforth refer to as relative protection.

Proposition 3 summarizes the equilibrium response of relative protection rates to the change in the degree of substitution $\sigma$.

**Proposition 3** If industry $1$ is organized, while industry $2$ is not, the equilibrium relative protection of the organized industry decreases with the degree of substitution:

$$\frac{dT_1 (\sigma, a, a)}{d\sigma} < 0.$$  

For the non-organized industry, the effect is the opposite – the relative protection increases with the degree of substitution:

$$\frac{dT_2 (\sigma, a, a)}{d\sigma} > 0.$$  

This result confirms the logic discussed in the previous section. Similarly to the PFS framework, an organized lobby is interested in raising its own price (and profit) and lowering the price of the other good. However, substitutability brings in a conflicting incentive: the organized industry has to control the gap between its own price and the price of a substitute good to limit a demand shift. This leads to less protection for the organized industry and more protection (or less disprotection) for the non-organized industry as the substitutability increases.

Moreover, the trade tariffs imposed on the organized and non-organized industries converge as the threat of a demand shift becomes more and more serious. For $\sigma \to 1$, the goods become indistinguishable from the consumer’s point of view, i.e., industries 1 and 2 face a joint demand for these goods. Hence, the foreign firms in sector 1 and sector 2 become identical competitors from the point of view of the domestic organized industry 1, so it lobbies for the same trade tariff in both industries.

It is worth noting that the result of Proposition 3 does not necessarily imply that the influence-driven protection rates will converge to the socially-optimal level as the degree of substitutability increases. In fact, for a larger value of $\sigma$, the organized industry is interested in reducing the difference between the price of its own good and the substitute, not in achieving the first-best outcome. For example, consider
the case of perfect substitutes $\sigma = 1$. As argued above, in this case both industries face the same trade tariff. However, depending on the ownership concentration of industry 1 (that is, of the share of population $\alpha$ that has claims to its profit) the trade tariff can differ from the first-best level in either direction. If $\alpha = 1/2$, industry 1’s members are exactly representative of the entire population – they own as much of the firm’s profit claims\(^{11}\) as does the average person. Therefore, the tariff for which the industry is lobbying perfectly matches the first-best tariff. If instead the ownership of industry 1 is more concentrated, so that $\alpha < 1/2$, it cares about the domestic profits, as opposed to consumer welfare, more than does the average person. Therefore, in this case, the equilibrium protection rates exceed the socially optimal ones. Similarly, if $\alpha > 1/2$, the resulting equilibrium protection falls below the socially optimal level.

Still, if the degree of substitution is not too high, the non-organized industry is always underprotected as compared to the first-best trade tariff, because

\[
T_2 (0, \alpha, a) = \frac{G (0, \alpha, a)}{D (0, \alpha, a)} / \frac{1}{3} = 1 - \frac{8 \alpha}{9a + 11a} < 1,
\]

and the relative protection rate changes continuously. The organized industry achieves more than the first-best protection, as

\[
T_1 (0, \alpha, a) = \frac{F (0, \alpha, a)}{D (0, \alpha, a)} / \frac{1}{3} = 1 + \frac{8 (1 - \alpha)}{9a + 11a - 2} > 1.\(^{12}\)
\]

Therefore, we have the following corollary:

**Corollary 4** If the degree of substitutability is not too high, an increase in $\sigma$ shifts the influence-driven protection rates towards the socially optimal levels.

As shown in the original PFS model, competition in the lobbying market under some conditions may improve efficiency by pushing the economy closer to the first-best trade protection level.\(^{13}\) For example, when every person owns shares of one or the other industry and both industries are organized, their lobbying efforts offset each other and the economy ends up with the socially-optimal trade tariffs. Corollary 4 demonstrates that competitive pressure in the product market (resulting from the rise in the degree of substitution) works in the same direction. Figure 1 illustrates the findings of Proposition 3, Corollary 4 and the related discussion.

The result of Corollary 4 suggests a new explanation for a puzzle commonly observed in the empirical studies of PFS. Most of the studies report unexpectedly low estimates of the weight government puts

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\(^{11}\)Note that in case of perfect substitutability, domestic firms in sectors 1 and 2 are identical as are their profit functions.

\(^{12}\)The necessary condition for the interior solution $a + 3a > 2$ implies that $9a + 11a - 2 > 0$.

\(^{13}\)This result also holds in our setting.
on campaign contributions relative to social welfare, see e.g. Goldberg and Maggi (1999), Gawande and Bandyopadhyay (2000) or Mitra, Thomakos and Ulubasoglu (2002). This implies that lobbies have little influence over the government decisions, and the government behaves almost as a social welfare maximizer. This finding has caused a concern about the empirical interpretation of the PFS model, as such an estimate would suggest that the protection is hardly "for sale". For example, Gawande and Krishna (2001) write that "it is enough to cast doubt on the value of viewing trade policy determination through this political economy lens".

This paper suggests that smaller deviations from the first-best trade policy may result from the weaker incentives to lobby, due to cross-product substitutability. Indeed, most of the studies cited above are based on 4-digit SIC industry data, which implies at least some degree of substitutability between the products of the different industries. Therefore, organized industries’ profits and lobbying decisions may well be affected by the potential demand shift effects. By not accounting for the cross-product substitutability, the existing empirical analysis overestimates the organised industries’ gains from protection. This may lead to a downward bias in the estimate of the relative weight the government puts on these gains (or, equivalently, resulting campaign contributions) when comparing them against social welfare losses from protection.

This argument implies that controlling for cross-product substitutability may improve the estimates of PFS. Notice that the above model only considers the case when all products are equally substitutable. However, it can be directly extended to a more realistic scenario with products being substitutable within (highly aggregated) sectors, and unrelated between these sectors. More specifically, one would interpret the utility function (2) as defined over a variety of goods produced by subindustries within a large sector, and the overall utility would be defined as a sum of these sectoral utilities. Then, other things equal, the model would predict more intense lobbying and higher level of protection in the (organized) subindus-
tries of sectors with lower within-sector product substitutability. In turn, in sectors where goods produces by the subindustries are closer substitutes, there would be less protection and lobbying. Bombardini and Trebbi (2009) suggest some evidence consistent with these predictions. They find that the lobbying expenditures are lower in 4-digit SIC industries producing more substitutable products. However, a more direct empirical verification of the substitutability effect is yet to be done.

5 Substitutability and lobbying activity

So far, as in the original PFS model, we assumed an exogenous lobby group structure. But why are some industries organized while other ones not? We argue that demand linkages may contribute to explaining this phenomenon. Indeed, in the previous section we demonstrated that if industry $i$ is organized, the import tax for the substitute product $-i$ increases with the degree of substitutability. Hence, a sufficiently high degree of substitutability may entail a situation where a non-organized industry becomes protected without paying for it. That may give rise to a free-riding behavior in lobbying.

Let us illustrate this reasoning with an example. We continue to consider the 2-sector-2-firm model of the previous section but impose several additional assumptions. First, assume that the non-numeraire products are perfect substitutes, $\sigma = 1$. Second, assume that the industries have the same size $a_1 = a_2 = a$. We allow some of the voters to own no claims to any of the industries’ profit but the labor only, so $a \leq 1/2$. To study the free-riding behavior, we extend the existing game by allowing for an initial stage 0 of costless lobby formation. In this stage, industries simultaneously decide whether they are getting organized. The industries deciding to form a lobby participate in the standard lobbying game in stage 1. For this stage we consider the case when organized industries play globally truthful strategies, that is when the contributions of lobby $i$ are (globally) equal to the excess of the lobby’s welfare over a certain threshold $B_j$, 

$$C_j(\tau, B_j) = \max \left[ 0, W_j(\tau) - B_j \right].$$

All truthful equilibria are also locally truthful, so in our argument we can rely on the results obtained in previous sections, such as equations (16) and (17). Once more, we concentrate on interior solutions.

We start by discussing stage 1. There could be 3 possible situations: 1) when none of the industries got organized in stage zero, 2) when only one industry got organized, and 3) when both industries 1 and 2 got organized. The former situation leads to the socially optimal protection level in both industries

$$\tau^0_1(1, c) = \tau^0_2(1, c) \equiv \tau^0 = \frac{1 - c}{4}.$$

Due to the symmetry, we may limit ourselves to considering the second situation only in the case when
industry 1 is organized. Denote the truthful equilibrium when a single industry (i.e. industry 1) is organized by $\Omega^i$ and the resulting tariffs by $\tau_i^i(1, a, a), i = 1, 2$. When both industries are organized we concentrate on a symmetric truthful equilibrium $\Omega^b$, and the tariffs will be denoted by $\tau_i^b(1, a, a)$.\(^{14}\)

If only industry 1 is organized, the trade tariffs in this economy are determined by equations (16) and (17) for $\sigma = 1$. This entails the equilibrium tariffs

$$\tau_i^1(1, a, a) = \tau_i^2(1, a, a) \equiv \tau^i = \frac{(1 - c)}{4} \frac{5a + 2a + 2}{5a + 7a - 1}. \quad (21)$$

As mentioned above, the import taxes on both goods are the same, even though only one of the industries is organized. Under perfect substitutability, industry 1 treats the foreign producers of both good 1 and good 2 as similar Cournot competitors. The symmetry of the setting implies that the preferred trade policy of the organized lobby does not differ with respect to the foreign firms in sectors 1 and 2. Thus, the government also sets identical trade tariffs for these two goods.

In this equilibrium, the non-organized industry 2 gains more from trade protection than the organized (and contributing) industry 1. Indeed, the industries are identical and so are their trade tariffs. Thus, their gross welfare levels, that is, the welfare before the lobbying contributions are paid, coincide

$$W_1(\tau^i) = W_2(\tau^i).$$

However, the trade tariffs in this equilibrium differ from the first best level, which implies that the organized industry makes a positive contribution to the government. Therefore, the net welfare level of industry 1 is below the net welfare level of industry 2:

$$W_1(\tau^i) - C_1(\tau^i) < W_2(\tau^i).$$

In this regime, industry 2 benefits from the trade policy while not paying for it.

Now, consider equilibrium $\Omega^b$ when the domestic producers of both good 1 and good 2 are organized. Solving system (8) for $\sigma = 1$ and $a_L = 2a \leq 1$ yields the equilibrium import tariffs

$$\tau_i^b(1, a, a) = \tau_i^2(1, a, a) \equiv \tau^b = \frac{(1 - c)}{4} \frac{5a + 2a + 4}{5a + 14a - 2}, \quad (22)$$

Note that with two industries actively lobbying, the import tax is further away from the first-best level

$$\tau^b \geq \tau^i \geq \tau^0. \quad (23)$$

\(^{14}\)In the original PPS setting, the interests of different industries are opposed to each other. So, if organized, each industry indeed chooses to buy protection. However, in the presence of substitutes, this outcome is not necessarily unique. If both industries are allowed to lobby, but each of them can get protected without paying for it, the game may admit equilibria where only one of two organized industries is active. Alternatively, there can be equilibria where both industries lobby but make different contributions. We study symmetric truthful equilibria, which seems to be natural given the symmetry of the setting.

\(^{15}\)This follows from the necessary condition for the interior solution $(a + 3a \geq 2)$, which also implies that the import tariffs are positive under both regimes.
Indeed, due to the perfect substitutability ($\sigma = 1$) and equal size ($\alpha_1 = \alpha_2 = \alpha$), industries 1 and 2 have exactly the same preferences. Hence, when they both actively participate in lobbying, the resulting policy is more biased towards the interest groups’ preferred import tax. However, now both lobbies contribute to the government. As we concentrate on a symmetric equilibrium both lobbies contribute the same amount to the government and end up with the identical net welfare:

$$W_1(t^b) - C_1(t^b) = W_2(t^b) - C_2(t^b).$$

We are interested in the trade-off between the costs and benefits of this policy bias for both industry 1, which is lobbying in either equilibrium, and for industry 2, which only contributes in the second equilibrium. In other words, the aim is to compare the lobbies’ payoffs in these two equilibria. We start by evaluating the amount of contributions and the government surplus in these two equilibria.

**Lemma 5** *The total contributions to the government are greater in* $\Omega^b$ *than in* $\Omega^s$

$$C_1(t^b) + C_2(t^b) > C_1(t^s).$$

The intuition behind this lemma is as follows: The trade tariffs in the equilibrium with both industries being active in lobbying are further away from the first-best level than in equilibrium with one lobbying industry. This implies that government would need a higher lobby contribution in the former equilibrium to compensate for the social welfare loss.

Thus, we see that the equilibrium with two organized industries provides the lobbies with higher gross welfare, but requires additional lobbying contributions. However, these contributions are now paid by two lobbies, as compared to the equilibrium with a single organized industry. So lobby 1, which was active in both equilibria, benefits more (or loses less) than lobby 2, as it can now share the costs. Still, it is not clear whether either of them actually wins or loses in the equilibrium with both firms lobbying. This question is answered in the following proposition.

**Proposition 6**

a) *Lobby 1 has a higher net welfare in* $\Omega^b$ *than in* $\Omega^s$ *for any admissible parameter values*

$$W_1(t^b) - C_1(t^b) > W_1(t^s) - C_1(t^s).$$

b) *Lobby 2 has a lower net welfare in* $\Omega^b$ *than in* $\Omega^s$, *if and only if the size of the industry* $\alpha > 1/7$

$$W_2(t^b) - C_2(t^b) < W_2(t^s) \iff \alpha > 1/7.$$  

As long as the share of population that owns the industry is sufficiently large, it loses from participating in lobbying. The intuition behind this result is as follows: if the industries are very concentrated
(α ≤ 1/7), they highly benefit from an increase in protection as the loss in their members’ consumer welfare resulting from higher prices is negligible. With decreasing ownership concentration (α > 1/7), more and more consumers in the industry lose from the price increase which results in a decrease in protection rates and industry welfare. And it may be the case that the gain of the industry from increased protection (due to this industry participation in lobbying) is not high enough to cover the necessary contribution for this increase.

We characterized the outcome of the symmetric truthful equilibrium when both industries are organized, given its existence. The following lemma establishes that it indeed exists.

**Lemma 7** In the lobbying game with perfect substitutability (σ = 1) and industries of identical size, there exists a symmetric truthful equilibrium.

Now, we proceed backwards to the lobby formation stage. In this stage, each industry decides whether it gets organized (O) and buys influence in the next stage of the game, or stays non-organized (N) and remains passive in the lobbying game. The original PFS setup entails a single equilibrium of a type (O,O): getting organized is a dominant strategy in that game as different lobbies’ interests are strictly opposed to each other. The same happens in our setting if the industry’s ownership structure is very concentrated (α < 1/7). Then, again, getting organized is a dominant strategy and a single (O,O) equilibrium emerges.

However, if an industry has a dispersed ownership structure (higher α), it prefers to commit not to lobby as long as the substitute industry will be lobbying. That is, the lobby formation stage is a ”chicken game”, where each industry prefers to be organized when the other is not and vise versa. The payoffs of the game are given by Table 1.

<table>
<thead>
<tr>
<th>Ind 1/Ind 2</th>
<th>Organized</th>
<th>Non-organized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Organized</td>
<td>W₁(rᵇ) - C₁(rᵇ), W₂(rᵇ) - C₂(rᵇ)</td>
<td>W₁(rˢ) - C₁(rˢ), W₂(rᵇ)</td>
</tr>
<tr>
<td>Non-organized</td>
<td>W₁(rˢ), W₂(rˢ) - C₂(rˢ)</td>
<td>W₁(r⁰), W₂(r⁰)</td>
</tr>
</tbody>
</table>

Table 1: The payoffs of the lobby formation game

As established in Proposition 6,

Wᵢ(rˢ) > Wᵢ(rᵇ) - Cᵢ(rᵇ) > Wᵢ(rᵇ) - Cᵢ(rˢ) > Wᵢ(r⁰).

Therefore, this game has two pure strategy Nash equilibria, (O,N) and (N,O), and one mixed strategy equilibrium. The outcome (O,O) is no longer an equilibrium of the game. In other words, in the presence
of substitutability, fewer industries get organized, and lobbying becomes less intense. This discussion is summarized in the following proposition.

**Proposition 8** In the game extended by the participation decision stage, a dispersed ownership structure weakens the industries’ incentives to get organized which, in turn, leads to less intensive lobbying.

This proposition can be given an alternative interpretation: if an industry is sufficiently concerned about its members’ consumer welfare, it is less interested in getting organized. This is in line with the result in Bombardini and Trebbi (2009), who show that sectors with higher labor-to-capital ratio are characterized by lower levels of lobbying expenditures.

One natural question to ask is whether this less intensive lobbying behavior is efficient from the industries’ joint point of view. The answer is no. Recall that the equilibrium when both lobbies are active is characterized both by higher import taxes (and associated higher welfare of the lobbies), and by higher lobby contributions, as compared to the equilibrium with only one organized industry. It turns out that the total gain in industries’ welfare resulting from the joint participation in lobbying is not fully offset by increasing cost of lobbying.

**Corollary 9** The aggregate net welfare of industries 1 and 2 is larger in equilibrium $\Omega^b$ when they both participate in lobbying than in equilibrium $\Omega^s$ when only one of the lobbies is active

$$\left[ W_1(\tau^b) - C_1(\tau^b) + W_2(\tau^b) - C_2(\tau^b) \right] - \left[ W_1(\tau^s) - C_1(\tau^s) + W_2(\tau^s) \right] > 0.$$  

Finally, by continuity the results of this section also extend to markets with high, but not perfect substitutability.

To sum up, this section illustrates the mechanism through which demand linkages can caused the free-riding problem in lobbying: The product substitutability produces a positive inter-industry externality from protection which, in turn, may lead to dilution of the incentives to organize and less protection than would be jointly optimal for the industries.

### 6 Conclusion

This paper studies the impact of the demand linkages on the political rivalry among the interest groups in the Grossman and Helpman (1994) "Protection for Sale" framework. It is known that the political competition in the original PFS model is purely due to the interest groups concerned with the well-being of their members as consumers. This paper introduces product substitutability and oligopolistic competition into the PFS framework to provide a more compelling justification for the political rivalry
among different lobbies. It analyses the determination of trade policy and the intensity of inter-industry lobbying competition in the presence of these demand linkages.

The paper shows that the product substitutability weakens organized interest groups’ incentives to lobby. The threat of losing demand to the substitute product makes organized industries’ lobbying strategy less aggressive, which results in lower protection rates. Therefore, by not accounting for product substitutability, the original model overstates the organized groups’ desire for protection. This result may explain why the empirical investigations of the PFS model have found that the government is predominantly concerned with welfare rather than contributions by lobbies. The paper then turns to the endogenous lobby formation. It demonstrates that product substitutability also weakens industries’ incentives to get organized due to the possibility to free-ride on the substitute-producing industry’s lobbying effort.

The suggested framework can be extended in a number of directions. In particular, one can use it to analyze strategic interactions between the domestic and foreign government. Alternatively one can apply the framework to the trade associations’ lobbying. Bombardini and Trebbi (2009) provide both empirical and theoretical analysis of the trade-off between individual lobbying vs. lobbying through trade associations. However, the question of the determinants of the scope of the trade association is left unexplored. We believe it can be answered with the help of the suggested framework. Another important question that is left behind in the paper is the empirical verification of the suggested model. These extensions are part of the future research agenda.

References


### A Appendix

#### A.1 Proof of Lemma 1

The FOC of the profit maximization problem for each of the \( n_k \) symmetric domestic firms/each of the \( n_k^* \) foreign firms in sector \( k \) are respectively

\[
\begin{align*}
1 - n_k q_k - n_k^* q_k^* - \sigma \sum_{j \neq k} (n_j q_j + n_j^* q_j^*) & = c_k + q_k, \quad k = 1, \ldots, m \tag{24} \\
1 - n_k q_k - n_k^* q_k^* - \sigma \sum_{j \neq k} (n_j q_j + n_j^* q_j^*) & = c_k + q_k^* + \tau_k, \quad k = 1, \ldots, m. \tag{25}
\end{align*}
\]

It follows that \( q_k = q_k^* + \tau_k, \forall k \). So the systems (24), (25) can be rewritten in a matrix form as

\[
\begin{align*}
Rq & = 1 - c + n^* \tau \\
Rq^* & = 1 - c - n \tau,
\end{align*}
\]

where the respective matrices are given by

\[
R = \begin{pmatrix}
1+N_1 & \sigma N_2 & \cdots & \sigma N_m \\
\sigma N_1 & 1+N_2 & \cdots & \sigma N_m \\
\vdots & \vdots & \ddots & \vdots \\
\sigma N_1 & \sigma N_2 & \cdots & 1+N_m
\end{pmatrix}, \quad n^* = \begin{pmatrix}
n_1^* & \sigma n_2^* & \cdots & \sigma n_m^* \\
\sigma n_1^* & n_2^* & \cdots & \sigma n_m^* \\
\vdots & \vdots & \ddots & \vdots \\
\sigma n_1^* & \sigma n_2^* & \cdots & n_m^*
\end{pmatrix}, \quad n = \begin{pmatrix}
1+n_1 & \sigma n_2 & \cdots & \sigma n_m \\
\sigma n_1 & 1+n_2 & \cdots & \sigma n_m \\
\vdots & \vdots & \ddots & \vdots \\
\sigma n_1 & \sigma n_2 & \cdots & 1+n_m
\end{pmatrix}.
\]

Matrix \( R \) is invertible as its determinant is positive:

\[
\det R = (1 + N_1) \prod_{i=2}^{m} (1 + (1 - \sigma) N_i) + \sigma \sum_{i=2}^{m} N_i \prod_{j=1, j \neq i}^{m} (1 + (1 - \sigma) N_j) > 0.
\]

Therefore, the partial derivatives \( \partial q_k / \partial \tau_j \) are given by the respective elements of the matrix \( S = R^{-1} n^* \).

The diagonal elements of \( S \) are positive, e.g. for \( k > 1 \)

\[
S_{kk} = \frac{1}{\det R} \left( (1 + N_1) \prod_{i=2, i \neq k}^{m} (1 + (1 - \sigma) N_i) + \sigma \sum_{i=2, i \neq k}^{m} N_i \prod_{j=1, j \neq i, k}^{m} (1 + (1 - \sigma) N_j) \right) n_k^* \\
- \sum_{i=1, i \neq k}^{m} \left( \sigma N_i \prod_{j=1, j \neq i, k}^{m} (1 + (1 - \sigma) N_j) \right) \sigma n_k^* \\
\geq \frac{1}{\det R} \prod_{i=2, i \neq k}^{m} (1 + (1 - \sigma) N_i) \left[ (1 + N_1) n_k^* - \sigma^2 N_1 n_k^* \right] > 0
\]

The off-diagonal elements of \( S \) are nonnegative

\[
S_{kl} = \frac{1}{\det R} \left( (1 + N_1) \prod_{i=2, i \neq k}^{m} (1 + (1 - \sigma) N_i) + \sigma \sum_{i=2, i \neq k}^{m} N_i \prod_{j=1, j \neq i, k}^{m} (1 + (1 - \sigma) N_j) \right) \sigma n_i^* \\
- \sum_{i=1, i \neq k, l}^{m} \left( \sigma N_i \prod_{j=1, j \neq i, k}^{m} (1 + (1 - \sigma) N_j) \right) \sigma n_i^* - \left( \sigma N_l \prod_{j=1, j \neq l, k}^{m} (1 + (1 - \sigma) N_j) \right) n_l^* \right)
\]
Similarly, the partial derivatives $\partial q^*_k / \partial \tau_j$ are given by the respective elements of the matrix $S^* = -R^{-1} n$. The diagonal elements of $S^*$ are negative

$$S^*_{kk} = \frac{(-1)}{\det R} \left[ (1 + N_1) \prod_{i=1, i \neq k}^n (1 + (1 - \sigma) N_i) + \sigma \sum_{i=1, i \neq k}^m N_i \prod_{j=1, j \neq i, k}^m (1 + (1 - \sigma) N_j) \right] (n_k + 1)$$

$$- \sum_{i=1, i \neq k}^m \left( \sigma N_i \prod_{j=1, j \neq i, k}^m (1 + (1 - \sigma) N_j) \right) \sigma n_k$$

$$\leq - \frac{1}{\det R} \prod_{i=2, i \neq k}^m (1 + (1 - \sigma) N_i) \left[ (1 + N_1) (n_k + 1) - \sigma^2 N_1 n_k \right] < 0.$$

The off-diagonal elements of $S^*$ are nonnegative

$$S^*_{kl} = \frac{(-1)}{\det S} \left[ (1 + N_1) \prod_{i=1, i \neq k}^n (1 + (1 - \sigma) N_i) + \sigma \sum_{i=1, i \neq k}^m N_i \prod_{j=1, j \neq i, k}^m (1 + (1 - \sigma) N_j) \right] \sigma n_l$$

$$- \sum_{i=1, i \neq k, l}^m \left( \sigma N_i \prod_{j=1, j \neq i, k, l}^m (1 + (1 - \sigma) N_j) \right) \sigma n_l - \left( \sigma N_l \prod_{j=1, j \neq k, l}^m (1 + (1 - \sigma) N_j) \right) (n_l + 1)$$

$$= - \frac{1}{\det S} \prod_{j=2, j \neq k, l}^m (1 + (1 - \sigma) N_j) \left( n_l - N_l \right) \geq 0$$

which proves the first part of Lemma 1.

From the FOC (24) it follows that the profit of a domestic firm in sector $k$ is given by $\pi_k(\tau) = q^2_k$, which immediately proves the second part of Lemma 1.

### A.2 Proof of Lemma 2

From the proof of Lemma 1 and definition (6) it follows that

$$M = \begin{pmatrix} n_1^* & 0 & \ldots & 0 \\ 0 & n_2^* & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & n_m^* \end{pmatrix} \left( \frac{\partial q^*_k}{\partial \tau_j} \right) = \begin{pmatrix} n_1^* & 0 & \ldots & 0 \\ 0 & n_2^* & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & n_m^* \end{pmatrix} R^{-1} n.$$

Matrices $n$ and $R$ have identical structure, so by the same argument as in the proof of Lemma 1, we have $\det n > 0$. Thus, $M$ is invertible as a product of invertible matrices, and

$$H = -M^{-1} = n^{-1} R \begin{pmatrix} n_1^* & 0 & \ldots & 0 \\ 0 & n_2^* & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & n_m^* \end{pmatrix}^{-1} = m \begin{pmatrix} n_1^* & 0 & \ldots & 0 \\ 0 & n_2^* & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & n_m^* \end{pmatrix}^{-1}. \quad (26)$$
Similarly to the proof of Lemma 1 one can show that the diagonal elements of $m = n^{-1} R$ are positive, and the off-diagonal elements of $m$ are nonnegative, which completes the proof.

### A.3 Derivation of equation (8)

Substituting the expressions (4) and (5) into the equation (7) yields

$$
(a + a_L)\sum_{k=1}^{m} \tau_k n_k^* q_k^*(\tau) - \sum_{k=1}^{m} p_k d_k(p(\tau)) + \hat{U}(d_1(p(\tau)), \ldots, d_m(p(\tau)))
$$

$$
+ \sum_{k=1}^{m} (I_k + a)n_k \nabla \pi_k(\tau) = 0,
$$

Taking the respective derivatives, substituting the definition $m_k = n_k^* q_k^*$ and rearranging yields

$$
- \sum_{k=1}^{m} \tau_k \frac{\partial m_k(\tau)}{\partial \tau_j} = \sum_{k=1}^{m} \frac{(I_k + a)}{(a + a_L)} n_k \frac{\partial \pi_k(\tau)}{\partial \tau_j} - \sum_{k=1}^{m} \frac{\partial p_k}{\partial \tau_j} d_k(p(\tau)) + n_j^* q_j^*(\tau).
$$

or, equivalently, in a matrix form

$$
\tau = H \left( \sum_{k=1}^{m} \frac{(I_k + a)}{(a + a_L)} n_k \frac{\partial \pi_k(\tau)}{\partial \tau_j} - \sum_{k=1}^{m} \frac{\partial p_k}{\partial \tau_j} d_k(p(\tau)) + n_j^* q_j^*(\tau) \right)_{k,j=1, \ldots, n},
$$

where $H = -(M)^{-1}$. Equation-by-equation version of (29) yields the system (8).

### A.4 Derivation of equation (16), (17) and (19)

The output levels of the foreign and domestic firms in sectors 1 and 2 are given by equations (14) and (15). Using them, we obtain the following formulas for the tariff sensitivity of the imports

$$
\frac{\partial m_k(\tau)}{\partial \tau_k} = -2 \left( 3 - \sigma^2 \right) \frac{9 - 4\sigma^2}{9 - 4\sigma^2}, \quad \frac{\partial m_k(\tau)}{\partial \tau_{-k}} = \frac{\sigma}{9 - 4\sigma^2};
$$

domestic profits

$$
\frac{\partial \pi_k(\tau)}{\partial \tau_k} = 2 \left( 3 - 2\sigma^2 \right) \frac{(1 - c)(3 - 2\sigma) + (3 - 2\sigma^2) \tau_k + \sigma \tau_{-k}}{(9 - 4\sigma^2)^2},
$$

$$
\frac{\partial \pi_k(\tau)}{\partial \tau_{-k}} = 2\sigma \frac{(1 - c)(3 - 2\sigma) + (3 - 2\sigma^2) \tau_k + \sigma \tau_{-k}}{(9 - 4\sigma^2)^2};
$$

price

$$
\frac{\partial p_k(\tau)}{\partial \tau_k} = \frac{(3 - 2\sigma^2)}{9 - 4\sigma^2}, \quad \frac{\partial p_k(\tau)}{\partial \tau_{-k}} = \frac{\sigma}{9 - 4\sigma^2};
$$

and aggregate domestic consumption in sectors 1 and 2

$$
d_k(p(\tau)) = \frac{2(3 - 2\sigma)(1 - c) - 3\tau_k + 2\sigma \tau_{-k}}{9 - 4\sigma^2}.
$$

Substituting these relations into the system (28), collecting terms and multiplying by a common factor, we get an equivalent system.
\[ -2 \left( 3 - 2\sigma^2 \right)^2 \frac{(I_k + a)}{(a + a_L)} - 2\sigma \left( 3 - 2\sigma^2 \right) \frac{(I_k + a)}{(a + a_L)} - 76\sigma^2 + 16\sigma^4 + 99 \right] \tau_k \]

\[ -\sigma \left[ 2 \left( 3 - 2\sigma^2 \right) \frac{(a + I_k)}{(a + a_L)} + 2\sigma \left( a + I_k \right) \frac{(a + a_L)}{(a + a_L)} + 15 - 4\sigma^2 \right] \tau_{-k} \]

\[ = (1 - c) \left( 3 - 2\sigma \right) \left[ 3 - 2\sigma + 2\sigma \frac{(a + I_k)}{(a + a_L)} + 2 \left( 3 - 2\sigma^2 \right) \frac{(a + I_k)}{(a + a_L)} \right], \]

which determines trade tariffs \( \tau_1 \) and \( \tau_2 \).

The first-best trade policy benchmark corresponds to the government tariff choice in the absence of organized lobbies, \( I_1 = I_2 = a_L = 0 \). Therefore, \( (I_k + a)/(a + a_L) = 1 \) for \( k = 1, 2 \), and the system can be simplified into two symmetric equations

\[ (-20\sigma - 4\sigma^2 + 4\sigma^3 + 27) \tau_k - \sigma (7 - 4\sigma) \tau_{-k} = (1 - c) \left( 3 - 2\sigma \right)^2 \]

for \( k = 1, 2 \). Solving this yields equation (19)

\[ \tau_1 = \tau_2 = (1 - c)/(3 + \sigma). \]

Formulas (16) and (17) follow from solving system (30) for \( I_1 = 1, a_L = a \), and \( I_2 = 0 \).

### A.5 Proof of Proposition 3

To show that the ratio \( \frac{\tau_k(a,a,a)}{\tau_k(a)} \) decreases with \( \sigma \) for any \( (a, a) \), we proceed in six steps.

1. Describe the necessary condition for the interior solution for the tariff.

2. Represent \( T_k(a,a,a) \) as a ratio of two polynomials \( T_k(a,a,a) = \frac{N(a,a,a)}{D(a,a,a)} \) and show that both the numerator \( N(a,a,a) \) and the denominator \( D(a,a,a) \) decline in \( \sigma \).

3. Show that \( T_k(a,a,a) \) declines in \( \sigma \) on \( 0, 1 \) iff the ratio \( R(a,a,a) = N_\sigma(a,a,a)/D_\sigma(a,a,a) \) is higher than \( T_k(a,a,a) \) for all \( \sigma \in 0, 1 \) and \( (a, a) \) delivering an interior solution.

4. Introduce an auxiliary linear function \( A(a,a,a) \) and show that \( R(a,a,a) \geq A(a,a,a) \) for any admissible \( (a, a, a) \).

5. Show that \( A(a,a,a) \geq T_k(a,a,a) \) for any admissible \( (a, a, a) \).

6. From steps 3, 4 and 5, conclude that \( R(a,a,a) \geq T_k(a,a,a) \).

#### A.5.1 Necessary condition for the interior solution for the tariff

From equation (15) a necessary condition that the foreign firm produces a non-negative amount of the good at \( \sigma = 0 \) is:

\[ q^*(0, \tau_1, \tau_2) = \frac{(1 - c) - 2\tau_1}{3} \geq 0. \]
By using equation (16) we can rewrite it as
\[
\frac{1}{2} \geq \frac{\tau_1(c, 0, \alpha)}{(1 - c)} = \frac{F(0, \alpha, \alpha)}{D(0, \alpha, \alpha)} = \frac{3\alpha + \alpha + 2}{9\alpha + 11\alpha - 2} \\
\iff a + 3\alpha \geq 2 \tag{31}
\]

Assume that the necessary condition for the interior solution (31) always holds.

A.5.2 Ratio \( T_1(\sigma, \alpha, a) = N(\sigma, \alpha, a) / D(\sigma, \alpha, a) \)

By the use of equations (16), (19) and (20) we rewrite
\[
T_1(\sigma, \alpha, a) = \frac{\tau_1(\sigma, \alpha, a)}{\tau_0(\sigma)} = \frac{(\sigma + 3)F(\sigma, \alpha, a)}{D(\sigma, \alpha, a)}.
\]

where \( F(\sigma, \alpha, a) \) and \( D(\sigma, \alpha, a) \) are defined in (18). Denote the numerator of \( T_1(\sigma, \alpha, a) \) by
\[
N(\sigma, \alpha, a) = (\sigma + 3)F(\sigma, \alpha, a).
\]

Both \( N(\sigma, \alpha, a) \) and \( D(\sigma, \alpha, a) \) are decreasing in \( \sigma \) for any \( a, \alpha \). Indeed \( N(\sigma, \alpha, a) \) is a 4-th degree polynomial in \( \sigma \), and \( \partial N(\sigma, \alpha, a) / \partial \sigma < 0 \) both at \( \sigma = 0 \) and \( \sigma = 1 \). Now prove that \( \partial N(\sigma, \alpha, a) / \partial \sigma \) does not change sign on \( \sigma \in [0, 1] \). Note that \( \partial N(\sigma, \alpha, a) / \partial \sigma \) is a 3-rd degree polynomial in \( \sigma \), negative as \( \sigma \to -\infty \) and positive as \( \sigma \to \infty \). If \( \partial N(\sigma, \alpha, a) / \partial \sigma \) were to change sign on \( [0, 1] \), then it should first increase and then decrease, and there would be 2 roots on \( [0, 1] \). As \( \partial^2 N(\sigma, \alpha, a) / \partial \sigma^2 \) is negative at \( \sigma = 0 \), there is no roots of \( \partial N(\sigma, \alpha, a) / \partial \sigma \) on \( [0, 1] \). Therefore, \( \partial N(\sigma, \alpha, a) / \partial \sigma \) is negative and \( N(\sigma, \alpha, a) \) decreases in \( \sigma \).

Similarly, \( D(\sigma, \alpha, a) \) is a 4-th degree polynomial in \( \sigma \), \( \partial D(0, \alpha, a) / \partial \sigma = 0 \) and \( \partial D(1, \alpha, a) / \partial \sigma < 0 \), as long as the necessary condition for the interior solution (31) holds. Note that \( \partial D(\sigma, \alpha, a) / \partial \sigma \) is negative as \( \sigma \to -\infty \) and positive as \( \sigma \to \infty \). The second derivative \( \partial^2 D(\sigma, \alpha, a) / \partial \sigma^2 \) is a quadratic function of \( \sigma \), has its minimum at \( \sigma = 0 \), and is negative at \( \sigma = 1 \). Therefore it is also negative at \( \sigma = 0 \), which, as above, implies that \( \partial D(\sigma, \alpha, a) / \partial \sigma \) is non-positive at \( \sigma \in [0, 1] \). Therefore \( D(\sigma, \alpha, a) \) decreases in \( \sigma \) on \( [0, 1] \).

A.5.3 Necessary and sufficient condition for \( T_1(\sigma, \alpha, a) \) to decline in \( \sigma \)

The derivative of \( T_1(\sigma, \alpha, a) \) is negative if and only if
\[
\frac{\partial N(\sigma, \alpha, a)}{\partial \sigma} / \frac{\partial D(\sigma, \alpha, a)}{\partial \sigma} > \frac{N(\sigma, \alpha, a)}{D(\sigma, \alpha, a)} \iff T_1(\sigma, \alpha, a)
\]
as both \( \partial N(\sigma, \alpha, a) / \partial \sigma \) and \( \partial D(\sigma, \alpha, a) / \partial \sigma \) are negative. Denote
\[
R(\sigma, \alpha, a) \equiv \frac{\partial N(\sigma, \alpha, a)}{\partial \sigma} / \frac{\partial D(\sigma, \alpha, a)}{\partial \sigma}.
\]

With this notation, \( T_1(\sigma, \alpha, a) \) declines in \( \sigma \) if and only if \( R(\sigma, \alpha, a) > T_1(\sigma, \alpha, a) \).
A.5.4 Auxiliary function $A (\sigma, \alpha, a)$, such that $R (\sigma, \alpha, a) \geq A (\sigma, \alpha, a)$

Define $A(a, \alpha, \sigma)$ - a linear function of $\sigma$

$$A(a, \alpha, \sigma) = \frac{3a + \alpha + 2}{9a + 11\alpha - 2} - \frac{8a + \alpha + 8a\alpha + 6\alpha^2 + 2}{(9a + 11\alpha - 2)(37a + 49\alpha - 6)} \sigma.$$ 

Consider the difference $\Delta_{RA}$ between $R (\sigma, \alpha, a)$ and $A(a, \alpha, \sigma)$:

$$\Delta_{RA} = R (\sigma, \alpha, a) - A(a, \alpha, \sigma)$$

$$= \frac{2(\sigma - 1)}{\sigma (9a + 11\alpha - 2)(37a + 49\alpha - 6)} L(\sigma, \alpha, a) *$$

$$* (14a + 22\alpha - 118a\alpha - 81\alpha^2 - 45\alpha^2) + 8\sigma^2 (a + 2\alpha - 1)(a + 2\alpha)^{-1}$$

where $L(\sigma, \alpha, a)$ is a cubic polynomial with respect to $\sigma$,

$$L(\sigma, \alpha, a) = (3a + 5\alpha + 9a\alpha + 7\alpha^2)(37a + 49\alpha - 6)(9a + 11\alpha - 2)$$

$$+ \sigma (37a + 49\alpha - 6)(2a + 6\alpha + 176a\alpha + 105\alpha^2)$$

$$- 39a^3 - 34aa^2 + 9a^2 + 63a^2)$$

$$+ 8\sigma^2 (a + 2\alpha)(21a + 3\alpha - 17a\alpha - 9\alpha^2 + 10)(9a + 11\alpha - 2)$$

$$+ 32\sigma^3 (a + 2\alpha)(a + 2\alpha - 1)(a + 8\alpha + 6\alpha^2 + 2).$$

Consider the signs of components of the product in (32). Note that

$$\frac{2(\sigma - 1)}{\sigma (14a + 22\alpha - 118a\alpha - 81\alpha^2 - 45\alpha^2) + 8\sigma^2 (a + 2\alpha - 1)(a + 2\alpha)^{-1}} = \frac{4(\sigma - 1)}{D_4(\sigma, \alpha, a)} \geq 0.$$ 

Also $(9a + 11\alpha - 2) > 0$ and $(37a + 49\alpha - 6) > 0$ (by the condition (31)). Similarly, the coefficients of the polynomial $L(\sigma, \alpha, a)$ are all positive. Thus, for any $\sigma \in (0, 1)$

$$\Delta_{RA} = R (\sigma, \alpha, a) - A(a, \alpha, \sigma) \geq 0.$$ (34)

A.5.5 Proof of $A(\sigma, \alpha, a) \geq T_1(\sigma, \alpha, a)$

Consider the difference $\Delta_{TA}$ between $T_1(\sigma, \alpha, a)$ and $A(a, \alpha, \sigma)$:

$$\Delta_{TA} = T_1(\sigma, \alpha, a) - A(a, \alpha, \sigma) = \frac{4\sigma M(\sigma, \alpha, a)}{(9a + 11\alpha - 2)(37a + 49\alpha - 6) D(\sigma, \alpha, a)}$$

(35)

where $M(\sigma, \alpha, a)$ is a 4th degree polynomial with respect to $\sigma$,

$$M(\sigma, \alpha, a) = (9a + 11\alpha - 2)(54\alpha + 74a - 238a\alpha - 181\alpha^2 - 211a^3 - 416aa^2 - 189a^2 - 93a^3)$$

$$- 2\sigma (37a + 49\alpha - 6)(2a + 4\alpha + 29a\alpha + 16\alpha^2 - 29\alpha^3 - 49aa^2 - 18a^2 + 9\alpha^3)$$

$$+ \sigma^2 (68a + 124\alpha + 449a^2\alpha^2 - 912a\alpha - 760a^2 + 1555\alpha^3 - 433a^4)$$
Again, study the signs of the components of the product in (35). By the condition (31) \[(9a + 11a - 2) (37a + 49a - 6) > 0.\]

Also, using the condition (31) and the fact that \(D (\sigma, a, a)\) decreases in \(\sigma\) on \([0, 1]\), we have \(D (\sigma, a, a) \leq D (1, a, a) = 8 (a + a) (5a + 7a - 1) > 0.\)

Consider the remaining component \(M (\sigma, a, a)\). Its second derivative \(\frac{\partial^2 M (\sigma, a, a)}{\partial \sigma^2}\) is a convex quadratic parabola with the minimum at some \(\sigma < 0\). Thus, it is increasing at \(\sigma \in 0, 1\). It is positive at \(\sigma = 0\), so we conclude that for any \(\sigma \in 0, 1,\)

\[\frac{\partial^2 M (\sigma, a, a)}{\partial \sigma^2} > 0.\]

It follows that \(M (\sigma, a, a)\) is a convex function of \(\sigma\) at 0, 1 for any \((a, a)\) and reaches its maximum at (either of) the corner points of the segment. But \(M (\sigma, a, a)\) is negative both at \(\sigma = 0\) and \(\sigma = 1\). Therefore \(M (\sigma, a, a) \leq 0,\) which is equivalent to

\[\Delta_{TA} = T_1 (\sigma, a, a) - A (a, a, \sigma) \leq 0.\]  

(37)

A.5.6 Conclusion: \(R (\sigma, a, a) \geq A (\sigma, a, a) \geq T_1 (\sigma, a, a)\)

From (34) and (37), it follows that \(R (\sigma, a, a) \geq T_1 (\sigma, a, a),\) which implies that \(\frac{v_1 (\sigma, a, a)}{v_1^2 (\sigma)}\) decreases with \(\sigma\) for any admissible \((a, a)\). The result that the ratio \(\frac{v_2 (\sigma, a, a)}{v_2^2 (\sigma)}\) increases with \(\sigma\) for any \((a, a)\) is proven in a similar way.

A.6 Derivation of equation (22)

Equation (22) results from solving system (30) for \(I_1 = I_2 = 1, \sigma = 1\) and \(a_L = 2a\). For these parameters, the system consists of two symmetric equations

\[
\left[ -4 \frac{(1 + a)}{(a + 2a)} + 39 \right] \tau_k - \left[ 4 \frac{(1 + a)}{(a + 2a)} + 11 \right] \tau_{-k} = (1 - c) \left[ 1 + 4 \frac{(1 + a)}{(a + 2a)} \right],
\]

which yields

\[v_1^b (1, a, a) = v_2^b (1, a, a) = \frac{(1 - c) 5a + 2a + 4}{4} \frac{5a + 14a - 2}{5a + 14a - 2}.\]
A.7 Proof of Lemma 5

By definition, in a truthful equilibrium, each lobby \( j \) chooses a scalar anchor \( B_j \) so that the government would be just indifferent between choosing the equilibrium policy \( \tau \) and the policy \( \tau^{-j} \), chosen by the government if the contributions of lobby \( j \) were zero:

\[
\tau_{-j} = \arg \max_{\tau \in \mathcal{T}} \sum_{i \in L, i \neq j} C_i(\tau, B_i) + a W(\tau). \tag{38}
\]

Then, the contribution of lobby \( j \) solves

\[
\sum_{i \in L, i \neq j} C_i(\tau_{-j}, B_i) + a W(\tau^{-j}) = \sum_{i \in L} C_i(\tau, B_i) + a W(\tau). \tag{39}
\]

As a consistency check, each lobby \( j \) should make no contribution at the tariff vector \( \tau_{-j} \)

\[
W_j(\tau_{-j}) \leq B_j, \tag{40}
\]

as otherwise it can increase its reservation utility \( B_j \) at no cost.

Let’s show that the government is equally well off in \( \Omega^s \) and \( \Omega^b \). Consider equilibrium \( \Omega^s \). If industry 1 were to contribute zero, the government would get no contributions at all, and would thus maximize the gross social welfare

\[
\tau^s_{-j} = \arg \max_{\tau \in \mathcal{T}} a W(\tau) = \tau_0. \tag{41}
\]

This observation and condition (39) imply that in \( \Omega^s \) the government gets the first-best payoff \( G(\tau_0) \).

Now, turn to \( \Omega^b \). By construction, condition (40) holds and industry 1 does not make any contribution at the tariff vector \( \tau^b_{-1} \):

\[
C_i(\tau^b_{-1}, B_i) = \max \left[ 0, W_i(\tau^b_{-1}) - B_i \right] = 0. \tag{42}
\]

As the goods are perfect substitutes and the two industries are exactly alike, their gross welfare is the same under policy \( \tau^b_{-1} \). As the equilibrium \( \Omega^b \) is symmetric, the anchors of two industries are the same \( (B_1 = B_2) \). Therefore, industry 2 does not contribute anything at the tariff vector \( \tau^b_{-1} \) either. Hence, if the contributions of lobby 1 were zero, the government would choose the free-trade policy

\[
\tau^b_{-1} = \arg \max_{\tau \in \mathcal{T}} a W(\tau) = \tau^0. \tag{43}
\]

The same result holds for trade policy \( \tau^b_{-2} \). Condition (39) implies that in \( \Omega^b \) the government’s payoff is the same as in the first-best equilibrium without any lobbying.

The statement of Lemma 5 follows from this result and condition (23). Higher equilibrium tariffs are further from the first-best outcome. Thus, the government requires higher contributions to compensate for the larger social welfare loss.
A.8 Proof of Proposition 6

Start by calculating lobbying contributions in equilibrium $\Omega^b$. From (39) and (41), it follows that

$$C_1(\tau^*, B_1^*) = aW(\tau^0) - aW(\tau^*).$$

Hence, the net welfare of lobby 1 is

$$V_1(\tau^*) \equiv B_1^* = W_1(\tau^*) - C_1(\tau^*, B_1^*) = W_1(\tau^*) - aW(\tau^0) + aW(\tau^*). \tag{44}$$

As industry 2 is not organized, its net welfare is equal to its gross welfare,

$$V_2(\tau^*) = W_2(\tau^*). \tag{45}$$

In equilibrium $\Omega^b$, conditions (39), (42) and (43) determine the aggregate contributions

$$C_1(\tau^b, B_1^b) + C_2(\tau^b, B_2^b) = aW(\tau^0) - aW(\tau^b).$$

As this equilibrium is symmetric, the anchors of both lobbies are the same and so is their gross welfare. Hence, their contributions are also equal and comprise half of the aggregate amount

$$C_1(\tau^b, B_1^b) = C_2(\tau^b, B_2^b) = \frac{1}{2} \left( aW(\tau^0) - aW(\tau^b) \right).$$

The net payoff of either lobby group is thus

$$V_i(\tau^b) \equiv B_i^b = W_i(\tau^b) - C_i(\tau^b, B_i^b) = W_i(\tau^b) - \frac{1}{2} \left( aW(\tau^0) - aW(\tau^b) \right), \quad i = 1, 2. \tag{46}$$

So in order to determine whether industry 1 gains from the equilibrium with two lobbies, payoffs (44) and (46) must be compared. The difference between the two payoffs is

$$V_1(\tau^b) - V_1(\tau^*) = W_1(\tau^b) - \frac{1}{2} \left( aW(\tau^b) - aW(\tau^*) \right) - \left( W_1(\tau^*) - aW(\tau^0) + aW(\tau^*) \right)
= W_1(\tau^b) - W_1(\tau^*) + \frac{1}{2} \left( W(\tau^0) + W(\tau^b) - 2W(\tau^*) \right). \tag{47}$$

Substituting the expressions for the outputs, profits, volume of imports and domestic consumption into the formulas for the lobby $i = 1, 2$, welfare (4) and aggregate social welfare (5), and simplifying the resulting expressions yields

$$W_i(\tau^*) = \frac{9}{4} (1-c)^2 (a + a) \frac{a - a + 7a^2 + 3aa}{(5a + 7a - 1)^2},$$

$$W_i(\tau^b) = \frac{9}{4} (1-c)^2 (a + 2a) \frac{a - 2a + 14a^2 + 3aa}{(5a + 14a - 2)^2},$$

$$W(\tau^0) = \frac{9}{20} (1-c)^2,$$
\[
W(\tau') = \frac{9}{4} (1 - c)^2 (a + a) \frac{5a + 9a - 2}{(5a + 7a - 1)^2},
\]
\[
W(\tau^b) = \frac{9}{4} (1 - c)^2 (a + 2a) \frac{5a + 18a - 4}{(5a + 14a - 2)^2}.
\]

Inserting into (47), we obtain
\[
V_1(\tau^b) - V_1(\tau') = \frac{9}{20} \frac{(1 - c)^2 a (1 - 2a)^2}{(5a + 7a - 1) (5a + 14a - 2)^2} \geq 0.
\]

Similarly, equations (45), (46) and the formulas above imply that the welfare difference of industry 2 is given by
\[
V_2(\tau^b) - V_2(\tau') = W_1(\tau^b) - \frac{1}{2} (a W(\tau^0) - a W(\tau^b)) - W_2(\tau')
\]
\[
= \frac{9}{20} \frac{(1 - c)^2 a (1 - 2a)^2}{(5a + 14a - 2)} \frac{1}{(5a + 7a - 1)^2} \left\{ \begin{array}{ll}
> 0, & \alpha < 1/7; \\
\leq 0, & \alpha \geq 1/7. 
\end{array} \right.
\]

A.9 Proof of Lemma 7

If industry 2 chooses \( B_2^b \) defined by expression (46), industry 1’s best response is precisely setting \( B_1^b = B_2^b \). Clearly, it is not in industry 1’s interest to decrease \( B_1^b \) (and hence its payoff, in case \( B_1^b \) does not bind). We need to consider two cases. Assume first that \( \alpha > 1/7 \) so that
\[
B_1^b = W_1(\tau^b) - \frac{1}{2} (a W(\tau^0) - a W(\tau^b)) < W_1(\tau'),
\]
which implies that the government gets positive contributions both at policies \( \tau^b \) and \( \tau' \). If industry 1 chooses \( B_1^b > B_2^b \), the government prefers tariff \( \tau^0 \) to \( \tau^b \) and \( \tau' \). Indeed, by equation (7) \( \tau^b \) maximizes the sum of social welfare and the lobbies’ welfare, so that the government prefers \( \tau^b \) to \( \tau^r \):
\[
G(\tau^r) = a W(\tau^r) + (W_1(\tau^r) - B_1^b) + (W_2(\tau^r) - B_2^b)
\]
\[
< a W(\tau^b) + (W_1(\tau^b) - B_1^b) + (W_2(\tau^b) - B_2^b) = G(\tau^b).
\]

In turn, due to our assumption \( B_1^b > B_2^b \), policy \( \tau^b \) does not pay the government enough to deviate from the first-best policy:
\[
G(\tau^b) = a W(\tau^b) + (W_1(\tau^b) - B_1^b) + \frac{1}{2} (a W(\tau^0) - a W(\tau^b))
\]
\[
= (W_1(\tau^b) - B_1^b) + \frac{1}{2} (a W(\tau^0) + a W(\tau^b)) < a W(\tau^0).
\]

If instead \( \alpha \leq 1/7 \), implying that
\[
B_1^b = W_1(\tau^b) - \frac{1}{2} (a W(\tau^0) - a W(\tau^b)) > W_1(\tau'),
\]

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the government never sets $\tau^*$ as it does not get any contributions for this policy. Similarly to above, the government prefers tariff $\tau^0$ to $\tau^b$. Therefore, by increasing $B^b_1$ industry 1 receives a payoff $W_1(\tau_0) < B^b_1$, so this cannot be a best response.

### A.10 Proof of Corollary 9

From (48) and (49) it follows that

$$[V_1(\tau^b) + V_2(\tau^b)] - [V_1(\tau^*) + V_2(\tau^*)] = \frac{9}{4} \frac{a^2 (1 - 2a)^2 (1 - c)^2}{(5a + 14a - 2) (5a + 7a - 1)^2} > 0.$$