Kandidatspeciale
Christian Eriksen Stassen

Testing the Efficient Market Hypothesis
A General Equilibrium Approach to Asset Pricing

Vejleder: Katarina Juselius
Afleveret den: 18/08/09
Abstract

The basic notion of the Efficient Market Hypothesis is that no agent can consistently achieve higher returns than the market return. Previous attempts to model the Efficient Market Hypothesis have been, and still are, plenty. The notion of "not being able to beat the market", meaning that no agent can achieve returns consistently above what the "average" investor can, is theoretically appealing. Most financial models have been built around the EMH, including the famous Black Scholes model and the multiple valuation models, and the theory is used in many other fields of economics, including macroeconomic growth models (often prices bubbles are just assumed not to exist as "markets are efficient").

Empirical results, however, have shown that the EMH is violated in several cases - most of the models focus on showing that asset prices are not random walks, which to some extent shows that asset prices are predictable, and hence an investor who can predict prices could potentially beat the market. None of these empirical models have offered an alternative explanation of what drives financial markets. The sheer notion of how to actually test the EMH is confusing when we disregard the notion of asset prices following random walks, and the discussion of the EMH has to a large degree been abandoned, and current research now focus on behavioral finance explanations for asset prices. With this paper I intend to reopen the discussion.

The purpose of the paper is to propose a model that first of all show a case in which
asset prices are not random walks, but that it still isn’t possible to achieve above-market returns. The model builds on Tobin’s q model, in which asset prices in the long run should follow the prices of the cost of replacement of capital, as agents should invest if capital can be sold for a higher price than it is acquired.

I formulate the model in a single sector model, in which I show that in the long run, asset prices do indeed follow the cost of capital replacement. I also show that in the short run, non-sophisticated investors might cause market anomalies, which, when tested, could lead to some tests showing a lack of market inefficiency. Other authors have tried to include market rigidities.

I extend the model to a multi-sector setting, in order to be able to predict stock indices. The main conclusion is, that each stock index can be treated in separation if mild conditions are satisfied. From this model, I arrive at a testable hypothesis, as the stock indices have a common stochastic component.

I start out the empirical investigation by finding a representation that takes care of a few large outliers, caused by the stock market shocks in 1987, 1998 and 2001. Hereafter I determine the cointegration rank. The theoretical model predicted at least one relation, but my conclusion, though being borderline between a rank of one and a rank of two, ended up with a rank of two.

With this rank, I continued the investigation to find a parsimonious representation, by doing tests for long run excludability, long run exogeneity and endogeneity. The empirical investigation is concluded by a test for long run homogeneity. I accept the test, which indicate the validity of the theory, however with many preconditions which are discussed. The paper then ends by discussing how the model can be extended to be used in financial models and in future research.
Preface

By choosing a finance elective during ones studies, one will without a doubt come across the Efficient Market Hypothesis (EMH), which will be discussed in detail later. The same happened to me when I did my first finance course in 2006, and I was (and still am) a firm believer of the theory. In the spring of 2007, I did a course in macroeconometrics and cointegration, and the exam paper was an empirical investigation of the EMH via a cointegration analysis, using a modified version of the dataset I’m using here. We tested the EMH by testing if share prices followed a random walk, which is the most common way of testing the EMH. As will be explained later, the findings were a rejection of the EMH, which surprised me immensely. During a summer school course a month later, I heard about Tobin’s q, which, as will also be explained later, offer an explanation for why one might reject a random walk without rejecting the EMH.

I found this very interesting, and started looking into research about applications of Tobin’s q in relations to efficient markets. I found that very little research had been done (most applications of Tobin’s q are tests of microeconomic theories of industry supply), and the amount that had been done had been very unsuccessful. Most of the problems arose from measurements of Tobin’s q, as it is not directly observable from aggregated data.

I found this to be very intriguing. If one could ”statistically prove” (whatever that means) a model showing that rejecting a random walk isn’t equal to rejecting the
EMH, a large amount of arguments against the EMH could be discarded (most arguments against the EMH build upon specific examples in which random walks are found not to describe asset prices), and at the same time one might even be able to build a model that could potentially be used to predict future asset prices. I decided to try to combine the results of my exam with the theories I had learned to achieve this goal.

In some respects my idea seemed stupid - most researchers have now moved on from the old discussion of random walks (as there are simply too many evidences against this), and moved into the realm of behavioral finance, where they explain asset price movements to be a product of game theoretical and behavioral features of investor behavior. However, most of these models only explain short run peculiarities, and I believe that a long run model like Tobin’s \( q \) model should offer a better description of asset price movements than these short run model, just like long run growth models describe long run GDP better than most business cycle models. This long run thinking was what ultimately lead me to combine my exam results with the theory, as cointegration is ideal for long run investigations.

I would like to thank first my advisor, Katarina Juselius, both for providing help and advice during the writing process but also to open my eyes for the cointegration approach without which I would have never come across this idea. Also, I would like to thank Jacob Brøchner Madsen, who during my summer school course repeatedly told me to forget ”fancy-shmansy explanations” for asset price movements and convert to the Tobin-belief. Finally, I would like to thank Doug Swanson, a physics graduate student during my year abroad at Princeton University, who was able to point me in the right direction when I ran into problems with a multidimensional system of optimizations. Besides the aid of these three persons, this paper was written without further assistance, and any error or statement within is solely my responsibility.
Contents

1 Introduction 1
  1.1 The Efficient Market Hypothesis ........................................... 1
  1.2 Ketchup Economics .............................................................. 3
  1.3 Macroeconomy vs. Microeconomy ............................................ 4
  1.4 Aim and Structure ............................................................... 5

2 Existing Literature 6
  2.1 Theoretical Literature .......................................................... 7
    2.1.1 Efficient Markets and Mean Reversal ................................. 7
    2.1.2 Extensions of Tobin’s q .................................................... 9
  2.2 Empirical Literature ............................................................ 11
    2.2.1 Testing Efficient Markets and Mean Reversal ...................... 11
    2.2.2 Problems when Testing Tobin’s q ..................................... 13
  2.3 Summary ................................................................................. 14

3 The Single Sector Model 15
  3.1 Setting Up the Model ............................................................. 15
  3.2 The Long Run Dynamics .......................................................... 22
  3.3 The Short Run Dynamics .......................................................... 24

4 The Multi Sector Model 27
  4.1 Extending the Model ............................................................... 27
  4.2 Conclusions of the Theoretical Models ...................................... 30
Chapter 1

Introduction

1.1 The Efficient Market Hypothesis

In 1900, Louis Bachelier submitted his Ph.D thesis "The Theory of Speculation" (see [Bachelier, 1900]), in which he described share prices as following random walks modeled by Brownian motions. His theory thereby suggests that share prices would be unpredictable, as they are local Martingales. Bachelier’s work was largely ignored until the 1960s; however beginning in the 1930s scattered, independent work corroborated his thesis. A small number of studies in the 1930s and 1940s, including work by Alfred Cowles (see [Cowles, 1944]), indicated that US stock prices followed a random walk, and that professional investors were unable to earn excess returns when compared to the market return.

In the early 1960s, Paul Samuelson began circulating Bachelier’s thesis and Cowles’s work among economists, which sparked a renewed interest the area. In 1965 Eugene Fama published his Ph.D thesis (see [Fama, 1965]), arguing for the "Random Walk Hypothesis" (RWH), thereby stating that share prices followed random walks. At the same time, Samuelson published a "proof" (see [Samuelson, 1965]) for a version of the "Efficient Market Hypothesis" (EMH). At the intuitive level, the EMH states that assets reflect their fundamental value, thus rendering it impossible for investors to earn
abnormal returns compared to the market return from investing via simple statistical analysis - the RWH is thus a special case of the EMH. This definition is however somewhat ambiguous, and has later proved to be the critical point when discussing market efficiency.

The theoretical arguments justifying the RWH was the assumption that most economic agents are rational and utility maximizing with unbiased expectations of future asset prices. The mispricing that did occur from irrational and uninformed agents should immediately be arbitrated away by rational investors who would then earn excess returns, increasing their utility. The effects from the irrational and uninformed agents should thus be canceled out quickly, therefore showing up only as "white noise" in the data. Hence, asset prices would behave as random walks.

When translating the RWH into a testable hypothesis, economists distinguished between three different but nested forms of the RWH (see [Fama, 1970]). "Weak form market efficiency" argued that investors shouldn’t be able to earn excess returns by observing only historical asset prices - if that was possible, there would be an arbitrage opportunity which rational investors should exploit. "Semi-strong form efficiency" argued that all publicly available information, such as company earnings and market conditions, should be reflected in asset prices, making it impossible to earn excess returns without having insider knowledge. A market that was efficient in the semi-strong form would therefore also be weak form efficient. "Strong form efficiency" argued that it should be impossible to earn excess returns from both public and private information, thus also excluding the possibility to gain from insider trading. These three forms of the RWH were tested in numerous papers (see [Taylor] for an overview), and were generally thought of as confirming the RWH and hence the EMH.
1.2 Ketchup Economics

Given the theoretical appeal of the RWH, and the empirical findings confirming it, the RWH remained relatively unchallenged as the hypothesis when discussing asset pricing for almost twenty years. Many key asset pricing models that are still used in both theory and practice, including the famous Black-Scholes formula (see [Black and Scholes, 1973]) and the Discounted Cash Flow model (see [Copeland and Weston, 2004]), were built directly upon the assumption of the RWH. This, however, changed in 1986 when Lawrence Summers published his papers on the EMH (see [Summers, 1986a] and [Summers, 1986b]).

He termed the use of the RWH when arguing for market efficiency as "Ketchup Economics" - the no arbitrage logic only allows you to get relative pricing relationships right, like "pricing a quart bottle of ketchup if you know the price of a pint". Determining the overall level of asset prices, and hence the fundamental value of assets, doesn’t follow without specifying risk-preferences and preference aggregation for the economy as a whole. One would have to take a stand on what the required return of investors was before being able to test for market efficiency.

When sceptics argued that statistical "evidence" was clearly in favor of the RWH, Summers asked what one would actually expect to find if asset prices weren’t random walks. Summers proposed an alternative model, in which a mispricing term was present. Even if the model was correct, testing it proves extremely difficult. If the mispricing term displayed large persistence, one would need 5000 years of data in order to reject the RWH despite its invalidity, as the power of the test for inefficiency is very low.
1.3 Macroefficiency vs. Microefficiency

"Ketchup Economics" is perhaps more relevant now than ever before. Paul Samuelson stated in his famous 1998 "Dictum" that "markets seem to exhibit a large amount of efficiency when examined on a micro-level, but to a less extent on a macro-level" (see [Samuelson, 1998]), indicating that assets might be priced correctly relative to each other simultaneously to the overall price level being mispriced. The recent breakdown in the world’s financial markets has shown that this still holds to a great extent; the RWH models that were used to price assets in day-to-day transactions failed to detect the great overpricing that had occurred across all asset classes.

An alternative to the RWH models was given by James Tobin (see [Tobin, 1969]), who proposed a model in which asset prices tends towards a constant (Tobin’s $q$) through a general equilibrium. This means that asset prices are predictable at least in the long run, indirectly invalidating the RWH. Tobin’s theory does not, however, invalidate the EMH - the predictability of returns does not mean that one can earn excess returns. Tobin’s model has been extended in multiple ways, one of which was by Lawrence Summers (see [Summers, 1981]), extending the model into a more realistic framework, in which taxes and subsidies are incorporated into the framework.

![Figure 1.1: Tobin’s q as a predictor, compared to traditional models. [Siegel, 2002]](image)
Despite the appealing nature of Tobin’s model (which I will explain in detail later), and despite the agreement among theorists that the model offered several advantages to existing asset pricing models (see e.g. [Schaller, 1990]), the implementation of the theory has generally been disappointing, with evidence of misspecification of the model, omitted variables and lacking goodness of fit. Many of these problems arise from problems measuring Tobin’s $q$, leading to the RWH being more well documented empirically, and the $q$ model is therefore not much used as a practical tool for asset pricing. The RWH models are therefore still the industry standard when pricing assets, despite its obvious flaws.

1.4 Aim and Structure

The overall aim with this thesis is to formulate a model building on the $q$ theory that is more feasible for practical uses. I will introduce a multi sector version of Summers model, proposing a steady state relation between sectoral share prices. The model will then be used to show why the RWH models fail to predict share prices and why this doesn’t imply that the EMH fails. The main conclusion of the model is a testable hypothesis that overcome the problems that are prevalent when measuring Tobin’s $q$. The model will then be estimated by a cointegrated VAR analysis, using US sectoral share indices, based on data from 1926 to 2006, and compared to the RWH. Here I find cointegration, enabling me to reject the RWH and maybe validate the model.

The rest of the paper is organized as follows: In chapter 2 I discuss other authors’ findings and relate my own methodology to these. In chapter 3 I set up a $q$ model for share prices. Chapter 4 extends the model to a multidimensional setting. In chapter 5 I link the model to the data and explain how to test the theory I’ve built in comparison with the RWH. In chapter 6 I estimate the model and perform the test for cointegration rank. Chapter 7 is the conclusion.
Chapter 2

Existing Literature

In this chapter I will address existing literature on efficient markets and the RWH. The amount of literature is vast, and almost every author writing about the subject presents a unique solution to the problem. I will here list a number of surveys, theoretical as well as empirical, that all offer one or more ideas that are relevant to my paper. A short summary of the findings of the literature presented below is given in the end of this chapter.

As my theoretical finding is that share prices should mean revert in the long run via Tobin’s $q$, I will look at literature on both mean reversion in share prices and on Tobin’s $q$. I will first discuss prior theoretical findings on the subjects. A tax-embedded version of the standard Tobin’s $q$ model is presented in chapter 3; in this chapter I will thus focus on other attempts to model mean reversion in share prices, including extensions of Tobin’s model. Hereafter I will look at literature addressing the empirical issues that arise when attempting to test the RWH and Tobin’s $q$. Using cointegration to test the $q$ model is not widely done, so I will mainly focus on other approaches that have been suggested.
2.1 Theoretical Literature

2.1.1 Efficient Markets and Mean Reversal

As mentioned in the introduction, the literature on the RWH commenced with Bachelier in the early 20th century and was continued by e.g. Samuelson and Fama in the 1960es. Their main argument was that market inefficiencies, in the form of predictability of asset returns, could not exist, as utility maximizing agents would exploit these inefficiencies and thereby increase their utility. The argument was carried on by Fisher Black and Myron Scholes in their nobel winning article on the Black Scholes model (see [Black and Scholes, 1973]) thus starting the era of "modern finance", and ultimately the idea of efficient markets was made public by Burton Malkiel in his book "A Random Walk Down Wall Street" (see [Malkiel, 1973]), in which he let’s a chimpanzee throw darts at stock quotes and show that the chimpanzee performs as well as any mutual fund manager. A thorough survey on the classical literature on the RWH as well as more recent literature can be found in [Malkiel, 2003].

In the mid-eighties criticism of the RWH gained momentum. One of the central contributions was by Lawrence Summers (see [Summers, 1986a]), who proposed the following model for asset prices:

\[ P_t = P_t^* + u_t \]
\[ u_t = \alpha \cdot u_{t-1} + e_t \]

(2.1)

Here, \( P^* \) is the "true", fundamental price of the asset, and \( u \) is a mispricing term. If the mispricing term is close to being non-stationary (i.e., \( \alpha \) close to 1), the test for zero autocorrelation in the returns has very low power (see also [Granger and Roll, 1977]). In order to have a 50% probability of rejecting market inefficiency, one need approximately 5000 years of data.

Summer’s idea quickly became popular although most literature focused on how to
show return predictability could be tested more efficiently without dealing with the question of where these inefficiencies arose from. Campbell and Shiller showed that by imposing a simple transversality condition on asset returns, one can write the log dividend price ratio, $\delta$, as a function of future returns, $r$ and dividend increases $\Delta d$ (see [Campbell and Shiller, 1988]):

$$\delta_t = E \left[ \inf_{i=1}^{\inf} \rho_i \cdot (r_{t+i} - \Delta d_{t+i}) + \text{constants} \right]$$

(2.2)

Here $\rho$ is the autocorrelation of asset prices, which is assumed to be close to but smaller than one. A high log dividend price ratio is thus possibly an indicator of high future returns - this should thus lead to an increased demand for the asset, pushing up price and hence decreasing returns. The power of the test for autocorrelation in the returns is higher than the test proposed by Summers (see below for empirical results of this theory). Fisher Black showed that if risk premia is a decreasing function of wealth (corresponding to a DARA utility function), one would have mean reversion in share returns as the demand for shares goes up and down in resemblance of the business cycle. Share returns exhibiting autocorrelation is thus according to this model not necessarily a consequence of market inefficiency (see [Black, 1990] for a further explanation).

The last decade, the literature has focused on modeling potential sources of apparent inefficiencies through behavioral finance. An example is Hong and Stein; they show that an increase in dispersion of beliefs between analysts and short sales constraints can drive prices further away from fundamentals, thus making analysts disagree even more. Volatility is thus endogenous, and can be used to measure when assets are mispriced (see [Hong and Stein, 2003]). The literature on behavioral finance is vast and rapidly increasing (other examples are [Harrison Hong, 2006] and [Kubik and Stein, 2005]). However, most of these articles address specific market inefficiencies, and do not offer a unified framework in which to explain share return predictability. I will therefore now turn to general equilibrium models (in the form of Tobin's q) to address the RWH from a more general standpoint.
2.1.2 Extensions of Tobin’s q

The standard $q$ model is presented in chapter 3. As discussed below, the $q$ model exhibit weak performance when tested empirically. Here, I will look at extensions made to the $q$ model in order to overcome these difficulties. Most models have addressed how to estimate $q$ for single companies on cross sections, and only a few for aggregate $q$ using time series data. I here look at two papers for single firm $q$ as their ideas generalize to aggregated $q$, and a third paper on how to estimate aggregate $q$.

As I will explain in further detail in chapter 3, $q$ is the ratio of market value of assets to the replacement costs thereof. One of the main problems when testing the $q$ model is the discrepancy between marginal $q$ and average $q$ - when we have to estimate replacement costs for firm capital, we only have accounting figures to rely upon. As the US tax rules abide to conservatism, the GAAP (the accounting standard in the US) often prescribe using historical costs instead of market value when valuing inventory etc. Thus, the figure we get is an ”average” historical cost and not the ”marginal” cost of new capital. Hayashi shows that by assuming that the firm has pricing power, that the cost of installing new capital is linear in $K$ and that a simple transversality condition hold, one obtain a quasi-linear relationship between marginal $q$ (denoted $q$) and average $q$ (denoted $a$) of the following form:

$$q_t = \frac{a_t}{p_t} \cdot \int_0^t f(c,u) \cdot du \quad (2.3)$$

where $p$ is the price level and $f$ is a function of adjustment costs and time (see [Hayashi, 1982] for a further explanation). This means that from a given investment trajectory, we can obtain marginal $q$ by observing average $q$ and the investment trajectory. As can be seen from the paper's expression for $q$ in comparison with the expression I obtain in chapter 3, this model extends without further ado to the aggregate case. The empirical results of this model is discussed below.
Ericson and Withed [Erickson and Whited] looks at another explanation for \( q \)'s bad performance in predicting asset prices. \( q \) is often estimated by the following relationship:

\[
q = \frac{D + E - R}{K}
\]  

(2.4)

where \( D \) is the value of the firm's debt, \( E \) is the value of equity, \( R \) is the replacement costs of the firm inventory and \( K \) is the replacement value of capital stock. The paper shows how both the numerator and the denominator can have measurement errors, and that there is high autocorrelation in these errors. They thus induce a functional form that takes account of the errors, thus making it possible to test the model using measurement error consistent GMM (the empirical results are discussed below). Again, this conclusion extends to the aggregate case.

[Schaller, 1990] looks at potential problems when looking at aggregated data. Schaller shows that differences in investment behavior of firms facing different levels of competition can lead to measurement errors - if the fraction of firms facing imperfect and perfect competition changes over time, one will observe a change in \( q \) even though the investment decisions of individual firms have remain stable. Schaller also shows that when using aggregate data, even when data for marginal \( q \) is available, one might find large serial correlation in the \( q \) values, as aggregate data is an aggregation of flow variable in balance sheet form (through the aggregation of national accounts).

With these theoretical models for how to test the efficiency of capital markets in hand, I will proceed to look at other empirical studies.
2.2 Empirical Literature

2.2.1 Testing Efficient Markets and Mean Reversal

The early literature on tests of efficient markets all focused on looking for autocorrelation in daily, weekly or monthly returns. All of these studies found zero autocorrelation as mentioned above. One of the first studies to find autocorrelation in returns was [Fama and French, 1988]. Using the same data set as I’m using (although a shorter period), they also show that daily, weekly and monthly returns exhibit zero autocorrelation. However, looking at longer horizons, they find negative autocorrelation on the 3-5 year horizon. Hereafter, the autocorrelation goes to zero. Contrary to [Klaus Gugler and Yurtoglu, 2004] (see below), they find that the autocorrelation is even more prone for small cap firms. They also show that for both short and long horizons, variance in returns is much more predictable than returns; a finding that later led to the development of the (G)ARCH model (see e.g. [Bollerslev, 1986]). Fama and French argue that the predictability is most probably due to time varying equilibria. This argument was challenged by [Poterba and Summers, 1986]. They look at half-life’s for shocks to pricing equilibria, and show that this effect die out too fast to explain the findings on 3-5 years. Combined with the argument by Summers stated above, these two papers contradicting beliefs ignited discussion about the validity of the RWH and how to test it.

As mentioned, Campbell and Shiller [Campbell and Shiller, 1987a] found a way to test for autocorrelation without having the problem of low power. They consider two approaches to see if it is possible to find autocorrelation patterns on shorter horizons. A simple VAR shows that a moving average of earnings and the current real price predict the future dividends. They also show that the model is relatively stable against mis-specifications of the definition of the explaining variables - as mentioned above, financial data is often difficult to observe, so robustness is of great importance (and might become even more important as the financial markets in emerging markets become even
more integrated in the world economy). A problem with the model, however, is that we need a fairly long and stable moving average (app. 20 years). As discussed below, this can be difficult due to the changing nature of financial markets.

[Campbell and Shiller, 1987b] also consider NPV valuation models to test for autocorrelation in returns. They show that if a NPV valuation model is true, dividends and the model’s prices should cointegrate. However, the test for cointegration fail, which they conclude again is due to the need for a longer, stable time series. The difficulties of finding a reliable long time series was summarized by [John Campbell and Mackinlay, 1997]: "Overall, there is little evidence for mean reversion in long-horizon returns, though this may be more of a symptom of small sample sizes rather than conclusive evidence against mean reversion—we simply cannot tell." [Ronald Balvers and Gilliland, 2000] addresses this problem by looking at relative share index prices across countries (thus resembling my approach to some extent). They find that by looking at relative share indices, the degree of mean reversion is significant even on 6 month horizons.

Resembling the interest in behavioral finance, many recent studies have focused on particular market inefficiencies. A good overview is given in [Lamont and Thaler, 2003]. A good example of these “tests” is the case of the merger between Royal Dutch and Shell in 2005. New shares were issued in a 60/40 ratio in favor for Royal Dutch holders, and the two shares kept trading apart. Even though the share prices should stay in a 3:2 relationship, large and persisting deviations from this relationship still take place. Cases like this clearly show that market inefficiencies prevail, but offers no clear explanation on why they exist. Thus, I will now turn my attention to a few studies dealing with tests of Tobin’s q.
2.2.2 Problems when Testing Tobin’s q

As mentioned above, Hayashi [Hayashi, 1982] built a theoretical model for the relationship between marginal \( q \) and average \( q \). Using this relationship on Japanese firm data, [Hayashi and Inoue, 2003] shows that while average \( q \) has no significant explanation power in predicting firms’ returns, marginal \( q \) has significant explanation power. As found by Fama and French, the results seem even stronger for smaller firms, although this effect is vaning when the economy is decreasing and capital is more sparse (which is highly relevant in the current crisis). They also consider the \( q \) model in combination with serial correlation structures of technology shocks in the economy. By using this approach they show that \( q \) predicts cashflows more significantly than returns - although no theory has been developed on this, it might be interesting to look further into this, as cash flows are more tightly connected to the investment decision than returns (which is also affected by capex etc.). Equal results (although more significant) was obtained by [Erickson and Whited] using measurement consistent OLS as described above. They try to extend the analysis to time series data, but experience problems with returns being I(1) (in concordance with the surveys mentioned above), thus invalidating their approach.

[Klaus Gugler and Yurtoglu, 2004] extends the model by including R&D expenses in investment figures (instead of focusing on inventories as most other studies do), and obtain more significant results. An interesting conclusion is that the results in this case are worse for smaller firms than for larger firms, opposing the study by Fama and French. They also find that the results become more significant when excluding firms having experienced cash constraints - constrained firms are not free to choose the investment patterns most favorable to them, thus disabling the model underlying the \( q \) theory. These conclusions will be discussed below.
2.3 Summary

As mentioned above, the amount of literature on efficient markets is vast, and the papers presented above might not seem to be very strongly related. I will here summarize the findings:

The validity of the RWH is still much debated. Many tests have been offered in order to assess it, but the main problem when testing for inefficiency is how to construct the test. Models have been constructed to model mean reversion in share prices in many different forms, and other models show inefficiencies in other settings. Many recent empirical studies have found autocorrelation in share prices, but fail to explain this theoretically. One theoretical model for mean reversion in share prices that has been proposed is Tobin’s $q$ model, which I will present in further detail in chapter 3. However, this model has failed most tests due to problems with measuring $q$. Extensions of the model have been made in order to cope with the problems. Most of these extensions only worked when testing on cross sections and didn’t overcome the empirical problems when working with time series data. In chapters 4 and 5 I will present an extension that possibly enables me to test the RWH using Tobin’s $q$ model with a time series of US sectoral share prices. Chapter 6 shows the empirical results.
Chapter 3

The Single Sector Model

3.1 Setting Up the Model

Tobin’s $q$ is defined as the ratio of the market value of capital to the acquisition cost of new capital, which can also be interpreted as the real share price. The model that I intend to investigate is a model for Tobin’s $q$ on a sectoral level. In this chapter, I will set up a single sector version of the model (hence a model for the economy as a whole). As I will later show, the single sector model leads to the same long run conclusions as the multi sector model, and lets me explore the model’s properties and dynamics in a more simple setting. In the next chapter, the model will be extended to a multi sector version.

The single sector model does however suffer from empirical difficulties. Estimating Tobin’s $q$ to use it as an evaluation tool is difficult, as we need to estimate both the market value of capital, and the cost of acquiring new capital (will be shown below), thus estimating the ”marginal $q$”. This proves troublesome, as we only observe historical acquisition costs of capital (book values as reported in financial statements or a price index over last year’s prices for capital goods) and thus only observe the ”average $q$” (see also [Hayashi, 1982]). By extending the model to a multi sector setting, I overcome these obstacles in an elegant manner, rendering an empirical investigation more feasible.
CHAPTER 3. THE SINGLE SECTOR MODEL

The derivation of the model follows Summers (see [Summers, 1981]), although I have left out variables such as depreciation allowances and investment tax rebates in order to simplify the model (these variables are only interesting for short run policy analysis, which is not the purpose of this thesis). I have, however, not excluded all short run variables, as some of them will turn out to be of importance in my empirical investigation of the model. I have also put more emphasis on explaining the properties of the model, and how the model clinches with the RWH.

In the derivation of the single sector model, I assume the existence of a representative firm. The objective of the representative firm is to maximize its shareholder value. I also assume "going concern", that is, the representative firm operate forever, ultimately resulting in an infinite horizon model. The assumption of "going concern" is also underlying most common share valuation models, e.g. the discounted cash flow model and the multiple valuation model, and is indeed part of the accounting rules in most countries. The assumption of "going concern" should therefore not be too controversial (the assumption of a single firm representing the entire economy also indirectly results in the firm operating into infinity - or at least until the end of the world). For a thorough discussion of alternative share valuation models, see [Copeland and Weston, 2004].

Another key assumption of the model is that agents have full information, rendering the model deterministic. The agents thus know the movements in future earnings, GDP etc, and should be able to anticipate how share prices move according to the model. This assumption is of course very unrealistic, and will prove to be the perhaps largest challenge to the model’s conclusion. The assumption will therefore be discussed in length later. I develop the model in continuous time, but a discrete time version of the model yields the same qualitative results (see [Romer, 2006] for a discussion of discrete time versus continuous time). Finally, it is assumed that the economy exhibits decreasing marginal productivity in capital.
In this simple setting it is reasonable to assume that investment depends on the ratio of the market value of existing capital to its replacement cost. Unless an investment of one dollar increases the market value of the firm by more than one dollar, there is no reason to invest. Given the costs of adjustments when implementing new capital and lags in recognition and implementation, there is no reason to expect that all investments that increase market value by more than their cost will be made immediately (this will be explained in detail below). As Tobin argues in his original paper (see [Tobin, 1969]), these considerations lead to an investment equation being a function of Tobin’s \( q \) in the form:

\[
I_t = h \left( \frac{V_t}{P_t \cdot K_t} \right) \cdot K_t \equiv h(q) \cdot K_t
\]

\[h(1) = 0, h' > 0\] (3.1)

Where \( I_t \) is the level of investments, \( V_t \) is the value of the firm, \( P_t \) is the overall price level and \( K_t \) is the level of capital in the economy. In his original article, Tobin defines \( q \) as we have done here. \( q \) thus equals the ratio of the market value of capital to the acquisition cost of new capital or the real share price, as mentioned (the value of capital divided by the nominal capital level). Tobin’s main finding is that \( q \) will be constant equal to one in the long run equilibrium. As I will later show, the tax adjusted model yield an investment equation of similar form, but it is no longer the case that \( q = 1 \) when we include taxes, adjustment costs and debt financing; \( q \) doesn’t even have to be constant in the long run equilibrium despite the deterministic nature of the model, which is probably the root of a large part of the difficulties one meets when trying to measure the level of \( q \) from historical data.

The after-tax nominal returns to shares (assuming a representative firm) are given by the after-tax dividends plus the after-tax capital gains:

\[
(\rho + \pi) V_t = (1 - c_t) \dot{V}_t + (1 - \theta_t) \text{Div}_t
\]

(3.2)

Where \( \text{Div}_t \) is the total dividend payout, \( \rho \) is the required real return on shares, \( \pi \) is the inflation rate, \( c \) is the capital gain tax rate and \( \theta \) is the income tax rate. For later
use, it’s noted that neither of these have to be constant over time. The total dividends are given by the after-tax profits minus investment and depreciation costs plus debt depreciation due to inflation:

\[
Div_t = \left[ P_t \cdot F_t(K_t, L_t) - w_t \cdot L_t - P_t \cdot b_t \cdot i_t \cdot K_t \right] (1 - \tau_t) \\
- \left[ 1 - b_t + (1 - \tau_t) \phi_t \right] \cdot P_t \cdot I_t + P_t \cdot b_t \cdot K_t \cdot \pi_t - \delta_t^R \cdot P_t \cdot K_t
\]

(3.3)

Where \( F \) is (real) GDP as a function of capital \( K \) and labor \( L \), \( \tau \) is the corporate tax rate, \( i \) is the nominal interest rate, \( b \) is the fraction of investment financed by debt instruments, \( \delta^R \) is the depreciation rate of the capital stock, \( w \) is the wage and \( \phi \) is a convex adjustment cost of investments. The inclusion of convex adjustment costs stems from the realistic assumption that installing new capital is costly (and this cost increases with the amount invested) and ensures a unique solution to the maximization problem (see [Summers, 1981] or [Romer, 2006] for a further discussion of the inclusion of (convex) adjustment costs in the investment decision).

Equation (3.2) is a standard ordinary differential equation with the solution (see [Nohel, 1989]):

\[
V_t = \int_t^\infty \frac{1 - \theta}{1 - \epsilon} Div_s e^{-\int_s^t \left( \rho + \pi \right) u du} ds
\]

(3.4)

This equation is the continuous time version of the share value in the discounted cash flow model, essentially saying that the value of the firm equals the sum of all future dividends. The objective of the firm is hence to maximize the value with respect to its capital level (labor is assumed to be fixed).

Normal share valuation models, e.g. the discounted cash flow model and the multiple valuation model, are partial equilibrium models and look solely to this equation when valuating shares. However, contrary to the partial equilibrium models, I add a constraint on the capital level which must be fulfilled on a macro level, stating that the change in economy wide capital must equal the total investments minus depreciation of the existing capital:

\[
\dot{K}_t = I_t - \delta^R_t K_t
\]

(3.5)
The value function is maximized by the Pontryagin maximum principle (see [Sydsæter, 2004]). Suppressing time for easier notation, we substitute equation (3.3) into (3.4) and set up the current-value Hamiltonian:

\[
H = \frac{1 - \theta}{1 - c} \left[ [P \cdot F(K, L) - w \cdot L - P \cdot b \cdot i \cdot K] (1 - \tau) - [1 - b + (1 - \tau) \phi] \cdot P \cdot I + P \cdot b \cdot K \cdot \pi - \delta R \cdot P \cdot K \right] + \lambda \left( I - \dot{K} - \delta^R \cdot K \right)
\]  

(3.6)

where \( \lambda \) is the Lagrange multiplier and hence the shadow cost of new capital stock. Differentiating equation (3.6) with respect to \( I \), we arrive at the first order condition:

\[
-\lambda = \frac{1 - \theta}{1 - c} \left[ - [1 - b + (1 - \tau) \phi] P \cdot I - \frac{P \cdot I}{K} (1 - c) \phi' \right] \Leftrightarrow
\]

(3.7)

In equation (3.7) the left hand side is the after tax shadow price of capital (the effective after-tax value of an additional capital good) and the right hand side is the marginal cost of one extra unit of capital good. This equation must hold in order for us to be in an optimum.

To make the model operational we need to define the shadow cost of new capital stock, \( \lambda \). The shadow cost is a function of both capital and investment which is again a function of capital. Thus, we can invoke the envelope theorem as we have optimized investments with respect to capital (the effect on \( I \) when changing \( K \) is thus zero). We thereby know that the value of shares equals the shadow value of capital (see [Sydsæter, 2004]):

\[
V = \lambda \cdot K
\]

(3.8)

Rearranging this equation yields:

\[
\lambda = \frac{V}{K}
\]

(3.9)

Substituting this expression into equation (3.7) and rearranging we end up with the following expression, with \( q = \frac{V}{pK} \):

\[
\frac{I}{K} = \frac{\frac{V}{pK} \frac{1-c}{1-\theta} - 1 + b - (1 - \tau) \phi}{(1 - c) \phi'} \equiv h(q)
\]

(3.10)
We find that investments are a function of $q$ just like Tobin’s findings. As opposed to Tobin’s original article, $q$ doesn’t necessarily have to equal one in order for investments to be zero, as we now have effects from taxes, subsidies etc. From the equation we see, that if $h(q) > 0$ then it is valuable for investors to increase capital as the tax adjusted market value of capital is larger than the acquisition cost of new capital, and vice versa. In equilibrium we must therefore have $h(q) = 0$ to have $I = 0$, as any other value of $h$ would lead to positive/negative investment. In this way, we can predict investments, and hence profitability, leading to asset prices being predictable.

This is the main difference from the RWH models. In the general equilibrium model, a large profitability leads to increased investments. This is very realistic, as we often observe successful companies’ products being copied by other firms trying to make an easy buck. In a partial equilibrium model, this effect is often ignored, and we then loose the predictability in asset prices, hence the RWH. An example is the multiple valuation model - here, a company is valued by comparing it to its peers based on different multiples, but ignoring potential effects that a high sectoral profitability might have on the capital inflows in the sector.

Now we are in the position to analyze the effects on share prices and investment of tax reforms. Since investment depends on share prices and share prices depend on investment we need to treat the two markets simultaneously. We thus need two first-order differential equations. The first one is the capital market equilibrium condition (equation (3.10)): 

$$\frac{I}{K} = h(q) = \frac{\dot{K}}{K} + \delta^R$$  \hspace{1cm} (3.11)

The steady-state condition for $K$ is then given by setting this equation equal to zero (ignoring the rate depreciation, which is however just a local constant):

$$\dot{K} = 0 \iff q \cdot \frac{1-c}{1-\theta} - 1 + b - (1 - \tau) \phi = 1$$ \hspace{1cm} (3.12)
Rearranging, we get:

\[ q_{t=0} = \frac{1 - \theta}{1 - c} (2 - b + (1 - \tau) \phi) \quad (3.13) \]

We have thus defined the steady state condition for the capital market, and it is easily verified that the steady state \( q \) is a constant horizontal locus given the parameters - this is in some respect a surprising conclusion as it means that share prices are mean reverting to their tax adjusted acquisition costs, and will be discussed in detail later.

The second first-order differential equation is the share market equilibrium condition (equation (3.2)):

\[
(\rho + \pi) \cdot V = (1 - c) \dot{V} + (1 - \theta) \cdot Div 
\quad (3.14)
\]

Inserting our definition of \( q \) we can derive the steady state condition for \( q \):

\[
(\rho + \pi) \cdot q \cdot P \cdot K = (1 - c) \Delta (q \cdot P \cdot K) + (1 - \theta) \cdot Div \iff \\
\dot{q} = \frac{\rho + \pi}{1 - c} q - \frac{1 - c}{1 - \theta} \frac{Div}{P \cdot K} - q \left[ \frac{\pi}{K} + \dot{K} \right] 
\quad (3.15)
\]

The steady-state condition for \( q \) can be found directly from here by setting \( \dot{q} = 0 \), and is given by:

\[
q_{t=0} = \frac{1 - \theta}{\rho + \pi} \frac{Div}{P \cdot K} = \frac{1 - \theta}{\rho + \pi} MP_K 
\quad (3.16)
\]

The last equality follows from the definition of the dividends per share. In steady state, these dividends equals the profit of the firm (in equilibrium, the firm is not accumulating more capital). It then follows from standard microeconomic theory that the firm’s profits equals the marginal product of capital.

I have now derived the dynamics of the model, and the next section will show how these dynamics determine the long run equilibrium. The conclusions will then be used as the foundation when we move into the multi sector setting.
3.2 The Long Run Dynamics

With the $q_{K=0}$ and $q_{q=0}$ loci defined, the long run dynamics of the system follows easily. Mathematically, it is easily verified that the system exhibits saddle path stability (see e.g. [Nohel, 1989]), and has the form shown in figure (3.1) (see [Romer, 2006] or [Barro and i Martin, 2003] for a log-linearization of the model, resulting in a closed form solution for the saddle path):

![Figure 3.1: The phase diagram](image)

The intuition of the dynamics is also clear. Above the $q_{K=0}$ locus following equation 3.13, the value of $q$ is higher than the steady state value on the capital market. Thus, investments are profitable as the market value of a unit of capital is higher than the acquisition costs, and hence investments will be made, thus increasing the amount of capital, and likewise below the $q_{K=0}$ locus. Only on the locus, capital will remain constant.
From equation 3.16, we see that above the $q_{t=0}$ locus, the value of shares is higher than the steady state value on the share market. Thus, given the dividends, the returns are lower than the required return rate, and the share price thus has to increase to yield a capital gain to raise the return rate. Thus, $q$ will increase, hence pushing the value of shares even higher, resulting in a positive bubble in share prices destabilizing the system (a negative bubble will occur when starting out below the $q_{t=0}$ locus).

On the saddle path, the economy will converge against the equilibrium point $E$. From the point $A$ the transition goes as follows: as we are above the $q_{K=0}$ locus, capital will be increased following the dynamics described above. As capital increases, the marginal product decrease (due to the assumption of decreasing marginal returns to capital), lowering profitability and hence dividends. As the dividends are now lower than in the share market equilibrium, the share price will fall to ensure that investors get their required return per share.

The saddle path stability also makes sense intuitively: From the equations (or just by examining the diagram), it can be inferred that paths above/below the saddle path are divergent paths. A point above the saddle path would lead to explosive share prices (a bubble), which isn’t sustainable on the long run. A point below the saddle path would lead to share prices becoming negative, which per definition is impossible. The divergent paths must then at some point jump to the saddle path, but as the model is deterministic, jumps are irrational as all investors would try to avoid the jump by acting before the other agents, correcting the divergence instantaneously. It thus follows that at any given time, the saddle path is the only rational path for the economy. Hence, the long run conclusion is that the economy will follow the saddle path to the long run equilibrium point, $E$. See Summers [Summers, 1981] for a further discussion of the properties of the model.
The long run solutions for \( q \) and \( K \) are easily found from equations (3.13) and (3.16):

\[
K^* = M P_K^{-1} \left( \frac{\rho + \pi}{1 - c} (2 - b + (1 - \tau)) \right)
\]  
(3.17)

Where \( M P_K \) is the marginal product of capital. Since share prices are determined entirely by the \( \dot{K} = 0 \) line the steady state (real) share price is given by:

\[
q^* = \frac{1 - \theta}{1 - c} (2 - b + (1 - \tau) \phi) \equiv \frac{1 - \theta}{1 - c} \cdot M
\]  
(3.18)

The interaction between the capital stock and share prices is clearly seen from the diagram - the long run share price is completely determined by the \( \dot{K} = 0 \) line. This leads to predictability in returns, and thereby an invalidation of the RWH. The predictability does not, however, invalidate the EMH - still, all agents should act rationally and move along the saddle path. It should thereby still be impossible to earn excess returns when compared to the market returns.

The theoretical predictions of equation 3.18 and how it can be tested empirically will be further explained in chapter 5.

### 3.3 The Short Run Dynamics

Although the long-run conclusion of the model is the central point of this paper, I will briefly analyze a demand shock to the economy in order to show how the partial equilibrium models break down. For an analysis of various shocks, see [Summers, 1981] or [Romer, 2006].

As can be seen from equation (3.3), a higher demand yields higher profit and, hence, higher dividends. From equation (3.16), this leads to an upward jump of the \( q_{i=0} \) curve. As the capital level is fixed in the short run, and as the saddle path is the only rational path for the development of the economy, the share price will jump from the old equilibrium, \( E \), to point \( A \), which is shown in figure 3.2 below.
From $A$, the economy will follow the differential equations, moving down to the new equilibrium, $C$, via the saddle path through the dynamics described in the precious section. An interesting point is, that if investors were myopic and hence only considered equation (3.3) without the capital constraint as is done in the partial equilibrium models, the economy would initially jump to point $B$, and follow the $q_{it} = 0$ curve towards equilibrium. The investors would thus in the adjustment period consistently overvalue the share price, and hence investors would consistently earn returns below their required return as the capital losses are greater than expected.

Theoretically, this feature of the model seems to be in great conjunction with observed asset price movements. Studies repeatedly find, that in an economic upturn, investors tend to overestimate profitability and ignore the effect from inflow of capital and inflation (see [IMF, 2003]). A great example of this is the "dotcom" bubble in the late 1990s and the recent housing bubble; in both cases a specific asset class (shares
and real estate, respectively) seemed to outperform the market by a large margin. This lead to an inflow of capital to the asset class that wasn’t recognized by investors, hence creating a bubble. This corresponds to some degree to the predictions from the model - when profitability increases, myopic investors “forget” about the fact that this will lead to increased capital inflow and hence misprice assets (although the deterministic nature of the model is unable to explain bubbles).

As discussed earlier, however, this apparent predictability of the behavior of returns has not showed up in empirical studies. An explanation for this might be the wedge between the observed average $q$ and the actual marginal $q$ - as discussed in e.g. [Romer, 2006], this leads to biased estimates of $q$. Using a biased estimator when trying to predict returns will result in the - perhaps fallacious - conclusion that returns are unpredictable. Hence, the use of $q$ models have not been a great succes in practical work, as the conclusions of the model seem inconsistent with empirics.

The model has now been derived in a single-sector setting. I showed how the model would be able to argue that the RWH doesn’t hold whereas the EMH has not been invalidated. I also discussed some problems that arise when trying to test the model empirically. The next section will show the model functions in a multi-sector setting, which will then be tested.
Chapter 4

The Multi Sector Model

4.1 Extending the Model

The main conclusion from the above model was that the ratio of the market value of capital to the acquisition costs of new capital stock should tend towards a constant; Tobin’s $q$. The idea is now to extend the model to a multi sector setting - the extension enables me to perform an empirical investigation of the model’s properties, overcoming the aforementioned problems arising from the single sector model.

As we saw in the single sector setting, the steady state real share price (Tobin’s $q$) is given by the following expression:

\[ q^* = \frac{V}{P \cdot K} = \frac{1 - \varphi}{1 - c} \cdot M_i \]  

(4.1)

We now assume that there exist $n$ different sectors in the economy. This could be generalized to including different sectors in different countries. This would, however, incorporate currency risk, making the already complex problem even more complex, and will not be discussed here. Each sector, $i = 1, \ldots, n$, will have an individual (index) share price, capital level, price level of capital, and possibly also individual tax rates (an example is the windfall profit tax on US oil companies that was much debated during the 2008 US presidential campaign).
We thus have a state-space system for every sector which should be solved simultaneously. Following the set up in the single sector model, I thus have to solve the following system of optimal control problems:

$$\max_K \left( V_{t,i} = \int_t^{\infty} \frac{1 - \theta_i}{1 - \epsilon_i} \text{Div}_{s,i} e^{-\int_t^s \frac{(\rho_i + \pi_i) u,i}{1 - \epsilon_i} ds} \right) \forall i = 1, \ldots, n$$

(4.2)

This problem should in theory be solvable as it is just a vector valued version of the original one dimensional problem which has already been solved (see [Nohel, 1989]). I have unfortunately not been able to obtain a closed form solution for the system.

However, if certain regularity conditions are fulfilled, the $n$ different systems can be solved individually for the long run equilibrium, leading us back to the single sector model (see [Kamyad et al., 1992]). It is not straightforward to check these conditions, so instead I have done a numerical solution for $n = 2$ using Mathematica, for certain values of the parameters, in order to see whether or not the regularity conditions can be assumed to hold (the simulation code is included in appendix 9). This is of course not a rigorous check, but should on the other hand give a reasonable picture of the dynamics of the system and hence allow me to check the conclusions. In order to check the regularity conditions more thoroughly I ran more simulations in Mathematica, but as they didn’t show other results (no matter which parameters I shocked, the conclusion was the same), I didn’t include these.

I have tried two scenarios - in the first scenario the two sectors have the same tax rates etc., and in the second I have changed them for sector two. The results are shown below in figure (4.1):
As we see, in the above example the regularity conditions seem to be fulfilled. The change in sector two doesn’t seem to affect the long run dynamics of sector one, as this has the same solution in both. The regularity conditions therefore seem like a reasonable assumption, and thus the long run conclusions are straightforward (as will be shown below).

The literature doesn’t answer the question of the short run dynamics (see [Kamyad et al., 1992]), but from the system it is difficult to assess if the short run dynamics in sector one get obstructed by a change in sector two. The short run dynamics are thus very complicated, as a shock to one sector potentially affects every sector, and they will be left out of the analysis as they are irrelevant for the long run conclusion, which is the matter of essence here.

The conclusion about the independent long run dynamics of each sector of the system leads to a sector wise expression for the long run equilibrium (nominal) share price by rewriting equation (4.1), were I’ve assumed that the tax rate is constant across both time and sectors (this will be discussed later):

\[
\frac{V_i}{K_i} = P_i \cdot \frac{1 - \varphi}{1 - c} \cdot M
\]
4.2 Conclusions of the Theoretical Models

I have now derived the model in a single sector setting, and explored the dynamics herein. The main theoretical conclusion was that share prices (value of the company per unit of capital, $q$) in the long run tends toward a constant. Putting the model in a multi sector framework I showed by simulation that this conclusion also holds for the multi sector model, although the short run dynamics now gets very complicated.

The intuition for this result is the following: the value of the company is the (discounted) value of all future dividends. Dividends are usually a fraction of profit (see [Copeland and Weston, 2004]). On an aggregate level, the overall profit in an economy is the GDP of the country. Most long run growth models (e.g. the Solow model - see [Romer, 2006] for a discussion) shows that in the long run, the economy will grow at a steady rate determined by savings and investment in capital and human capital, often in the form of exponential growth. Thus, in the long run, the dividends will also grow at a steady (exponential) rate, with a certain amount of GDP/profit invested in capital. Investors take this into account, and thus the company value will reflect this future rate of growth, and the company value will therefore grow at the same rate, determined solely by the growth of capital. When looking at $q$, we would expect $q$ to be constant.

This was the same conclusion that Tobin reached in his original paper, and the idea that nominal share prices is to be determined only by capital growth caused a big stir in finance academia. After the publication, the model was tested time and time again, but as mentioned it yielded disappointing results.

Any deviation from the long run equilibrium should thus revert back. In the next chapter I will explain how this hypothesis can be tested using cointegration analysis, and how the results clinch with the EMH and the RWH. Chapter 6 shows the results from the estimation.
Chapter 5

Taking the Model to the Data

A small re-arrangement of equation 4.3 gives the following expression:

\[ V_i = K_i \cdot P_i \cdot \frac{1 - \varphi}{1 - c} \cdot M \]  

(5.1)

Taking logarithms, the steady state log-share value for each sector, \( v_i \), is thus a function of the sectoral capital (in nominal terms), \( k_i \), given by the following expression (lower case letters indicating the log of the variable):

\[ v_i = k_i + \mu , \quad \mu = \log \left( \frac{1 - \varphi}{1 - c} \cdot M \right) \]  

(5.2)

As mentioned in chapter 4, most long run growth models predict that the share value will grow at an exponential rate and that capital is a fraction hereof. As investment equals savings in the long run (neglecting net exports), all sectoral share values is thus a function of the overall level of savings/investments (see [Copeland and Weston, 2004] for a further discussion hereof). This leads to a steady state relation between (nominal) savings \( s \), and the share index price:

\[ v_i = f_i(s) \]  

(5.3)

That is, every sectoral (log) share index can be written as a function of savings. This means that the sectoral share indices share one common stochastic component. Thus, we should expect one cointegration relations between the share indices stemming from
the model ([Hayashi, 2000]). In chapter 7, I will discuss the possibility of having even more cointegration relations and the implications hereof in the empirical analysis. Furthermore, unless the function is superlogarithmic, the growth in asset prices should be exponential. A logarithmic correspondence between savings and asset prices is highly implausible, and will not be discussed further (for a discussion, see [Romer, 2006]).

In relation to my discussion of the EMH versus the RWH, this is clearly opposing the RWH. With a cointegration relation it is possible to predict future share prices by deviations from the relation. This would be a clear rejection of the RWH. However, it is not a contradiction of the EMH - as long as we move along the saddle path, noone can get superior returns (since we are comparing the returns to market returns), but we still have predictability of share prices. As will be discussed later, this has great implications for the debate on the EMH.

Without further components in the model, a test of the model would then be a test for cointegration rank of one, whereas the RWH, stating that it is impossible to predict future share prices only from publicly available information (the past price levels), would imply a zero rank. Following [Juselius, 2006], this implies long run homogeneity in the cointegration relations, meaning that one of the $\beta$ vectors (to be defined below) is of the form $(a, a, ..., a)$. In the MA formulation of the model, this means that one of the columns in $\beta_\perp$ has a number of 1’s corresponding to the number of variables in the homogeneity analysis. This is thus the main empirical conclusion of the model, which I will test in chapter 6.

It might seem that the model I have proposed doesn’t offer anything new - after all, I still look at relative pricing between the different sectors. This is however not the case. As opposed to the "Ketchup" models, I have at no point assumed relative mispricings to be arbitraged away, nor have I ignored the overall capital constraint as is done in the
"Ketchup” models. The long run relation between the different variables came out as a natural consequence of the maximization of rational agents.

Many assumptions and simplifications have been made in the development of the model, and even more when linking the model to data. These issues will be discussed in later chapters. With the potential problems in mind, I will now proceed with the empirical analysis.
Chapter 6

Estimating the Model

6.1 Introducing the Data

The data consist of monthly US sectoral share index observations collected from the Fama\&French database from 1926 to 2008 (see [French and Fama]). The data vector contains the following variables, which are all share price indices from June 1926 to June 2008:

\[ x_t' = [L\text{durbl}_t; L\text{nodur}_t; L\text{manuf}_t; L\text{hitec}_t; L\text{telcm}_t; L\text{shops}_t; L\text{utils}_t] \]

where Ldurbl is the log price of durables, Lnodur is the log price of nondurables, Lmanuf is the log price of manufacturing, Lhitec is the log price of high-tech, Ltelcm is the log price of tele-com, Lshops is the log price of shops, and Lutils the log price of utilities. More indices are provided in the database, but as none of these indices had any effect on the results and as they were all long-run excludable, I excluded them to be more parsimonious in my representation. Figure 6.1 presents some of the variables in levels and differences. The rest can be found in appendix 1.
Performing empirical investigations on financial data often yields numerous problems. I initially included the entire sample, but the analysis turned out to be unsatisfying using all this data. Given the discussion in chapters four and five, the tax rates were assumed constant, which is not true for the entire sample period. Furthermore, any functional relation between the variables is almost surely changed during this time period, as both the Great Depression in 29-36, WW2 in 39-45 and the unstable growth in the 60es followed by the oilshocks and high inflation in the 70es were highly unstable periods and led to many reforms and changes in the economic landscape.

This lead to very non-constant cointegration relations as I performed the analysis using the entire sample period. I tried a few sub-periods, and ultimately settled on the period January 1983 to June 2008. This period is characterized by a very stable economic and monetary regime following Volcker’s fight against inflation (although it was not yet completely finished in 1983), and the relevant tax rates were close to constant as well (only major exceptions are Reagan’s tax reform in 1986 and Bush’s dividend tax cut in 2003). I cut off the sample period in June 2008 - the months following were marked by the fall of Lehman Brother’s, the collapse in the Dow Jones Industrial from 13000 to almost 6000 and volatility levels higher than seen ever before. Upon conduction of the empirical analysis in March 2009, the economy continued to register daily shocks that either spurred optimism that markets would rebound or made investors believe in a coming apocalypse. I found it to be difficult to conduct my analysis when the
sample period ended with enormous volatility levels. The abbreviated sample has the advantage that it gives more constant parameters as this period was very stable relative to other periods in the last century, but obviously this sample doesn’t take all available information into account. I will comment hereon in chapter 7. Figure 6.2 presents some of the variables in levels and differences for the abbreviated sample (the rest are in appendix 1).

![Figure 6.2: Levels and differences, short sample](image)

Figures 6.1 and 6.2 presents some of the variables in levels and differences for both periods. First we observe that the variables in levels exhibit a linear trend (although with a build up followed by a collapse around 2001), whereas the differences seem to vary around a constant level, although there are large outliers at a number of dates. The linear trend in the logarithms, corresponding to exponential growth mentioned in chapter 5, is as expected.

We also observe that the manufacturing sector had a much more stable development throughout the period than the high-tech sector - the crash of the dotcom bubble in 2001 barely shows up in the manufacturing sector index whereas it took off almost one third of the high-tech sector index. This illustrates the difference between more defensive sectors (such as manufacturing, shops and utilities) and more aggressive sectors (telecom and high-tech). In relation to the discussion in chapter 4, this would make an investigation of the short-run dynamics very difficult, but hopefully the proposed common stochastic component is strong enough for the long-run analysis to be feasible.
Figures 6.3 show the residuals of the same series from a baseline VAR (the remaining is found in appendix 2):

![Figure 6.3: Residuals, short sample](image)

It is fairly evident that the residuals are far from being a gaussian white noise - we both have a large degree of persistence and several significant outliers (see appendix 3 for skewness and kurtosis tests showing the same conclusion). This lead me to include a range of dummies to be introduced below, which is part of my analysis from here on. Now having described the data, I will proceed with the VAR analysis. I will start out by describing the data and finding a model that fits the data reasonably well. Hereafter, I will test the cointegration rank and use this to find my final model. Ultimately, I will test my theoretical prior by testing Long Run Homogeneity.

The cointegration analysis will be done with the statistical package CATS in RATS, and I will use standard notation when discussing theoretical concepts (see [Juselius, 2006] for a further explanation of the notation).
6.2 The VAR Analysis

6.2.1 Estimation of Baseline VAR

The baseline VAR model is given by the following specification:

$$\Delta x_t = \Gamma \Delta x_{t-1} + \alpha \beta' x_{t-1} + \mu_0 + \mu_1 t + \Phi D_t + \epsilon_t$$

$x_t$ is the 7x1 datavector. $\Gamma$ is the short-term matrix, measuring how an increase/decrease (in the form of a positive/negative difference, respectively) in last period’s log price affect this period’s log price. $\alpha$ is the long-run adjustment matrix, measuring how a shock to the cointegration relationships affect this period’s log price (the “weights” of the cointegrating variables), and $\beta$ is the cointegration matrix. $\mu_0$ and $\mu_1$ are a deterministic constant and trend (which we can decompose as $\alpha \beta_0 + \gamma_0$ and $\alpha \beta_1 + \gamma_1$), respectively, and $\Phi D_t$ are the dummies I will include (their specification will be discussed below).

The linear trends in the levels of the log prices were as expected. Quadratic trends in the log-prices are, however, from a theoretical viewpoint unreasonable as this would lead to explosive growth in the prices, and will thus not be discussed further. Excluding quadratic trends, $\mu_1$ is restricted to be in the cointegration relationships (thus excluding the possibility for quadratic trends in the levels but enabling the possibility for drift in the cointegration relations), and $\mu_0$ is unrestricted, leaving us with a possibility for trends in the levels, corresponding to the theoretical prior (in [Johansen, 2006] this is denoted as case $H^*(r)$). In the ECM representation, this is generally equivalent to $\gamma_1 = 0$ and $\gamma_0, \beta_0, \beta_1 \neq 0$. In CATS, we model the specification of the dummy using the command CIdrift - a priori, I have thus not excluded that the trend cancels in the cointegration relations. With this specification, we have similarity in the test procedure for cointegration rank (see [Nielsen and Rahbek, 2000]).

As mentioned in the initial data analysis, we see a number of large outliers (measured as standardized residuals above 4 in a baseline model). In order to determine how to
CHAPTER 6. ESTIMATING THE MODEL

specify the dummies, I will look at the economic calendar at the given dates. We see very large residuals in the periods October 1987, August 1998 and September 2001. The outliers in October 1987 correspond to the stock market crash on October 19th 1987, "Black Monday", a day that saw a still unexplained decrease in financial markets. The Asian financial crisis, starting in 1997 and expanding into 1998, combined with the Russian financial crisis starting with the state default August 19th 1998 led to the collapse of the hedge fund Long Term Capital Management is probably what is causing the shock in 1998, and finally the terrorist attacks on World Trade Center September 11th 2001 combined with the insecurities faced upon the burst of the dotcom bubble gave a negative shock here. Had I continued with the full sample, numerous shocks should have been included in the 1930ies, 40ies and 70ies.

It is unlikely that these shocks have contributed to savings and investment behavior - although the Asian financial crisis led to a change in Asian behavior, none of the shocks affected the US economy. It is therefore unlikely that the dummies have contributed to a "broken trend" in the levels. I therefore insert a shift-dummy on the given dates, but restrict this to the cointegration relations. I also insert an unrestricted blip-dummy, thus enabling me to test for whether the shocks have changed the cointegration relations (as an example, the crash of the dotcom bubble led to a decrease in the inflow of "easy money" for financing in that sector, and at the same time president Bush gave tax cuts to construction and manufacturing sectors, which could mean that the replacement costs of capital (and hence $q$) would change for these sectors compared to the others). Thus, the shift-dummy accounts for the possible jump in the cointegration relations, whereas the blip-dummy accounts for the "blip" in the variables' differences. Following the standard notation, I thus have to include $\phi_s D_{s,t} + \phi_p D_{p,t}$ on the given dates, where $\phi_s D$ is restricted to the cointegration relation and $\phi_p D_{p,t}$ is unrestricted. The final CATS code for this specification is included in appendix 5, which also shows the CATS results from this estimation without rank imposed.
6.2.2 Test of Model Fit

As noted above, the initial model showed that the model suffered from both skewness and kurtosis problems, residual homoscedasticity and AR effects in the data. Having included the dummies, I will now redo these tests (results are only in the appendix as these tests are not the main focus here). I furthermore tests for parameter constancy.

As can be seen in appendix 4, the model now no longer suffer from severe skewness effects. There is still evidence of kurtosis, but as noted in [Juselius, 2006], this does not interfere with the asymptotic results of the model. The model exhibit moderate ARCH effects; however, [Anders Rahbek and Dennis, 2002] have shown that the tests for cointegration rank (which is a main point in the estimation) are robust to moderate ARCH effects, indirectly eliminating this problem. A likelihood based lag length determination shows that a lag length of two seems to be a correct specification (appendix 6).

I performed a range of tests for parameter constancy. One is shown below, the rest are in appendix 7. A common result is that, despite the now shorter sample period, parameter constancy is not really present (although the parameters are much more stable than in the longer period). This conclusion is independent of the base sample used and the tests used. As mentioned above, one should not expect great stability when discussing financial data. I will therefore not discuss the lack of constancy further in this chapter, but I will return to the discussion in the conclusion.

![Figure 6.4: Constant Log Likelihood Test](image)
6.2.3 Determining Cointegration Rank

In order to determine the cointegration rank, I initially perform the Johansen trace test. As the model contains a large number of dummies, the asymptotic tables (e.g. tables 15.1-4 in [Johansen, 2006]) should be corrected with a $\chi^2(1)$ for each dummy. Instead, I have simulated the distribution in table 1. As shown in [Juselius, 2006], the “top-down” procedure for rank-determination, starting with the hypothesis that $r = 0$ and moving down until we exceed the acceptance threshold, yields the most asymptotically correct result. The I(1) analysis based on the simulated critical values gives the following table:

<table>
<thead>
<tr>
<th>p-r</th>
<th>r</th>
<th>Eig.Value</th>
<th>Trace</th>
<th>Trace*</th>
<th>Frac95</th>
<th>P-Value</th>
<th>P-Value*</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0</td>
<td>0.216</td>
<td>217.452</td>
<td>209.971</td>
<td>192.378</td>
<td>0.001</td>
<td>0.004</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.162</td>
<td>157.665</td>
<td>143.403</td>
<td>155.123</td>
<td>0.041</td>
<td>0.180</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>0.142</td>
<td>113.837</td>
<td>101.322</td>
<td>121.526</td>
<td>0.099</td>
<td>0.301</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0.088</td>
<td>74.316</td>
<td>71.617</td>
<td>91.052</td>
<td>0.595</td>
<td>0.713</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.045</td>
<td>32.815</td>
<td>29.704</td>
<td>64.258</td>
<td>0.976</td>
<td>0.993</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.043</td>
<td>18.848</td>
<td>16.638</td>
<td>41.663</td>
<td>0.940</td>
<td>0.977</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>0.018</td>
<td>5.582</td>
<td>4.748</td>
<td>21.177</td>
<td>0.970</td>
<td>0.987</td>
</tr>
</tbody>
</table>

We have a relatively large amount of observations (306 pr. variable) in the sample period, so the approximation with the asymptotic values should be relatively good. Even so, we see that the conclusion of cointegration rank depends on whether I consider the small sample Bartlett corrected limit values or the regular limits. If I use the Bartlett corrected values, I get $r = 1$ with a p-value of 18%, whereas the non-corrected yields $r = 2$ with a p-value of 9.9% (given a confidence bound of 5%, which might even be too ambitious when dealing with financial data).

This ambiguity of the rank is possibly a consequence of the non-constancy of the relations. As the rank is of crucial importance to my paper, I will therefore incorporate...
further tests and information in order to get a more firm determination the cointegration rank (for a discussion of the methodology, see [Johansen, 2006] or [Juselius, 2006]).

By considering the graphs for the cointegration relations, we see that the first two relations display a fairly high amount of mean-reversion (yet still with some persistence, especially in relation 2), while the third is very persistent. This speaks for setting $r = 1$ or $r = 2$.

Figure 6.5: cointegration relation 1-4
The roots of the companion matrix show, that for \( r = 3 \) the largest root is 0.91, which in practice is difficult to distinguish from a unit root, whereas for \( r = 2 \), the largest root is 0.82, which is more acceptable (although again with some uncertainty). For \( r = 1 \), the largest root is 0.74. This again points towards setting \( r = 1 \) or \( r = 2 \) (see appendix 8 for a list of the companion matrix roots).

By considering the coefficients in the \( \alpha \) matrix, we see, that \( \alpha_1 \) has many coefficients that are significant on a 5% or 10% level in a standard t-test (although in case of a unit-
CHAPTER 6. ESTIMATING THE MODEL

root the test is Dickey-Fuller distributed, rendering the conclusion uncertain), which is also the case for $\alpha_2$ (and $\alpha_3$ has some too). This is again an indicator pointing towards setting $r = 2$.

As a last test, I looked at the recursive trace-test. This test should theoretically grow linearly for $i = 1, \ldots, r$ and be constant for $i = r + 1, \ldots, p$ as it is a function of the eigenvalues (the eigenvalues can be interpreted as the squared canonical correlation coefficient, which are different from zero for the cointegration relations and zero elsewise). The graphs can be seen in appendix 7. Again, the conclusion is ambiguous. In the X-form, where the graphs are not corrected for short term fluctuations, it is more or less impossible to give a conclusion, whereas the R-form with the graphs corrected for short term fluctuations seem to have a trending behavior for especially $r=1$ and a very weekly partial trending behavior for $r=2$. This somehow points towards $r=1$.

All in all, the conclusion of cointegration rank is - as expected from the outset - somewhat unclear, but most tests that gave a clear conclusion pointed towards $r = 2$ (see appendix 5 for the restricted estimate), which is my choice of rank. I performed the analysis with $r = 1$ also, but didn’t reach much different conclusions. This will be discussed in detail in chapter 7.

6.2.4 Model Specification

In this section I work on the model to potentially get a more parsimonious representation. More specifically, I test for whether some of the variables are long run excludable, whether some of the variables are weakly exogenous (corresponding to a zero-row in $\alpha$) and whether some variables are exclusively adapting (corresponding to a unit-vector in $\alpha$). All the tests were performed using built in procedures in CATS unless otherwise stated.
Initiality, I tested if one, two or all of the dummies could be excluded from the cointegration relations (and I did the test for different choices of rank to assess the robustness). All tests were rejected with a p-value of 0 (see appendix 10 for the results). Hereafter I tested if one or more of the dummies were long run excludable, but these tests were also rejected with a p-value of 0 (see appendix 10).

Next, I tested weak exogeneity for each variable individually (appendix 10). This is again an automated test in CATS. I found that all variables except LMANUF and LSHOPS were weakly exogenous (and these two variables were exclusively adapting). I thus test if the remaining five variables are jointly weakly exogenous (appendix 10). We can accept at most $p - r = 5$ zero-rows in $\alpha$, as we need as many free parameters as restrictions (we have $m = 5$ restrictions), which is exactly the number we are testing for here. Additionally we can only have 2 unit vectors in $\alpha$ as its dimension is $7 \times 2$. This is done via the following H matrix:

$$H = \begin{pmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

The test is $\chi^2$ distributed with $m \cdot r = 10$ restrictions. The p-value is 17% (appendix 11), and I thus accept the null of the five variables being jointly weakly exogenous (this is identical to test if LMANUF and LSHOPS are jointly exclusively adapting). I will now go on to perform the test for long run homogeneity, where I will use the results just found.

6.2.5 Test for Long Run Homogeneity

I now perform the test for Long Run Homogeneity. This test is of central performance in this paper - a cointegration rank of 1 or above is already telling us that the RWH is not describing financial markets correctly, but as this is "old news" qua the discussion in chapter 2, the LRH test will show us if Tobin's $q$ model can be said to describe movements in financial markets. As shown below, none of the dummies are excludable. The homogeneity hypothesis, from the theoretical prior, is only restricting one cointe-
CHAPTER 6. ESTIMATING THE MODEL

The first structure is just identified, while the second is over identified. Generic identification is therefore achieved, but economic identification is less clear, as the treatment of the second relation is ambiguous. This gives me the following set of H matrices for the two tests (for a justification of the H matrices, see [Juselius, 2006] section 12.4):

\[
\text{TEST1 } H_1 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\text{TEST2 } H_1 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]

The results from the tests can be seen in table 3 (with the restricted estimation values in appendix 11). Test 1 is accepted, however borderline, with a p-value of 5.1% (5.7% if we Bartlett correct), whereas test 2 is accepted with a p-value of 6.9% (or 9.9% if we Bartlett correct).
The above test was the main test of my paper, and this concludes the empirical investigation. In the next chapter, I will summarize my findings and explain the consequences of my results, as well as look at unresolved issues and ways to proceed for future research.
Chapter 7

Conclusion

7.1 Implications of the Results

The conclusion of the theoretical part of the paper was clear. I formulated the model in a single sector model, in which I show that in the long run, asset prices do indeed follow the cost of capital replacement, however it did not invalidate the EMH as the predictability should be viable to all investors, leaving "everyone” to achieve market returns on their investments. I also show that in the short run, non-sophisticated investors might cause market anomalies, which, when tested, could lead to some tests showing a lack of market inefficiency. I extended the model to a multi-sector setting, in order to be able to predict stock indices. The main conclusion is that each stock index could be treated in separation if mild conditions are satisfied. From this model, I arrive at a testable hypothesis, as the stock indices have a common stochastic component.

The empirical part showed two very important conclusions: first, I found a rank of two, which is higher than the expected zero from the random walk hypothesis. This is "just another proof in the line” that the RWH does not hold, but it is one of the few examples that focus on predictability in broad stock indices (another example is the aforementioned Campbell-Shiller result).
However, as mentioned above, predictability does NOT imply that the EMH is invalidated, but could be a proof for the model (more about this in the next section). I then went on to perform the test for long run homogeneity. It was borderline accepted, indicating that the model could explain the data.

7.2 Unresolved Issues

Having found this result is far from proving that the EMH holds. One thing that needs to be addressed is to see whether or not an application of the model would yield above market returns. In order to assess this, one would have to recursively estimate the model, specify the short run structure and predict returns and hereafter relate the achieved return to the market return.

Another big issue with the results is some of the tests showing that the model fit is far from perfect - even after excluding large chunks of the data (I both excluded the insignificant indices and shortened the data period, as discussed) and including numerous dummies, there were still large problems with constancy of the data.

Also, the borderline acceptance of the test is worrisome - I did try other data periods and other choices of rank, and the results were similar (ranging from slight acceptance to slight rejectance), but having such a borderline conclusion tells us that we still need to work on the model to be more sure in the conclusion. Another feature that was completely ignored in the model was changing tax rates - I do, however, believe this issue is of minor importance, as tax rates tend to vary very little over time. Only major exception would be intrasectoral tax rates, as some sectors (like real estate) tend to get heavily subsidized during a crisis (just like we experience now with the US housing and greentech sectors), which would add an extra stochastic element to the analysis.
The last unresolved issue I would mention here is the fact that we found two cointegration relations, unlike the single one expected from the model. This could be a data issue, but could easily be a result of more present relations. An example of a possible relation derives from the dynamic CAPM - here, all stock returns are equal to the risk free rate plus a stochastic risk premium over the market return. The risk premium could be equal to individual risk plus systemic risk, and hence all indices would share the systemic risk as another common stochastic component, which could increase the rank. This could perhaps be modeled by including a volatility measure, like the VIX, in the model.

7.3 Concluding Remarks

Research has come a long way since Bachelier. Unfortunately most research has been driven away from the core, and tried to model market behavior through more complicated models of behavioral finance. I think it is important not to stray too far away from the core question: what is an efficient market, and how do we test it? The current financial crisis has turned the world upside down, and most modern and complicated models have failed to describe reality. The purpose of the paper was to shed more light on the EMH, and I hope the paper can help put focus on this very fundamental question before researchers and practitioners, having recovered from the crisis, continues their search for ever more complicated models.
Chapter 8

Appendices

8.1 Appendix 1 - Variables in lvls and differences

Levels and differences of the variables:

Figure 8.1: Levels and differences, whole sample

Figure 8.2: Levels and differences, whole sample
Figure 8.3: Levels and differences, whole sample

Figure 8.4: Levels and differences, part sample

Figure 8.5: Levels and differences, part sample

Figure 8.6: Levels and differences, part sample
8.2 Appendix 2 - Residuals

Figure 8.7: Levels and differences, whole sample

Figure 8.8: Levels and differences, whole sample

Figure 8.9: Levels and differences, whole sample
8.3 Appendix 3 - Residual Analysis, corrected model

RESIDUAL ANALYSIS NO DUMMIES

Residual S.E. and Cross-Correlations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Residual S.E.</th>
<th>Cross-Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>DLDURBL</td>
<td>0.0545</td>
<td>1.000</td>
</tr>
<tr>
<td>DLNODUR</td>
<td>0.0397</td>
<td>0.562</td>
</tr>
<tr>
<td>DLMANUF</td>
<td>0.0431</td>
<td></td>
</tr>
<tr>
<td>DLHITEC</td>
<td>0.0704</td>
<td></td>
</tr>
<tr>
<td>DLTELCM</td>
<td>0.0489</td>
<td></td>
</tr>
<tr>
<td>DLSHOPS</td>
<td>0.0465</td>
<td></td>
</tr>
<tr>
<td>DLUTILS</td>
<td>0.0374</td>
<td></td>
</tr>
</tbody>
</table>

DLDURBL 1.000
DLNODUR 0.562 1.000

54
DLMANUF 0.813 0.731 1.000
DLHITEC 0.660 0.422 0.720 1.000
DLTELCM 0.552 0.564 0.593 0.608 1.000
DLSHOPS 0.714 0.739 0.798 0.672 0.627 1.000
DLUTILS 0.323 0.533 0.397 0.122 0.384 0.320 1.000
LOG(|\Sigma|) = -47.741
Information Criteria: SC = -45.629
H-Q = -46.453
Trace Correlation = 0.106
Tests for Autocorrelation
Ljung-Box(75): ChiSqr(3577) = 9665.981 [0.000]
LM(1): ChiSqr(49) = 46.811 [0.562]
LM(2): ChiSqr(49) = 48.440 [0.496]
TEST for Normality: ChiSqr(14) = 89.426 [0.000]
Test for ARCH:
LM(1): ChiSqr(784) = 1421.358 [0.000]
LM(2): ChiSqr(1568) = 2549.307 [0.000]
Univariate Statistics
Mean Std.Dev Skewness Kurtosis Maximum Minimum
DLDURBL 0.000 0.055 -0.584 4.615 0.176 -0.257
DLNODUR 0.000 0.040 -0.519 5.573 0.127 -0.208
DLMANUF 0.000 0.043 -0.929 7.330 0.133 -0.262
DLHITEC 0.000 0.070 -0.359 4.096 0.180 -0.259
DLTELCM 0.000 0.049 -0.289 4.281 0.207 -0.155
DLSHOPS 0.000 0.047 -0.735 6.100 0.105 -0.270
DLUTILS 0.000 0.037 -0.458 3.574 0.100 -0.129
ARCH(2) Normality R-Squared
DLDURBL 1.195 [0.550] 21.178 [0.000] 0.145
8.4 Appendix 4 - Residual Analysis, corrected model

RESIDUAL ANALYSIS DUMMIES
Residual S.E. and Cross-Correlations

\[
\begin{array}{cccccc}
DLDURBL & DLNODUR & DLMANUF & DLHITEC & DLTELCM & DLSHOPS & DLUTILS \\
0.04976432 & 0.03672467 & 0.03777150 & 0.06481295 & 0.04652430 & 0.04223902 & 0.03689809 \\
DLDURBL & 1.000 \\
DLNODUR & 0.493 & 1.000 \\
DLMANUF & 0.774 & 0.688 & 1.000 \\
DLHITEC & 0.625 & 0.362 & 0.684 & 1.000 \\
DLTELCM & 0.544 & 0.542 & 0.592 & 0.593 & 1.000 \\
DLSHOPS & 0.667 & 0.699 & 0.757 & 0.635 & 0.617 & 1.000 \\
DLUTILS & 0.307 & 0.542 & 0.394 & 0.087 & 0.383 & 0.311 & 1.000 \\
\end{array}
\]

\[\text{LOG}(|\Sigma|) = -48.297\]
Information Criteria: SC = -45.393
H-Q = -46.525
Trace Correlation = 0.168

Tests for Autocorrelation Ljung-Box(75): ChiSqr(3577) = 9509.298 [0.000]
LM(1): ChiSqr(49) = 62.392 [0.095]
LM(2): ChiSqr(49) = 65.161 [0.061]
Test for Normality: ChiSqr(14) = 55.036 [0.000]

Test for ARCH:
LM(1): ChiSqr(784) = 1286.445 [0.000]
LM(2): ChiSqr(1568) = 2438.333 [0.000]

Univariate Statistics

Mean Std.Dev Skewness Kurtosis Maximum Minimum
DLDURBL 0.000 0.050 -0.131 2.982 0.160 -0.132
DLNODUR 0.000 0.037 -0.071 3.803 0.123 -0.136
DLMANUF 0.000 0.038 -0.164 3.157 0.112 -0.117
DLHITEC 0.000 0.065 -0.216 3.614 0.162 -0.237
DLTELCM 0.000 0.047 -0.198 4.045 0.193 -0.146
DLSHOPS 0.000 0.042 -0.201 2.961 0.105 -0.118
DLUTILS 0.000 0.037 -0.437 3.632 0.102 -0.124

ARCH(2) Normality R-Squared
DLDURBL 0.834 [0.659] 0.903 [0.637] 0.290
DLNODUR 0.261 [0.878] 8.939 [0.011] 0.256
DLMANUF 0.482 [0.786] 1.788 [0.409] 0.345
DLHITEC 59.921 [0.000] 6.296 [0.043] 0.224
DLTELCM 20.670 [0.000] 13.195 [0.001] 0.174
DLSHOPS 6.478 [0.039] 2.159 [0.340] 0.330
DLUTILS 6.221 [0.045] 9.741 [0.008] 0.108
8.5 Appendix 5 - Final CATS code and estimate

calendar 1926 1 12 ;
allocate 0 2008:07 ;
open data data.xls ;
data(format=xls,org=obs) / Ldurbl Lnodur Lmanuf LHiTec Ltelem Lshops Lutils

*shift dummies
set dum2910s = (t>=1929:10);
set dum3109s = (t>=1931:09);
set dum3208s = (t>=1932:08);
set dum3304s = (t>=1933:04);
set dum3803s = (t>=1938:03);
set dum4005s = (t>=1940:05);
set dum7311s = (t>=1973:11);
set dum8710s = (t>=1987:10);
set dum9808s = (t>=1998:08);
set dum0109s = (t>=2001:09);

*impulse dummies corresponding to the shift dummies
set dum2910p = (t==1929:10);
set dum3109p = (t==1931:09);
set dum3208p = (t==1932:08);
set dum3304p = (t==1933:04);
set dum3803p = (t==1938:03);
set dum4005p = (t==1940:05);
set dum7311p = (t==1973:11);
set dum8710p = (t==1987:10);
set dum9808p = (t==1998:08);
set dum0109p = (t==2001:09);
smpl 1983:01 2008:05
source c:/programmer/cats2/cats.src;
@cats(lags=2,dettrend=cdrift,break=level)
# Ldurbl Lnodur Lmanuf LHITec Ltelem Lshops Lutils
### The Unrestricted Estimates

<table>
<thead>
<tr>
<th></th>
<th>( \hat{\beta} )</th>
<th>( \hat{\alpha} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beta(1)</strong></td>
<td>2.220</td>
<td>8.371</td>
</tr>
<tr>
<td><strong>Beta(2)</strong></td>
<td>-4.699</td>
<td>-1.871</td>
</tr>
<tr>
<td><strong>Beta(3)</strong></td>
<td>-6.408</td>
<td>-5.957</td>
</tr>
<tr>
<td><strong>Beta(4)</strong></td>
<td>4.916</td>
<td>-0.924</td>
</tr>
<tr>
<td><strong>Beta(5)</strong></td>
<td>0.633</td>
<td>-4.947</td>
</tr>
<tr>
<td><strong>Beta(6)</strong></td>
<td>-4.516</td>
<td>7.689</td>
</tr>
<tr>
<td><strong>Beta(7)</strong></td>
<td>-3.502</td>
<td>-2.780</td>
</tr>
</tbody>
</table>

### II

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beta(1)</strong></td>
<td>-6.016</td>
<td>0.131</td>
<td>-0.286</td>
<td>0.098</td>
<td>-0.019</td>
<td>-0.075</td>
<td>0.102</td>
<td>-0.020</td>
<td>0.041</td>
<td>0.029</td>
</tr>
<tr>
<td><strong>Beta(2)</strong></td>
<td>0.034</td>
<td>0.044</td>
<td>-0.165</td>
<td>0.052</td>
<td>0.007</td>
<td>-0.136</td>
<td>0.055</td>
<td>-0.010</td>
<td>-0.021</td>
<td>0.004</td>
</tr>
<tr>
<td><strong>Beta(3)</strong></td>
<td>0.034</td>
<td>0.100</td>
<td>-0.249</td>
<td>0.058</td>
<td>0.031</td>
<td>-0.140</td>
<td>0.071</td>
<td>-0.034</td>
<td>-0.027</td>
<td>0.024</td>
</tr>
<tr>
<td><strong>Beta(4)</strong></td>
<td>0.066</td>
<td>0.057</td>
<td>-0.036</td>
<td>-0.099</td>
<td>0.161</td>
<td>-0.131</td>
<td>-0.123</td>
<td>-0.116</td>
<td>0.004</td>
<td>-0.021</td>
</tr>
<tr>
<td><strong>Beta(5)</strong></td>
<td>0.075</td>
<td>0.094</td>
<td>-0.048</td>
<td>-0.034</td>
<td>-0.034</td>
<td>-0.098</td>
<td>-0.048</td>
<td>0.023</td>
<td>-0.084</td>
<td>0.001</td>
</tr>
<tr>
<td><strong>Beta(6)</strong></td>
<td>0.048</td>
<td>0.174</td>
<td>-0.260</td>
<td>0.071</td>
<td>0.014</td>
<td>-0.247</td>
<td>0.099</td>
<td>0.026</td>
<td>0.003</td>
<td>0.005</td>
</tr>
<tr>
<td><strong>Beta(7)</strong></td>
<td>-0.048</td>
<td>0.007</td>
<td>-0.024</td>
<td>0.013</td>
<td>0.022</td>
<td>-0.020</td>
<td>-0.048</td>
<td>-0.011</td>
<td>-0.011</td>
<td>0.001</td>
</tr>
</tbody>
</table>

### Rank \( r = 2 \) Imposed

#### THE EIGENVECTOR(s) (transposed)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beta(1)</strong></td>
<td>2.220</td>
<td>8.371</td>
<td>-18.898</td>
<td>8.969</td>
<td>-3.600</td>
<td>-9.040</td>
<td>12.164</td>
<td>2.072</td>
<td>-1.270</td>
<td>3.335</td>
<td>0.088</td>
</tr>
<tr>
<td><strong>Beta(2)</strong></td>
<td>-4.699</td>
<td>-1.871</td>
<td>-0.919</td>
<td>-1.579</td>
<td>9.739</td>
<td>-3.378</td>
<td>-3.516</td>
<td>-2.737</td>
<td>-1.324</td>
<td>3.532</td>
<td>0.067</td>
</tr>
</tbody>
</table>

The matrices based on 2 cointegrating vectors:

### \( \hat{\beta}^T \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beta(1)</strong></td>
<td>-0.117</td>
<td>-0.443</td>
<td>1.000</td>
<td>-0.475</td>
<td>0.190</td>
<td>0.478</td>
<td>-0.644</td>
<td>-0.110</td>
<td>0.087</td>
<td>-0.176</td>
</tr>
<tr>
<td><strong>Beta(2)</strong></td>
<td>-0.482</td>
<td>-0.192</td>
<td>-0.094</td>
<td>-0.162</td>
<td>1.000</td>
<td>-0.347</td>
<td>-0.361</td>
<td>-0.281</td>
<td>-0.136</td>
<td>0.363</td>
</tr>
</tbody>
</table>
CHAPTER 8. APPENDICES

8.6 Appendix 6 - Lag Length

LAG LENGTH DETERMINATION

Effective Sample: 1983:06 to 2008:05

MODEL SUMMARY

Model k T Regr Log-Lik SC H-Q LM(1) LM(k)

VAR(5) 5 300 43 7357.532 -43.327 -45.556 0.019 0.362
VAR(4) 4 300 36 7308.878 -43.935 -45.801 0.001 0.008
VAR(3) 3 300 29 7270.721 -44.612 -46.115 <0.001 0.112
VAR(2) 2 300 22 7246.541 -45.382 -46.523 0.122 0.074
VAR(1) 1 300 15 7199.714 -46.002 -46.779 <0.010 0.010

Lag Reduction Tests:

VAR(4) < VAR(5) : ChiSqr(49) = 97.308 [0.000]
VAR(3) < VAR(5) : ChiSqr(98) = 173.622 [0.000]
VAR(3) < VAR(4) : ChiSqr(49) = 76.314 [0.008]
VAR(2) < VAR(5) : ChiSqr(147) = 221.982 [0.000]
VAR(2) < VAR(4) : ChiSqr(98) = 124.674 [0.036]
VAR(2) < VAR(3) : ChiSqr(49) = 48.360 [0.499]
VAR(1) < VAR(5) : ChiSqr(196) = 315.637 [0.000]
VAR(1) < VAR(4) : ChiSqr(147) = 218.329 [0.000]
VAR(1) < VAR(3) : ChiSqr(98) = 142.015 [0.002]
VAR(1) < VAR(2) : ChiSqr(49) = 93.655 [0.000]

8.7 Appendix 7 - Recursive Tests

![Test for Constancy of the Log-Likelihood](image)

Figure 8.13: Constant Log Likelihood Test
CHAPTER 8. APPENDICES

Figure 8.14: Recursive Trace Test

Test of Beta(t) = 'Known Beta'

Figure 8.15: Test of known Beta
**Eigenvalue Fluctuation Test**

\[ \text{Tau(Ksi)} = C(T)[|Ksi(I) - Ksi(T)|] \]

Figure 8.16: Eigenvalue Fluctuation Test

**Transformed Eigenvalues**

\[ \text{Ksi} = \log(\text{Lambda}(1-\text{Lambda})), \quad \text{Sum(Ksi)} = \text{Ksi}(1) + \ldots + \text{Ksi}(r) \]

Figure 8.17: Transformed Eigenvalue Test
## 8.8 Appendix 8 - Roots of the Companion Matrix

The Roots of the COMPANION MATRIX // Model: H(1)

<table>
<thead>
<tr>
<th>Root</th>
<th>Real</th>
<th>Imaginary</th>
<th>Modulus</th>
<th>Argument</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root1</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root2</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root3</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root4</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root5</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root6</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root7</td>
<td>0.740</td>
<td>−0.000</td>
<td>0.740</td>
<td>−0.000</td>
</tr>
<tr>
<td>Root8</td>
<td>0.245</td>
<td>−0.000</td>
<td>0.245</td>
<td>−0.000</td>
</tr>
<tr>
<td>Root9</td>
<td>0.071</td>
<td>0.199</td>
<td>0.211</td>
<td>1.229</td>
</tr>
<tr>
<td>Root10</td>
<td>0.071</td>
<td>−0.199</td>
<td>0.211</td>
<td>−1.229</td>
</tr>
<tr>
<td>Root11</td>
<td>−0.101</td>
<td>0.076</td>
<td>0.126</td>
<td>2.496</td>
</tr>
<tr>
<td>Root12</td>
<td>−0.101</td>
<td>−0.076</td>
<td>0.126</td>
<td>−2.496</td>
</tr>
<tr>
<td>Root13</td>
<td>−0.001</td>
<td>−0.008</td>
<td>0.008</td>
<td>−1.642</td>
</tr>
<tr>
<td>Root14</td>
<td>−0.001</td>
<td>0.008</td>
<td>0.008</td>
<td>1.642</td>
</tr>
</tbody>
</table>

Roots of the Companion Matrix
### The Roots of the COMPANION MATRIX // Model: H(2)

<table>
<thead>
<tr>
<th></th>
<th>Real</th>
<th>Imaginary</th>
<th>Modulus</th>
<th>Argument</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root1</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root2</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root3</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root4</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root5</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root6</td>
<td>0.814</td>
<td>0.064</td>
<td>0.816</td>
<td>0.079</td>
</tr>
<tr>
<td>Root7</td>
<td>0.814</td>
<td>−0.064</td>
<td>0.816</td>
<td>−0.079</td>
</tr>
<tr>
<td>Root8</td>
<td>0.261</td>
<td>0.000</td>
<td>0.261</td>
<td>0.000</td>
</tr>
<tr>
<td>Root9</td>
<td>0.017</td>
<td>−0.186</td>
<td>0.186</td>
<td>−1.482</td>
</tr>
<tr>
<td>Root10</td>
<td>0.017</td>
<td>0.186</td>
<td>0.186</td>
<td>1.482</td>
</tr>
<tr>
<td>Root11</td>
<td>−0.119</td>
<td>−0.081</td>
<td>0.144</td>
<td>−2.544</td>
</tr>
<tr>
<td>Root12</td>
<td>−0.119</td>
<td>0.081</td>
<td>0.144</td>
<td>2.544</td>
</tr>
<tr>
<td>Root13</td>
<td>0.047</td>
<td>−0.017</td>
<td>0.050</td>
<td>−0.351</td>
</tr>
<tr>
<td>Root14</td>
<td>0.047</td>
<td>0.017</td>
<td>0.050</td>
<td>0.351</td>
</tr>
</tbody>
</table>

**Roots of the Companion Matrix**
The Roots of the COMPANION MATRIX // Model: H(3)

<table>
<thead>
<tr>
<th>Root</th>
<th>Real</th>
<th>Imaginary</th>
<th>Modulus</th>
<th>Argument</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root1</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root2</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root3</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root4</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Root5</td>
<td>0.902</td>
<td>-0.105</td>
<td>0.908</td>
<td>-0.116</td>
</tr>
<tr>
<td>Root6</td>
<td>0.902</td>
<td>0.105</td>
<td>0.908</td>
<td>0.116</td>
</tr>
<tr>
<td>Root7</td>
<td>0.669</td>
<td>0.000</td>
<td>0.669</td>
<td>0.000</td>
</tr>
<tr>
<td>Root8</td>
<td>0.302</td>
<td>-0.000</td>
<td>0.302</td>
<td>-0.000</td>
</tr>
<tr>
<td>Root9</td>
<td>-0.014</td>
<td>0.188</td>
<td>0.188</td>
<td>1.643</td>
</tr>
<tr>
<td>Root10</td>
<td>-0.014</td>
<td>-0.188</td>
<td>0.188</td>
<td>-1.643</td>
</tr>
<tr>
<td>Root11</td>
<td>-0.121</td>
<td>-0.087</td>
<td>0.149</td>
<td>-2.516</td>
</tr>
<tr>
<td>Root12</td>
<td>-0.121</td>
<td>0.087</td>
<td>0.149</td>
<td>2.516</td>
</tr>
<tr>
<td>Root13</td>
<td>0.076</td>
<td>0.003</td>
<td>0.076</td>
<td>0.036</td>
</tr>
<tr>
<td>Root14</td>
<td>0.076</td>
<td>-0.003</td>
<td>0.076</td>
<td>-0.036</td>
</tr>
</tbody>
</table>

8.9 Appendix 9 - Mathematica Code

Mathematica code used for simulation:

\[
\text{DSolve}[\frac{K'[t]}{K[t]} == \frac{A}{K[t]} - B, K[t], t]
\]

\[
\text{DSolve}[q'[t] == C*q[t] - D - q[t]*(F + \frac{K'[t]}{K[t]}), q[t], t]
\]

\[
\text{ParametricPlot}[q[t], K[t], t, 0, 2]
\]

\[
\text{SimulParPlot}
\]

function dy = rigid(t,y)

dy = zeros(3,1);

dy(1) = y(2) * y(3);

dy(2) = -y(1) * y(3);

dy(3) = -0.51 * y(1) * y(2);
options = odeset('RelTol',1e-4,'AbsTol',[1e-4 1e-4 1e-5]);
plot(rigid(T,Y))
plot(T,Y(:,1),'-',T,Y(:,2),'-',T,Y(:,3),'.') function dy = vdp1000(t,y)
dy = zeros(2,1);
dy(1) = y(2);
dy(2) = 1000*(1 - y(1))*y(2) - y(1);
plot(T,Y(:,1),'-o')
RunSensAnalysis
GO
Hereafter, a number of buttons in the graphical user interface is used to obtain the results displayed in the paper.

8.10 Appendix 10 - Exclusion and Weak Exo Tests

Test For Variable Exclusion, Weak Exogeneity and Unit Vector in Alpha

<table>
<thead>
<tr>
<th>TEST OF EXCLUSION</th>
<th>DGP</th>
<th>5% C.V.</th>
<th>LDUB</th>
<th>LDLB</th>
<th>LDOUB</th>
<th>LD</th>
<th>LMANUF</th>
<th>LMEPOL</th>
<th>LTELCE</th>
<th>LTELCO</th>
<th>LTELGB</th>
<th>LTELHO</th>
<th>LTELIS</th>
<th>LTELUS</th>
<th>C1(0.97,10)</th>
<th>C1(0.99,5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.841</td>
<td>0.35</td>
<td>0.030</td>
<td>0.025</td>
<td>0.003</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>5.001</td>
<td>0.350</td>
<td>0.001</td>
<td>0.025</td>
<td>0.003</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>7.835</td>
<td>2.75</td>
<td>0.190</td>
<td>0.001</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>9.626</td>
<td>0.05</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>11.070</td>
<td>0.61</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>12.592</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TEST OF WEAK EXOGENEITY</th>
<th>DGP</th>
<th>5% C.V.</th>
<th>LDUB</th>
<th>LDLB</th>
<th>LDOUB</th>
<th>LD</th>
<th>LMANUF</th>
<th>LMEPOL</th>
<th>LTELCE</th>
<th>LTELCO</th>
<th>LTELGB</th>
<th>LTELHO</th>
<th>LTELIS</th>
<th>LTELUS</th>
<th>C1(0.97,10)</th>
<th>C1(0.99,5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.841</td>
<td>0.35</td>
<td>0.030</td>
<td>0.025</td>
<td>0.003</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>5.001</td>
<td>0.350</td>
<td>0.001</td>
<td>0.025</td>
<td>0.003</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>7.835</td>
<td>2.75</td>
<td>0.190</td>
<td>0.001</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>9.626</td>
<td>0.05</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>11.070</td>
<td>0.61</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>12.592</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TEST OF UNIT VECTOR IN ALPHA</th>
<th>DGP</th>
<th>5% C.V.</th>
<th>LDUB</th>
<th>LDLB</th>
<th>LDOUB</th>
<th>LD</th>
<th>LMANUF</th>
<th>LMEPOL</th>
<th>LTELCE</th>
<th>LTELCO</th>
<th>LTELGB</th>
<th>LTELHO</th>
<th>LTELIS</th>
<th>LTELUS</th>
<th>C1(0.97,10)</th>
<th>C1(0.99,5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.841</td>
<td>0.35</td>
<td>0.030</td>
<td>0.025</td>
<td>0.003</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>5.001</td>
<td>0.350</td>
<td>0.001</td>
<td>0.025</td>
<td>0.003</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>7.835</td>
<td>2.75</td>
<td>0.190</td>
<td>0.001</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>9.626</td>
<td>0.05</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>11.070</td>
<td>0.61</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td>12.592</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
8.11 Appendix 11 - Long Run Homogeneity Tests

### 8.11 Appendix 11 - Long Run Homogeneity Tests

**test for joint weak exogeneity of the five variables**

**TEST OF RESTRICTED MODEL: CHISQR(10) = 14.029 [0.170]**

*** No Bartlett Correction for this test ***

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta(1)</td>
<td>0.211</td>
<td>-0.526</td>
<td>-1.000</td>
<td>-0.349</td>
<td>-0.414</td>
<td>0.311</td>
<td>-0.573</td>
<td>-0.558</td>
<td>0.793</td>
</tr>
<tr>
<td>Beta(2)</td>
<td>-0.578</td>
<td>-0.647</td>
<td>1.000</td>
<td>0.258</td>
<td>0.309</td>
<td>0.107</td>
<td>-0.344</td>
<td>-0.566</td>
<td>3.180</td>
</tr>
</tbody>
</table>

**o**

<table>
<thead>
<tr>
<th></th>
<th>Mps(1)</th>
<th>Mps(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DLDURB</td>
<td>0.200</td>
<td>0.000</td>
</tr>
<tr>
<td>DLNODU</td>
<td>0.600</td>
<td>0.000</td>
</tr>
<tr>
<td>DLNODU</td>
<td>0.000</td>
<td>0.200</td>
</tr>
<tr>
<td>DLHIT</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>DLMANU</td>
<td>0.143</td>
<td>0.182</td>
</tr>
<tr>
<td>DLTELC</td>
<td>0.649</td>
<td>0.800</td>
</tr>
<tr>
<td>DLUTIL</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**II**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>DLDURB</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>DLNODU</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>DLHIT</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>DLMANU</td>
<td>-0.186</td>
<td>-0.284</td>
<td>0.325</td>
<td>-0.207</td>
<td>0.084</td>
<td>-0.312</td>
<td>-0.199</td>
<td>-0.083</td>
<td>0.226</td>
</tr>
<tr>
<td>DLTELC</td>
<td>0.322</td>
<td>0.133</td>
<td>-0.414</td>
<td>-0.558</td>
<td>-0.207</td>
<td>-0.083</td>
<td>-0.199</td>
<td>-0.083</td>
<td>0.226</td>
</tr>
<tr>
<td>DLUTIL</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**test of Long Run Pricehomogeneity**

**TEST OF RESTRICTED MODEL: CHISQR(2) = 5.81 [0.031]**

**BARTLETT CORRECTION: CHISQR(2) = 5.580 [0.057] (Correction Factor: 1.008)**
test of Long Run Price homogeneity

<table>
<thead>
<tr>
<th>Test of Restricted Model: CHISQR(2) = 5.88 [0.069]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bartlett Correction: CHISQR(2) = 4.82 [0.090] (Correction Factor: 1.020)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.042</td>
<td>-0.019</td>
<td>0.030</td>
<td>0.012</td>
<td>0.035</td>
<td>-0.029</td>
<td>0.123</td>
<td>-0.216</td>
<td>-0.077</td>
<td></td>
</tr>
<tr>
<td>-1.667</td>
<td>-0.018</td>
<td>-0.328</td>
<td>0.169</td>
<td>0.629</td>
<td>1.702</td>
<td>1.013</td>
<td>1.097</td>
<td>-1.056</td>
<td></td>
</tr>
<tr>
<td>-0.005</td>
<td>0.106</td>
<td>0.027</td>
<td>-0.041</td>
<td>0.014</td>
<td>0.004</td>
<td>-0.000</td>
<td>0.003</td>
<td>-0.015</td>
<td></td>
</tr>
<tr>
<td>-0.219</td>
<td>0.006</td>
<td>1.129</td>
<td>-2.722</td>
<td>0.284</td>
<td>0.502</td>
<td>-0.656</td>
<td>0.081</td>
<td>-0.134</td>
<td></td>
</tr>
<tr>
<td>-0.003</td>
<td>0.003</td>
<td>0.000</td>
<td>-0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>-1.161</td>
<td>2.204</td>
<td>3.564</td>
<td>0.583</td>
<td>0.784</td>
<td>0.034</td>
<td>1.213</td>
<td>0.501</td>
<td>1.590</td>
<td></td>
</tr>
<tr>
<td>-0.171</td>
<td>0.128</td>
<td>0.000</td>
<td>-0.162</td>
<td>-0.041</td>
<td>-0.002</td>
<td>0.020</td>
<td>-0.067</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>-1.576</td>
<td>0.598</td>
<td>0.441</td>
<td>-2.122</td>
<td>-1.522</td>
<td>-0.575</td>
<td>-0.225</td>
<td>0.105</td>
<td>-0.044</td>
<td></td>
</tr>
<tr>
<td>-0.234</td>
<td>-0.000</td>
<td>0.120</td>
<td>0.041</td>
<td>-0.027</td>
<td>0.044</td>
<td>-0.000</td>
<td>-0.054</td>
<td>0.022</td>
<td></td>
</tr>
<tr>
<td>-1.764</td>
<td>-0.230</td>
<td>1.480</td>
<td>1.143</td>
<td>-0.529</td>
<td>-2.704</td>
<td>-0.768</td>
<td>-2.220</td>
<td>0.017</td>
<td></td>
</tr>
<tr>
<td>-0.013</td>
<td>-0.059</td>
<td>0.131</td>
<td>0.062</td>
<td>-0.018</td>
<td>0.042</td>
<td>-0.044</td>
<td>0.027</td>
<td>-0.078</td>
<td></td>
</tr>
<tr>
<td>-0.461</td>
<td>-2.238</td>
<td>1.878</td>
<td>1.301</td>
<td>-0.193</td>
<td>2.144</td>
<td>-1.566</td>
<td>0.373</td>
<td>-0.711</td>
<td></td>
</tr>
<tr>
<td>-3.032</td>
<td>0.214</td>
<td>0.550</td>
<td>0.339</td>
<td>-0.868</td>
<td>-0.230</td>
<td>0.070</td>
<td>0.487</td>
<td>1.665</td>
<td></td>
</tr>
</tbody>
</table>

70
Bibliography


Timothy Erickson and Toni Whited. Measurement error and the relationship between investment and.


Stephen Taylor.