October 2011

MEASURING THE DEADWEIGHT LOSS FROM TAXATION IN A SMALL OPEN ECONOMY

A general method with an application to Sweden

by

Peter Birch Sørensen

University of Copenhagen

Abstract: The paper develops a simple general equilibrium framework for calculating the marginal deadweight loss from taxation in a small open economy. The framework allows a decomposition of the deadweight loss from each tax instrument into the losses stemming from the contraction of the different tax bases. The paper describes a method of calibrating the model which exploits the links between the various factor supply elasticities implied by the standard life cycle model. It also develops a method of estimating effective tax rates that is consistent with optimising household and firm behaviour. To illustrate how the model works, it is calibrated to a data set for Sweden. The quantitative results highlight the importance of accounting for the interaction between the major tax bases when estimating deadweight loss.

Keywords: Deadweight loss, tax policy in a small open economy

JEL code: H21

Address for correspondence:

Peter Birch Sørensen

Department of Economics, University of Copenhagen

Øster Farimagsgade 5, 1353 Copenhagen K, Denmark

E-mail: pbs@econ.ku.dk

CALCULATING THE DEADWEIGHT LOSS FROM TAXATION IN A SMALL OPEN ECONOMY

A general method with an application to Sweden

by Peter Birch Sørensen[1]

I. BACKGROUND: THE GENERAL EQUILIBRIUM APPROACH TO THE MEASUREMENT OF TAX DISTORTIONS

Which tax structure will minimise the efficiency cost of raising the necessary revenue and achieving the desired amount of redistribution? This is a fundamental question for tax economists. To answer it we need to estimate the deadweight loss from the various types of taxes. About half a century ago Arnold Harberger (1964) developed a general equilibrium approach to the calculation of deadweight loss. According to his analysis the (approximate) deadweight loss from imposing a tax on a particular good *k* can be decomposed into the well-known "Harberger triangle" measure of the tax-induced distortion to the market for the taxed good itself and a sum of "tax interaction terms" reflecting the reduction or increase in tax distortions to other markets, as the tax on good *k* induces consumers to substitute towards or away from other taxed goods. For example, if the unit tax τ_k raises the relative price of good *k* by a corresponding amount, motivating consumers to increase their consumption of good *i* by the amount $\tau_k \cdot \left(dX_i / d\tau_k \right)$, the resulting social welfare gain is approximately equal to $\tau_i \cdot \tau_k \cdot \left(dX_i / d\tau_k \right)$. The reason is that, in a competitive market, the tax rate τ_i measures the difference between a consumer's marginal valuation of good *i* and the marginal cost of producing that good, so the tax wedge τ_i indicates the marginal social net gain from a unit increase in the consumption of good *i*.

The total welfare impact the tax interaction effects is $\sum_{i \neq k} \tau_i \cdot \tau_k \cdot \left(dX_i / d\tau_k \right)$ which must be substracted from the distortion imposed on the market for the taxed good itself to obtain a

[1] This paper grew out of a report (Sørensen, 2010a, 2010b) commissioned by the Expert Group on Public Economics (ESO) established by the Swedish Ministry of Finance. In preparing the report I benefited from valuable comments from Peter Englund, Lars Heikensten, Ann-Sofie Kolm, Claus Thustrup Kreiner, Anders Kristofferson and Agnar Sandmo. Åsa-Pia Järliden Bergström and Martin Hill also offered very useful comments and were most helpful in providing me with Swedish data. None of these persons should be held responsible for any remaining shortcomings.

comprehensive measure of the welfare effect of introducing (or raising) a tax. Yet over the years numerous studies of deadweight loss have taken a partial equilibrium approach, focusing only on the Harberger triangle, despite the general equilibrium methodology advocated by the master himself.[2] Sometimes the partial approach has been motivated by the scarcity of empirical knowledge of the cross price effects $dX_i / d\tau_k$ or by the fact that the error caused by ignoring tax interaction effects is likely to be small when the tax rates τ_i on other goods are low. However, as pointed out by Goulder and Williams (2003), if good X_i is leisure, the bias from ignoring the interaction between commodity markets and the labour market will almost surely be large. The reason is that labour is heavily taxed in all developed countries, so the tax wedge between the marginal product of labour and the marginal disutility of work is big. When a tax on some commodity induces substitution towards leisure, the resulting deadweight loss from reduced labour supply will therefore be relatively large. Indeed, in the context of the US tax system Goulder and Williams find that the simple Harberger triangle formula can underestimate the deadweight loss from commodity taxes by a factor of ten or even more by ignoring the interactions between the taxed commodities and the labour market. To make up for this shortcoming, Goulder and Williams develop a simple formula that can be used to produce an unbiased estimate of deadweight loss from commodity taxes when the taxed commodity is "average" in terms of its substitutability with leisure.

This paper is in the spirit of the work by Goulder and Williams. However, whereas they focus on commodity taxes and their interaction with the distortion from the labour income tax, the present analysis incorporates all the main tax instruments such as the labour income tax, consumption taxes, the corporate income tax and taxes on household savings. The goal of the paper is to develop empirically implementable formulae for the marginal deadweight losses from these main tax categories, allowing for tax interactions between the markets for goods, labour and capital. As we shall see, it is possible to write the marginal deadweight losses in terms of a few key own-price and cross-price elasticities and budget shares/income shares. The analysis utilizes the correspondence between the marginal deadweight loss from a tax and the change in net public revenue caused by the behavioural responses to the tax. Our deadweight loss formulae thus allow an estimate of the "degree of self-financing" (DSF) associated with a cut in some tax rate. The DSF is the fraction of the initial revenue lost which is recouped as economic activity responds positively to the tax cut,

[2] Hines (1999) surveys the history of the use of Harberger triangles to calculate welfare losses due to distorted prices.

once all tax interaction effects are accounted for. In policy making circles the DSF is often used to rank the efficiency gains from cuts in the different types of taxes, so a measure of deadweight loss which is tied directly to the DSF should help to broaden its appeal.

Modern analyses of the welfare effects of different taxes often rely on the use of computable general equilibrium (CGE) models of the type pioneered by Shoven and Whalley (1972) and Ballard et al. (1985). One advantage of using large-scale CGE models is that they allow a very detailed description of the economy. But big models sometimes take a "black box" character, containing complex general equilibrium effects that make it hard to identify the key mechanisms driving the simulation results. The present paper also adopts a general equilibrium approach to welfare analysis, but it relies on a simple and transparent model which makes it easy to identify the key parameters determining the welfare effects.

Although our model is simple, its use for quantitative analysis does require knowledge of the cross-price elasticities between saving and labour supply about which relatively little is known. To overcome this difficulty, the paper uses the standard life cycle model of saving and labour supply to derive links between the relevant cross-price elasticities and the wage elasticity of labour supply about which much more is known.

The paper focuses on a small open economy because most national economies are in fact small and open. With this focus we can highlight the important distinction between residence-based taxes on domestic saving (like the personal capital income tax) and source-based taxes on domestic investment (such as the corporate income tax). We can also illustrate the different welfare effects of taxing labour directly through the labour income tax and taxing workers indirectly through the source-based corporate income tax that tends to get shifted to wage earners in a small open economy. Indeed, it is well known from the contributions by Gordon (1986) and Razin and Sadka (1991) that is is inoptimal to levy a source-based tax on the normal return to capital in a small open economy with perfect capital mobility rather than taxing labour directly through the labour income tax. The analysis in this paper allows us to quantify how much more efficiency is lost by taxing labour indirectly through the corporate income tax. Thus, after having developed our general formulae for the marginal deadweight loss from the main types of taxes, we will illustrate how they can be used empirically by applying them to Swedish data. This exercise will highlight the quantitative importance of allowing for tax interaction effects in welfare analysis.

As mentioned, numerous studies of the welfare costs of taxation in an open economy have been based on large-scale CGE models (e.g., Jensen et al., 1994 and 1996), but relatively few studies have derived analytical expressions for deadweight losses in a general equilibrium context. The paper by Diewert (1983) focuses on tax distortions within the production sector of a small open economy without incorporating interactions with the household sector. Apart from the article by Goulder and Williams (2003) already mentioned, the two most direct sources of inspiration for the present work are the papers by Hansson (1984) and Benge (1999). Hansson (op.cit.) sets up a two-sector small open economy model to estimate the marginal cost of public funds associated with different tax instruments, but his modelling af savings behaviour is more rudimentary than here. Like the present paper, the study by Benge (op.cit.) allows an estimate of the additional efficiency loss caused by taxing labour indirectly through the source-based corporation tax rather than through the labour income tax, but his model does not incorporate consumption taxes and taxes on saving.

The rest of the paper proceeds as follows. Section II describes the theoretical general equilibrium framework underlying our analysis and section III uses that framework to derive formulae for the marginal deadweight loss from the various taxes. Section IV presents a general method for calibrating the parameters in the formulae. In section V we calibrate the model to Swedish data and present our estimates of the deadweight losses caused by the current Swedish tax system. The concluding section VI summarises the main findings of the paper.

II. A SIMPLE GENERAL EQUILIBRIUM FRAMEWORK FOR CALCULATING DEADWEIGHT LOSS IN A SMALL OPEN ECONOMY

This section presents our theoretical framework for estimating the marginal deadweight loss from taxation. Our model describes a long-run equilibrium in a small open economy where capital is perfectly mobile across borders whereas labour is immobile.[3] With perfect capital mobility the domestic equilibrium real interest rate is exogenously given from the world capital market, assuming that uncovered interest parity and relative purchasing power parity prevail in the long run. Firms maximise profits using inputs of capital and labour, taking product and factor prices as given. Households optimise their labour supply and the allocation of consumption over time. For concreteness, we model household behaviour by the standard two-period life cycle model where consumers work during the first period of life and save part of their income for the second period in which they are retired. The government levies a source-based business income tax on the return to

[3] Actually our framework can accommodate some international labour mobility since this can be modelled pragmatically as an increase in the wage elasticity of labour supply. The critical assumption is that labour is imperfectly mobile whereas capital mobility is perfect. This difference in the degree of mobility seems realistic.

domestic investment, a residence-based capital income tax on the return to domestic household saving, a labour income tax and an indirect tax on consumption. Part of the revenue from these taxes is used to finance transfers to workers and pensioners. The following subsections describe the model economy in more detail before deriving our measure of deadweight loss.

II.1. Households

The life cycle of the representative consumer is divided into two periods: during the first period she participates in the labour market, and in the second period she is retired and finances consumption by previous savings and by a government transfer. The consumer's lifetime utility is given by the well-behaved utility function

$$U = U\left(C_1, C_2, L\right), \tag{1}$$

where *L* is labour supply during young age, and C_1 and C_2 is total consumption during young and old age, respectively. When specifying the consumer's budget constraint, we choose the world market producer price of goods as our numeraire. Since we will focus on the effects of permanent changes in tax policy, we assume that, after some initial policy change, all tax rates remain constant over the life course of the cohort considered. With the producer price index serving as the numeraire, the consumer price index *P* (which includes indirect taxes) will therefore also be constant over the consumer's life cycle. The budget constraint during young age may then be written as

$$PC_1 = WL - T - PS, \tag{2}$$

where *W* is the wage rate, *T* denotes net direct taxes paid during the working career (taxes minus transfers), and *S* is real saving. During retirement, the consumer's budget constraint is

$$PC_2 = \left[1 + r\left(1 - t^r\right)\right]PS + B_2. \tag{3}$$

The variable *r* is the pre-tax real interest rate, t^r is the effective capital income tax rate on the real return to saving, and B_2 represents (after-tax) government transfers to retirees. When analysing the marginal deadweight loss from the taxation of labour income, we will consider the effect of an identical increase in the marginal tax rates of all taxpayers. For this purpose we may approximate the tax-transfer schedule faced by the working population by the linear tax schedule

$$T = t^w WL - B_1, \qquad 0 < t^w < 1, \qquad B_1 > 0, \tag{4}$$

where t^w is the effective marginal tax rate on labour income (including social security taxes as well as personal income tax), and B_1 is a lump-sum transfer to people of working age. Note that although the marginal tax rate is constant, (4) implies that taxation is progressive in the sense that the average tax rate $T/WL = t^w - B_1/WL$ is increasing in total labour income *WL*. In the empirical application of the model t^w is estimated as a weighted average of the effective marginal labour income tax rates across all taxpayers.

Combining (2) through (4), we obtain the consumer's lifetime budget constraint

$$PC_1 + pPC_2 = W\left(1-t^w\right)L + B_1 + pB_2, \qquad p \equiv \frac{1}{1+r\left(1-t^r\right)}, \tag{5}$$

where *p* is the relative price of future consumption. To maximise lifetime utility (1) subject to the lifetime budget constraint (5), the consumer must minimise the expenditure *E* needed to attain a given utility level $\bar{U}$, i.e., she must solve the problem

$$\underset{\text{w.r.t. } C_1, C_2, L}{\text{Minimize}} \quad E \equiv PC_1 + pPC_2 - W\left(1-t^w\right)L \quad \text{subject to} \quad U\left(C_1, C_2, L\right) = \bar{U}. \tag{6}$$

The solution to this problem yields an expenditure function of the form $E = E\left(P, p, W\left(1-t^w\right), \bar{U}\right)$. From standard duality theory, the derivatives of this expenditure function are

$$\frac{\partial E}{\partial W\left(1-t^w\right)} = -L, \tag{7}$$

$$\frac{\partial E}{\partial P} = C_1 + pC_2, \tag{8}$$

$$\frac{\partial E}{\partial p} = PC_2 = p^{-1}\left(PS + pB_2\right), \tag{9}$$

where the last equality in (9) follows from the consumer's second-period budget constraint (3). Given the lifetime budget constraint (5), optimal household behaviour implies a labour supply function of the form

$$L = L\left(w, p, I\right), \qquad w \equiv \frac{W\left(1-t^w\right)}{P}, \qquad I \equiv \frac{B_1 + pB_2}{P}, \tag{10}$$

where is the marginal after-tax real consumer wage and *I* is the present value of the household's exogenous real income.

II.2. Firms

Domestic output (Y) is produced by combining an aggregate capital good K with domestic labour L in the following production function where the subscripts indicate partial derivatives:

$$\begin{aligned} &Y = F(K,L), \qquad F_K > 0, \quad F_L > 0, \\ &F_{KK} < 0, \quad F_{LL} < 0, \quad F_{KL} = F_{LK} > 0. \end{aligned} \tag{11}$$

The function $F(K,L)$ is assumed to display constant returns to scale. The net profit Π of the representative domestic firm is

$$\Pi = Y - \rho K - WL, \qquad \rho \equiv r + \delta + t^k, \tag{12}$$

where ρ is the user cost of capital consisting of the sum of the net rate of return required by the international capital market (r), the rate of depreciation (δ), and the source-based tax on business capital (t^k), measured per unit of capital. In the following, we shall also refer to t^k as the investment tax wedge. The representative competitive production firm maximises (12) subject to (11), yielding the standard first-order conditions:

$$F_K(K,L) = \rho, \tag{13}$$

$$F_L(K,L) = W. \tag{14}$$

The user cost of capital is determined by the exogenous world interest rate and by the parameters of the business tax system. The consumer price index (P) is determined by the various indirect tax rates, as we shall lay out in detail in section IV, and the present value of transfers (I) and the marginal income tax rates t^w and t^r are likewise exogenous. Given these parameters, the system (10), (13) and (14) then determines K, L, and W. To calculate marginal deadweight losses, we will need to know how the direct and indirect tax rates (with the latter working through P) affect these variables. Exploiting the homogeneity properties of the production function, one can derive the following results:

$$\text{Effects of a change in } t^w\text{:} \qquad \frac{dL}{L} = \frac{dK}{K} = -\varepsilon_w^L \cdot \left(\frac{dt^w}{1-t^w}\right), \qquad \varepsilon_w^L \equiv \frac{\partial L}{\partial w}\frac{w}{L}. \tag{15}$$

$$\text{Effects of a change in } P\text{:} \qquad \frac{dL}{L} = \frac{dK}{K} = -\varepsilon_w^L \cdot \left(\frac{dP}{P}\right). \tag{16}$$

$$\text{Effects of a change in } t^r: \quad \frac{dL}{L} = \frac{dK}{K} = -\varepsilon_r^L \cdot \left(\frac{dt^r}{1-t^r} \right), \qquad \varepsilon_r^L \equiv \frac{\partial L}{\partial r\left(1-t^r\right)} \frac{r\left(1-t^r\right)}{L}. \tag{17}$$

$$\text{Effects of a change in } \rho: \quad \frac{dL}{L} = -\varphi \varepsilon_w^L \cdot \left(\frac{d\rho}{\rho} \right), \qquad \varphi \equiv \frac{\rho K}{WL}, \tag{18}$$

$$\frac{dK}{K} = -\left(\varepsilon_\rho^K + \varphi \varepsilon_w^L\right)\left(\frac{d\rho}{\rho}\right), \qquad \varepsilon_\rho^K \equiv -\frac{\partial K}{\partial \rho}\frac{\rho}{K} = -\frac{1}{F_{KK}}\frac{\rho}{K}. \tag{19}$$

The first equalities in (15) through (17) follow from the fact that, under constant returns to scale, the marginal products of capital and labour depend only on the capital-labour ratio. According to (13) *K/L* must therefore be constant – implying *dL/L=dK/K* – as long as the user cost of capital is constant. In other words, when labour supply changes as a result of a change in t^w, *P* or t_r, the capital stock must change by the same relative amount, and these changes in factor inputs will depend on the elasticities of labour supply with respect to the real after-tax wage rate and the real after-tax interest rate. To understand the results in (18) and (19), it is useful to decompose the total effect of a change in the user cost on the stock of capital into a "direct" and an "indirect" effect. The direct effect is the change in the capital stock that would occur if labour supply remained constant at its initial level. If the change in the user cost of capital is $d\rho$, it follows from (13) that this direct effect is

$$\frac{\partial K}{\partial \rho} d\rho = \frac{d\rho}{F_{KK}} \quad \Rightarrow \quad \left(\frac{dK}{K}\right)_{\text{direct}} = -\varepsilon_\rho^K \cdot \frac{d\rho}{\rho}, \qquad \varepsilon_\rho^K \equiv -\frac{\partial K}{\partial \rho}\frac{\rho}{K} = -\frac{1}{F_{KK}}\frac{\rho}{K} > 0, \tag{20}$$

where ε_ρ^K is the (numerical) elasticity of capital demand with respect to the user cost, calculated at the initial level of labour supply. But there is also an indirect effect on the capital stock since the rise in the user cost will discourage labour supply. According to (14) and (20) the impact on the producer real wage is

$$dW = F_{LK} \cdot \frac{\partial K}{\partial \rho} \cdot d\rho = \frac{F_{KL}}{F_{KK}} \cdot d\rho, \tag{21}$$

where we have used the fact that $F_{LK} = F_{KL}$. Since the production function $F\left(K,L\right)$ is homogeneous of degree one, the marginal product function $F_K\left(K,L\right)$ is homogeneous of degree zero, implying

$$K \cdot F_{KK} + L \cdot F_{KL} = 0 \quad \Leftrightarrow \quad \frac{F_{KL}}{F_{KK}} = -\frac{K}{L}, \tag{22}$$

so that (21) becomes

$$dW = -\frac{K}{L} \cdot d\rho. \tag{23}$$

As stated in (10), the marginal real consumer wage governing labour supply is $w \equiv W\left(1-t^w\right)/P$, so from (23) we get

$$dw = \left(\frac{1-t^w}{P}\right) dW = -\left(\frac{1-t^w}{P}\right) \frac{K}{L} \cdot d\rho. \tag{24}$$

Hence the change in labour supply induced by the change in the user cost of capital is

$$dL = \frac{\partial L}{\partial w} \cdot dw = -\left(\frac{1-t^w}{P}\right) \frac{K}{L} \frac{\partial L}{\partial w} \cdot d\rho, \tag{25}$$

which in turn leads to (18). Ceteris paribus, this drop in labour supply induces a similar relative drop in capital input as firms adjust the capital stock to maintain the optimal capital-labour ratio, so from (18) we get the indirect effect on *K* of a change in ρ:

$$\left(\frac{dK}{K}\right)_{\text{indirect}} = -\varphi \varepsilon_w^L \cdot \left(\frac{d\rho}{\rho}\right), \qquad \varphi \equiv \frac{\rho K}{WL}. \tag{26}$$

Adding the direct and the indirect effects in (20) and (26), we end up with (19). The important insight from this analysis is that a (tax-induced) rise in the cost of capital reduces labour supply because the fall in capital demand reduces the pre-tax wage rate, and the drop in labour supply in turn amplifies the initial drop in investment.

II.3. The government

We wish to derive a measure of deadweight loss that captures the effect of taxation on the lifetime utility of a representative cohort. For this purpose we shall need an expression for the present value of the net tax revenue collected from each cohort. It will be convenient to express the effective indirect tax rate on consumption (t^c) as a fraction of the consumer price index *P*. The present value of the net taxes paid by a cohort (*R*) is then given by

$$R = t^c \left(PC_1 + \frac{PC_2}{1+r\left(1-t^r\right)} \right) + t^w WL - B_1 + t^k K + \frac{t^r rPS - B_2}{1+r\left(1-t^r\right)}. \tag{27}$$

As we saw in section II.2, all of the burden of the tax on business investment (t^k) is carried by workers through a fall in wage rates. Hence equation (27) does not discount the investment tax revenue $t^k K$ since the investment tax is effectively paid during the first period of a cohort's life when they are active in the labour market. By contrast, a cohort does not start paying capital income tax until it has accumulated wealth, so equation (27) does discount the capital income tax revenue $t^r rPS$, just as the revenue from indirect taxes on consumption during old age must be discounted.

Inserting the household budget constraint (5) and using the definition $p \equiv 1/\left[1+r\left(1-t^r\right)\right]$, we may rewrite (27) as

$$R=\left[t^w+t^c\left(1-t^w\right)\right]WL-\left(1-t^c\right)\left(B_1+pB_2\right)+t^k K+p\left(t^r rPS-B_2\right). \tag{28}$$

Equation (28) shows that the consumption tax t^c works in part like a labour income tax and partly as a tax on transfer income.

II.4. The deadweight loss from taxation

We are now ready to specify the deadweight loss from taxation. Adopting an equivalent variation measure, we define the total deadweight loss from taxation as the difference between the maximum amount consumers would be willing to pay to get rid of all taxes (given the level of utility $\bar{U}$ prevailing after the imposition of taxes) and the actual tax revenue collected. Given our expenditure function $E=E\left(P, pP, W\left(1-t^w\right), \bar{U}\right)$, and recalling that the producer price index has been normalized at unity, the total deadweight loss from taxation (*DWL*) is therefore equal to

$$DWL=\overbrace{E\left(P, pP, W\left(1-t^w\right), \bar{U}\right)-E\left(1, \frac{1}{1+r}, W, \bar{U}\right)}^{\text{Equivalent variation}}-R. \tag{29}$$

The magnitude $E\left(1, \frac{1}{1+r}, W, \bar{U}\right)$ is the minimum (exogenous) income needed to attain the utility level $\bar{U}$ if there were no distorting taxes, since in that case we would have *P=1* and *p=1/(1+r)*. Hence the excess burden in (29) measures the additional revenue that could have been raised by a non-distortionary lump sum tax rather than through the existing distortionary taxes without leaving consumers worse off. The relative after-tax prices *p* and $W\left(1-t^w\right)$ depend on the tax rates t^r, t^w and t^k, where the effect of t^k stems from its impact on *W* through its influence on ρ, and the consumer price index *P* is determined by the effective indirect tax rate t^c. From (29) it then follows that the *marginal* deadweight loss from an increase in the some tax rate t^i is

$$\frac{dDWL}{dt^i} = \frac{dE}{dt^i} - \frac{dR}{dt^i} = \frac{dE}{dt^i} - \left(\frac{dR^s}{dt^i} + \frac{dR^d}{dt^i} \right), \qquad i = c,k,r,w. \tag{30}$$

To obtain the last equality in (30), we have split the total revenue change dR into the "static" revenue change dR^s that would occur if taxpayers did not change their behaviour, and the "dynamic" revenue change dR^d resulting from the behavioural responses to the change in the tax rate.[4] Note that the derivatives in (30) are calculated on the assumption that the taxpayer is compensated so as to maintain the utility level $\bar{U}$ prevailing before the tax increase.[5] The dynamic revenue change dR^d therefore stems exclusively from the substitution effects induced by the tax change. The static revenue gain dR^s is calculated by assuming that C_1, C_2 and *L* are unchanged. To maintain a clean distinction between static and dynamic revenue changes we also assume that the compensation paid out to consumers to preserve their utility level is distributed across the life cycle in a way that does not require any changes in first-period savings to keep C_1, C_2 and *L* constant.

The decomposition of the total revenue change into a "static" and a "dynamic" component is very useful because of the general result that the amount needed to compensated consumers for a tax increase equals the static revenue gain, as we shall see below. From (30) it then follows that the marginal deadweight loss equals the dynamic revenue loss. Intuitively, in a competitive market the initial marginal tax wedge on some activity *X* measures the difference between the marginal consumer benefit from that activity (MB_X) and its marginal cost (MC_X). When the behavioural response to a tax hike causes the tax base to shrink by the amount *dX*, the resulting dynamic revenue loss may therefore be written as $\left(MB_X - MC_X\right) \cdot dX$ which is a measure of the net welfare loss to society.[6]

---

[4] In the literature the static revenue change is sometimes referred to as the "mechanical" revenue change, and the dynamic revenue change is also known as the "revenue change from behavioural responses".

[5] While the *total* deadweight loss in (29) is calculated by means of the equivalent variation, we are thus effectively adopting a compensating variation measure of the *marginal* deadweight loss.

[6] This statement assumes that the taxed activity does not generate external effects. If there are externalities that are internalized via Pigovian taxes, the change in the revenue from these taxes induced by changes in taxpayer behaviour should be deducted from the total dynamic revenue change to obtain a correct measure of the welfare effect of a tax policy change. Sørensen (2010a, ch. 4) explains in detail how one can correct for external effects within the model framework presented in the present paper.

To obtain a measure of the efficiency loss that is independent of the units in which income and revenue are measured, it is useful to express the marginal deadweight loss as a fraction of the static revenue gain. When doing so, we obtain the so-called degree of self-financing associated (*DSF*) with the tax instrument t^i:

$$DSF_{t^i} \equiv \frac{dDWL/dt^i}{dR^s/dt^i} = -\frac{dR^d/dt^i}{dR^s/dt^i}, \tag{31}$$

where we have used (30) and the result that $dE/dt^i = dR^s/dt^i$. The *DSF* measures the fraction of the initial revenue gain from a tax increase which is lost again due to behavioural responses. In the case of a decrease in some tax, the *DSF* indicates the degree to which the tax cut pays for itself through behavioural changes that increase the tax base. A positive marginal deadweight loss is thus equivalent to a positive degree of self-financing.

The measure of marginal deadweight loss in (31) only includes changes in the revenue collected from the representative generation entering the labour market after the tax policy change. Equation (31) therefore captures the pure efficiency effect of the policy change without including intergenerational redistribution effects. For example, if the government raises the tax rate on savings income, a part of the immediate revenue gain will arise from heavier taxation of pre-existing wealth. If the tax hike is unanticipated, the higher tax on wealth accumulated in the past will work like a non-distortionary lump sum tax that does not generate a deadweight loss. An unanticipated rise in the consumption tax rate also works in part like a capital levy on existing wealth. However, these lump sum elements of tax increases reflect a redistribution away from the current old generations towards the current young and future generations. If the current old generations are shielded from the tax hike through grandfathering rules, the static revenue gain from the tax reform will not include a non-distortionary capital levy on these generations. Our measure of the efficiency effects of tax reforms therefore excludes such elements.

III. THE MARGINAL DEADWEIGHT LOSS FROM THE MAIN TYPES OF TAX

III.1. The marginal deadweight loss from a rise in the labour income tax rate

Given our assumption of a linear labour income tax, the marginal labour income tax rate t^w applies to the entire wage bill *WL*. In practice the labour income tax is rarely linear, but when analyzing the effect of a rise in t^w we may think of our model as simulating the effect of an identical rise in the marginal tax rate for all income groups in the economy.

From (7) it follows that $\partial E / \partial t^w = WL$, so the amount needed to compensate taxpayers for a unit rise in t^w is *WL*. According to (28), the static revenue gain that would result from a unit rise in t^w if labour supply and savings were unchanged is likewise equal to *WL*. From (30) we thus have

$$\frac{dDWL}{dt^w} = \frac{dE}{dt^w} - \left(\frac{dR^s}{dt^w} + \frac{dR^d}{dt^w} \right) = WL - \left(WL + \frac{dR^d}{dt^w} \right) = -\frac{dR^d}{dt^w}. \tag{32}$$

This confirms the general result reported above, i.e., that $dDWL / dt^i = -dR^d / dt^i$. To calculate the dynamic revenue change induced by a rise in the marginal labour income tax rate, we use the result in (15) that a change in labour supply will change the stock of business capital by the amount $dK = \left(K / L \right) \cdot dL$. Since $w \equiv W \left(1 - t^w \right) / P$, a rise in t^w changes the real after-tax consumer wage by the amount $dw = -\left(w / \left(1 - t^w \right) \right) \cdot dt^w$. From (30) it then follows that:

$$\frac{dR^d}{dt^w} = -\left(\frac{w}{1 - t^w} \right) \left\{ \left[t^w + t^c \left(1 - t^w \right) \right] W \frac{\partial L}{\partial w} + t^k \frac{K}{L} \frac{\partial L}{\partial w} + pt^r rP \frac{\partial S}{\partial w} \right\}. \tag{33}$$

Rewriting (33) in terms of elasticities and expressing the result as a fraction of the static revenue gain *WL*, we obtain the degree of self-financing associated with a small change in the marginal labour income tax rate:

$$\frac{dDWL / dt^w}{dR^s / dt^w} = -\frac{dR^d / dt^w}{dR^s / dt^w} = \frac{\left(m^w + m^k \theta^k \right) \varepsilon_w^L + t^r \theta^s \varepsilon_w^S}{1 - t^w},$$

$$\varepsilon_w^L \equiv \frac{\partial L}{\partial w} \frac{w}{L}, \quad \varepsilon_w^S \equiv \frac{\partial S}{\partial w} \frac{w}{S}, \quad m^w \equiv t^w + t^c \left(1 - t^w \right), \quad m^k \equiv \frac{t^k}{\rho - \delta}, \quad \theta^k \equiv \frac{\left(\rho - \delta \right) K}{WL}, \quad \theta^s \equiv \frac{prPS}{WL}. \tag{34}$$

The variable m^w is the total marginal effective tax rate on labour income, including the indirect taxes that work in part like a tax on labour income by eroding the real consumer wage. The variable m^k is the marginal effective investment tax rate, expressing the investment tax wedge as a fraction of the pre-tax return $\rho - \delta$ on the marginal investment. In addition, (34) includes the parameters θ^k and θ^s indicating the importance of investment and savings income relative to wage income. Recall that since our measure of deadweight loss assumes that consumers are compensated for the tax increase, the wage elasticities of labour supply and savings in (34) (ε_w^L and ε_w^S) are *compensated* elasticities. In section IV we shall see that the life cycle model implies a tight link between these two elasticities.

Using the definition of m^w, the deadweight loss in (34) may be decomposed into the dynamic revenue losses from the decline in the four tax bases considered:

$$\frac{dDWL/dt^w}{dR^s/dt^w} = \overbrace{\left(\frac{t^w \varepsilon_w^L}{1-t^w}\right)}^{\substack{\text{loss of labour} \\ \text{income tax revenue}}} + \overbrace{\left(\frac{t^c\left(1-t^w\right)\varepsilon_w^L}{1-t^w}\right)}^{\substack{\text{loss of consumption} \\ \text{tax revenue}}} + \overbrace{\left(\frac{m^k \theta^k \varepsilon_w^L}{1-t^w}\right)}^{\substack{\text{loss of business} \\ \text{income tax revenue}}} + \overbrace{\left(\frac{t^r \theta^s \varepsilon_w^S}{1-t^w}\right)}^{\substack{\text{loss of savings} \\ \text{tax revenue}}}. \quad (35)$$

Equation (35) highlights that the efficiency loss from a rise in the marginal tax rate on labour income does not only stem from the shrinking of the labour income tax base. As labour supply contracts, generating an upward pressure on real wages, domestic business investment becomes less profitable, and the resulting fall in domestic investment reduces the revenue from the business income tax. The fall in labour income also reduces consumption as well as household saving, thereby eroding the revenues from consumption taxes and the savings income tax.

III.2. The marginal deadweight loss from a rise in the consumption tax rate

Consider next the effect of a rise in the consumption tax rate. Since the consumer price index P includes indirect taxes amounting to $t^c P$, we have $\partial P/\partial t^c = P$. It then follows from (8) that the amount needed to compensate consumers for a unit rise in t^c is $PC_1 + pPC_2$. Now suppose that consumers are compensated for the tax increase by a rise in the transfer B_1 equal to the amount PC_1 and by a rise in B_2 amounting to PC_2, so that the present value of the rise in B_2 equals pPC_2. In total, these increases in transfers are just sufficient to fully compensate consumers. Further, since the increases in B_1 and B_2 exactly compensate for the rise in the tax-inclusive consumption expenditure in each of the two stages of the life cycle (given the initial consumption levels), the consumer does not have to change his first-period saving *PS* to maintain constant levels of C_1, C_2 and *L*.[7] From these observations it follows from (8), (27) and (30) that

[7] The consumer's second-period budget constraint is $PS = p\left(PC_2 - B_2\right)$. At the initial level of C_2, a unit rise in t^c raises PC_2 by the amount $\left(\partial P/\partial t^c\right)\cdot C_2 = PC_2$, but since B_2 goes up by a similar amount, *PS* is unchanged. A similar conclusion follows from the first-period budget constraint $PS = \left(1-t^w\right)WL + B_1 - PC_1$, since $dB_1 = PC_1 = \left(\partial P/\partial t^c\right)\cdot C_1$.

$$\frac{dDWL}{dt^c}=\frac{dE}{dt^c}-\left(\frac{dR^s}{dt^c}+\frac{dR^d}{dt^c}\right)=P\left(C_1+pC_2\right)-\left(P\left(C_1+pC_2\right)+\frac{dR^d}{dt^c}\right)=-\frac{dR^d}{dt^c}. \qquad (36)$$

The rise in the consumption tax rate induces behavioural changes through its impact on the real after-tax consumer wage $w\equiv W\left(1-t^w\right)/P$. Recalling that $\partial P/\partial t^c=P$, we have $\partial w/\partial t^c=-w$, and from (15) we still have $dK=\left(K/L\right)\cdot dL$. Using (28) and the definitions stated in (34), we then find:

$$\frac{dR^d}{dt^c}=-WL\left[\left(m^w+m^k\theta^k\right)\varepsilon_w^L+pt^r\theta^s\varepsilon_w^S\right]. \qquad (37)$$

Equation (28) also implies that the static revenue gain is:

$$\frac{dR^s}{dt^c}=\left(1-t^w\right)WL+B_1+pB_2. \qquad (38)$$

From (34), (37) and (38) it follows that

$$\frac{dDWL/dt^c}{dR^s/dt^c}=-\frac{dR^d/dt^c}{dR^s/dt^c}=\frac{\left(m^w+\theta^k m^k\right)\varepsilon_w^L+t^r\theta^s\varepsilon_w^S}{1-t^w+b_1+pb_2}$$

$$=\left(\frac{1-t^w}{1-t^w+b_1+pb_2}\right)\frac{dDWL/dt^w}{dR^s/dt^w}, \qquad b_1\equiv\frac{B_1}{WL}, \qquad b_2\equiv\frac{B_2}{WL}, \qquad (39)$$

where b_1 and b_2 are the replacement rates in the public transfer system for the young and the old, respectively. We see from the second line in (39) that the deadweight loss from a rise in the consumption tax rate is lower than the deadweight loss from a rise in the labour income tax rate involving the same static revenue gain. The reason is that the consumption tax is levied on a broader base which includes the consumption financed out of the public transfers B_1 and B_2. Since these components of income are exogenous to consumers, the consumption tax imposed on them is effectively a non-distorting lump-sum tax.

III.3. The marginal deadweight loss from a rise in the investment tax wedge

A rise in the effective source-based capital tax rate t^k could be implemented through a rise in the statutory corporate income tax rate or via measures to broaden the business income tax base, as we shall explain in section IV.2. The definition of the user cost of capital stated in (12) implies that

$\partial \rho / \partial t^k = 1$, since the small-open-economy assumption means that the required net rate of return r is given from abroad. According to (23) the burden of this rise in the cost of capital will be shifted to domestic workers through a fall in the wage rate equal to $dW / \partial t^k = -\left(K / L\right)$. Faced with this drop in the pre-tax wage rate, we see from (7) that workers will have to be compensated by the amount $-\left(1-t^w\right) L \cdot \left(\partial W / \partial t^k\right) = -\left(1-t^w\right) K$ to maintain the same utility level. According to (27) the static revenue gain from a unit rise in t^k will be $dR^s = K + t^w L \cdot \left(\partial W / \partial t^k\right) = K\left(1-t^w\right)$, assuming that workers are compensated through a rise in B_1 so that their savings PS can be kept unchanged as long as C_1, C_2 and L are unchanged. Thus we have

$$\frac{dDWL}{dt^k} = \frac{dE}{dt^k} - \left(\frac{dR^s}{dt^k} + \frac{dR^d}{dt^k}\right) = \left(1-t^w\right)K - \left(\left(1-t^w\right)K + \frac{dR^d}{dt^k}\right) = -\frac{dR^d}{dt^k}. \tag{40}$$

Using (18), (19), (28) and the definitions in (34) plus the fact that $d\rho = dt^k$, we find the dynamic revenue effect to be

$$\frac{dR^d}{dt^k} = -K\left[m^k \left(\tfrac{\rho-\delta}{\rho}\right)\varepsilon_\rho^K + \left(m^w + m^k\theta^k\right)\varepsilon_w^L + t^r\theta^s\varepsilon_w^S\right], \tag{41}$$

where we recall from (20) that the elasticity ε_ρ^K is calculated at the initial level of labour supply. Dividing (41) by the static revenue gain $dR^s / dt^k = \left(1-t^w\right)K$ and using (34), we get

$$\begin{aligned}\frac{dDWL / dt^k}{dR^s / dt^k} &= -\frac{dR^d / dt^k}{dR^s / dt^k} = \frac{m^k \left(\tfrac{\rho-\delta}{\rho}\right)\varepsilon_\rho^K + \left(m^w + m^k\theta^k\right)\varepsilon_w^L + pt^r\theta^s\varepsilon_w^S}{1-t^w} \\ &= \frac{m^k \left(\tfrac{\rho-\delta}{\rho}\right)\varepsilon_\rho^K}{1-t^w} + \frac{dDWL / dt^w}{dR^s / dt^w}.\end{aligned} \tag{42}$$

We see that the deadweight loss from a higher investment tax wedge consists of the revenue loss from the direct negative impact on domestic investment – represented by the first term in the second line in (42) – plus the revenue loss arising as the fall in wages induced by lower investment reduces labour supply and savings. Note that the direct negative impact on investment does not arise under a labour income tax which therefore generates a lower deadweight loss per unit of revenue than a tax on domestic investment, as shown by the second line in (42). This illustrates the well-known result that it is inoptimal to impose a source-based tax on the normal return to investment in a small open economy. The contribution of formula (42) is that it allows an estimate of the additional welfare lost by taxing labour indirectly through a source-based capital tax rather than directly through the labour income tax.

By analogy to (35), we can decompose the marginal deadweight loss in (42) into the losses stemming from the shrinkage of the various tax bases:

$$\frac{dDWL/dt^k}{dR^s/dt^k} = \overbrace{\left(\frac{t^w \varepsilon_w^L}{1-t^w}\right)}^{\substack{\text{loss of labour}\\ \text{income tax revenue}}} + \overbrace{\left(\frac{t^c\left(1-t^w\right)\varepsilon_w^L}{1-t^w}\right)}^{\substack{\text{loss of consumption}\\ \text{tax revenue}}} + \overbrace{\left(\frac{m^k\left(\varepsilon_\rho^K\left(\frac{\rho-\delta}{\rho}\right)+\theta^k\varepsilon_w^L\right)}{1-t^w}\right)}^{\substack{\text{loss of business}\\ \text{income tax revenue}}} + \overbrace{\left(\frac{t^r\theta^s\varepsilon_w^S}{1-t^w}\right)}^{\substack{\text{loss of savings}\\ \text{tax revenue}}}. \tag{43}$$

III.4. The marginal deadweight loss from a rise in the savings tax wedge

We finally consider the effects of a rise in the residence-based capital income tax rate t^r. From the consumer's second-period budget constraint $C_2 = \left[1+r\left(1-t^r\right)\right]PS + B_2$ it is clear that, when the rise in the capital income tax rate reduces after-tax savings income by the amount $rPS \cdot dt^r$, a compensating rise in the retirement benefit equal to $dB_2 = rPS \cdot dt^r$ will enable the consumer to maintain the initial level of old-age consumption at the initial level of savings. Indeed, using (9) one can show that a compensating rise in B_2 equal to this amount will enable the consumer to maintain the same level of lifetime utility when faced with the higher capital income tax rate.

Measured at the initial relative price of future consumption (p_0) which reflects the consumer's initial marginal rate of substitution between present and future consumption, the present value of the static revenue gain from the rise in the capital income tax rate is $dR^s = p_0 rPS \cdot dt^r$, while the compensating rise in B_2 implies a revenue loss with a similar present value. Once again we therefore find that the marginal deadweight loss is given by the revenue loss from the behavioural responses to the tax increase, that is, $dDWL/dt^r = -dR^d/dt^r$.

Let $r^a \equiv r\left(1-t^r\right)$ denote the after-tax real interest rate, implying $dr^a/dt^r = -r$. From (28) and the fact that $dK = \left(K/L\right)\cdot dL$ as long as the cost of capital is constant, we then find

$$\frac{dR^d}{dt^r} = -r\left(m^w W\frac{\partial L}{\partial r^a} + t^k\frac{K}{L}\frac{\partial L}{\partial r^a} + p_0 t^r rP\frac{\partial S}{\partial r^a}\right) \quad \Rightarrow$$

$$\frac{dR^d}{dt^r} = -\left(\frac{WL}{1-t^r}\right)\left[\left(m^w + m^k\theta^k\right)\varepsilon_r^L + t^r\theta^s\varepsilon_r^S\right], \qquad \varepsilon_r^L \equiv \frac{\partial L}{\partial r^a}\frac{r^a}{L}, \qquad \varepsilon_r^S \equiv \frac{\partial S}{\partial r^a}\frac{r^a}{S}. \tag{44}$$

As noted above, the static revenue gain equals $dR^s / dt^r = p_0 rPS$. Recalling that $\theta^s \equiv p_0 rPS / WL$, we therefore get

$$\frac{dDWL / dt^r}{dR^s / dt^r} = -\frac{dR^d \ / dt^r}{dR^s / dt^r} = \frac{\left(m^w + m^k\theta^k\right)\varepsilon_r^L + t^r\theta^s\varepsilon_r^S}{\left(1-t^r\right)\theta^s}, \tag{45}$$

which may be decomposed as follows:

$$\frac{dDWL / dt^r}{dR^s / dt^r} = \overbrace{\left(\frac{t^w\varepsilon_r^L}{\left(1-t^r\right)\theta^s}\right)}^{\substack{\text{loss of labour}\\ \text{income tax revenue}}} + \overbrace{\left(\frac{t^c\left(1-t^w\right)\varepsilon_r^L}{\left(1-t^r\right)\theta^s}\right)}^{\substack{\text{loss of consumption}\\ \text{tax revenue}}} + \overbrace{\left(\frac{m^k\theta^k\varepsilon_r^L}{\left(1-t^r\right)\theta^s}\right)}^{\substack{\text{loss of business}\\ \text{income tax revenue}}} + \overbrace{\left(\frac{t^r\varepsilon_r^S}{1-t^r}\right)}^{\substack{\text{loss of savings}\\ \text{tax revenue}}}. \tag{46}$$

IV. CALIBRATION METHODS

For the purpose of empirical application, we need to quantify the parameters entering the model presented above. Section IV.1 below explains how one can estimate the various factor supply elasticities, section IV.2 describes how one can pin down the parameter θ^s reflecting the timing of capital income relative to labour income over the life cycle, and section IV.3 lays out how the effective tax rates on the main tax bases can be calculated.

IV.1. Elasticities

To apply our formulae for deadweight loss we need estimates of the compensated cross price elasticities ε_w^S and ε_r^L and the compensated savings elasticity ε_r^S. Unfortunately the empirical evidence on the magnitude of these elasticities is scarce. To overcome this difficulty the technical working paper by Sørensen (2011) uses the life cycle model to derive links between these elasticities and the compensated net wage elasticity of labour supply (ε_w^L) which has been estimated in numerous empirical studies.[8] The working paper also derives the link between the compensated and the uncompensated interest elasticity of savings implied by the life cycle model. The cross-

[8] The working paper is downloadable from http://www.econ.ku.dk/pbs/.

restrictions are derived on the popular assumption that utility is separable in consumption and labour supply. We then obtain the following restrictions, where g^c is the growth rate of consumption over the consumer's life cycle, and c is the propensity to consume total (human plus non-human) wealth in the first period of the life cycle:[9]

$$\varepsilon_w^S = \left(\frac{1-t^w}{1-t^w+b_1-\left(\frac{b_2}{1+g^c}\right)} \right) \varepsilon_w^L, \qquad g^c \equiv \frac{C_2}{C_1} - 1, \tag{47}$$

$$\varepsilon_r^L = \left(\frac{r^a}{1+r^a} \right) (1-c)\, \varepsilon_w^L, \qquad c \equiv \frac{1}{1+p\left(1+g^c\right)}, \tag{48}$$

$$\varepsilon_r^S = \hat{\varepsilon}_r^S + \left(\frac{r^a}{1+r^a} \right) \left(\frac{1-t^w+b_1+\left(\frac{b_2}{1+r^a}\right)}{1-t^w+b_1-\left(\frac{b_2}{1+g^c}\right)} \right) \left(1-a\varepsilon_I^{C_2}\right), \tag{49}$$

$$\hat{\varepsilon}_r^S \equiv \frac{\partial S}{\partial r^a}\frac{r^a}{S}, \qquad \varepsilon_I^{C_2} \equiv \frac{\partial C_2}{\partial I}\frac{I}{C_2}, \qquad a = \frac{1-t^w+b_1+pb_2}{\left[1+\left(\frac{1+r^a}{1+g^c}\right)\right]\left(b_1+pb_2\right)}.$$

The formulae (47) and (48) allow an estimate of the compensated cross price elasticities of savings and labour supply, given an estimate of the compensated wage elasticity of labour supply and the empirically observable variables t^w, b_1, b_2 and r^a (coupled with an assumption on g^c). The parameter $\hat{\varepsilon}_r^S$ in (49) is the *uncompensated* interest elasticity of savings which is somewhat more familiar than the compensated elasticity, and $\varepsilon_I^{C_2}$ is the income elasticity of demand for future consumption. The latter parameter is not very well known, but Sørensen (2011) shows that if the elasticity of intertemporal substitution equals the constant $1/\gamma$, and if the Frisch wage elasticity of labour supply (which assumes that the marginal utility of consumption is kept constant) is equal to the constant $1/\eta$, the life cycle model implies that

$$\varepsilon_I^{C_2} = \frac{1}{1+\frac{2\gamma}{\eta}+\left(\frac{\eta+\gamma}{\eta}\right)\left(\frac{1-t^w}{b_1+pb_2}\right)}. \tag{50}$$

[9] Sandmo (1981) shows that the compensated savings elasticities in the two-period life cycle model will depend on whether the consumer is compensated through a change in first-period or in second-period income. The elasticities in (47) through (49) were derived on the "neutral" assumption that the consumer is compensated through changes in the transfers B_1 and B_2 that are calibrated such that they do not require a change in the level of saving if the consumer wants to maintain an unchanged level of consumption in the two periods. In section III we relied on a similar assumption.

One can also show that the compensated wage elasticity of labour supply, the Frisch elasticity and the elasticity of intertemporal substitution are linked to each other by the relationship

$$\varepsilon_w^L = \frac{1}{\eta + \frac{\gamma}{1+b_1+pb_2}}. \tag{51}$$

When calibrating our model, we make sure that the cross-restrictions (47) through (51) are met so that our calibration is consistent with the underlying theoretical model.

IV.2. The timing of capital income and labour income over the life cycle

To ensure that our measures of efficiency loss are not contaminated by intergenerational redistribution effects stemming from the overlap of generations, our formulae for deadweight loss focus on the impact of taxation on the lifetime welfare of a given cohort. As we have seen, this focus requires a calculation of lifetime tax payments in present value terms. In particular, when comparing the welfare loss from capital income taxes with the loss from other taxes, we must account for the fact that (the bulk of) capital income taxes tend to be paid at a relatively late stage in a cohort's life cycle. In our deadweight loss formulae this fact is captured by the parameter θ^s which measures the present value of a cohort's capital income relative to the present value of its labour income. In the simple two-period life cycle model we have $\theta^s = prPS/WL$, but there is a risk that the two-period model will underestimate the value of θ^s since it implies that no capital income is earned until the consumer has retired from the labour market. In practice, the representative consumer accumulates savings for many years before retirement and therefore starts to pay capital income taxes well before that date (at least if the return to retirement saving is taxed, as is the case in Sweden). To account for this, we will calibrate θ^s on the basis of a multi-period life cycle model where the representative consumer maximises a lifetime utility function of the additive and separable form

$$U = \sum_{t=0}^{T} \left(\frac{1}{1+d} \right)^t \left[u(C_t) - h_t \cdot v(L_t) \right], \qquad L_t = 0 \text{ for } t = n, n+1, ..., T, \tag{52}$$

subject to the lifetime budget constraint

$$\sum_{t=0}^{T} \frac{PC_t}{(1+r^a)^t} = \sum_{t=0}^{T} \frac{W_t L_t (1-t^w) + B_t}{(1+r^a)^t}. \tag{53}$$

The parameter d in (52) is the utility discount rate. The per-period utility function $u(C_t)$ is assumed to be concave and the disutility-of-work function $v(L_t)$ is taken to be convex. As indicated in (52), the (adult) consumer is active in the labour market until he retires at the age of n years. The length of the adult life is T years so $T-n$ is the number of years spent in retirement. Due to exogenous productivity growth, the producer real wage W_t is assumed to grow at the constant annual rate g^w over time, and the proportionality factor h_t in (52) is taken to grow at the constant annual rate g^h, reflecting that the value of the consumer's non-market activities rises over time. In the setting described by (52) and (53) the optimal evolution of consumption over time is given by the Euler equation

$$u'(C_t)=\left(\frac{1+r^a}{1+d}\right)u'(C_{t+1}), \tag{54}$$

and optimal labour supply behaviour implies

$$\frac{h_t\cdot v'(L_t)}{u'(C_t)}=\frac{W_t(1-t^w)}{P}, \qquad v'(L_t)=\left(\frac{1+r^a}{1+d}\right)\left(\frac{1+g^h}{1+g^w}\right)v'(L_{t+1}), \qquad t=0,....,n-2. \tag{55}$$

In the benchmark case of $\left(\frac{1+r^a}{1+d}\right)\left(\frac{1+g^h}{1+g^w}\right)=1$ on which we shall focus, it follows from (55) that labour supply is constant over the consumer's labour market career. Further, we assume that transfers are indexed to wages such that B_t/W_tL_t is equal to a constant b^w during the consumer's working career and that B_t/W_tL_t equals a constant b^r throughout his retirement period. With these assumptions Sørensen (2011) uses the budget constraint (53) and the optimum conditions (54) and (55) to show that the present value of pre-tax capital income relative to the present value of pre-tax labour income will be given by the expression

$$\begin{aligned}\theta^s=&\left(\frac{r}{1+r^a}\right)(1-t^w+b^w)\left[T+1+\frac{n}{(1+r^w)^n-1}-\frac{1}{r^w}\right]\\&+\left(\frac{r^a}{1+r^a}\right)\left[\frac{b^r}{(1+r^w)^n-1}\right]\left[T-n+\frac{r^w}{1+r^w}+\left(\frac{1}{1+r^w}\right)^{T+1-n}\right]\\&-\left(\frac{r^a}{1+r^a}\right)\left(\frac{\omega}{r^c}\right)(1-t^w+b^w+\beta^r)\left[\frac{r^c(T+1)-1+\left(\frac{1}{1+r^c}\right)^T}{1-\left(\frac{1}{1+r^c}\right)}\right],\end{aligned} \tag{56}$$

$$r^w\equiv\frac{1+r^a}{1+g^w}-1, \qquad r^c\equiv\frac{1+r^a}{1+g^c}-1, \qquad \beta^r\equiv b^r\left[\frac{1-(1+r^w)^{n-T-1}}{(1+r^w)^n-1}\right], \qquad \omega\equiv\frac{r^c}{1+r^c-(1+r^c)^{-T}}.$$

Here g^c is the annual real growth rate of consumption over the life cycle (determined by the Euler equation (54)) and the transfer rates b^w and b^r are fully analogous to the transfer rates b_1 and b_2 in the two-period model. In principle all parameters in (56) are thus empirically observable. The expression for θ^s in (56) reflects a situation where the consumer gradually builds up wealth until the date of retirement and gradually dissaves until the stock of wealth reaches zero at the end of the life cycle. Although the multi-period life cycle model underlying (56) is still rather stylized, it should allow a more accurate estimate of θ^s than the simple two-period model.

IV.3. Effective tax rates

To apply our deadweight loss formulae we finally need estimates of effective tax rates on labour income, savings income, business income and consumption. There are several ways to obtain such estimates, as discussed in Sørensen (2004). Drawing on the more detailed analysis in Sørensen (2010b), this section presents a set of formulae that will allow an estimate of effective tax rates based on national accounts data and information on statutory tax rates. The formulae are consistent with optimising consumer behaviour, given certain popular assumptions about the structure of preferences.

The effective marginal tax rate on labour income

Our variable t^w includes the marginal social security tax rate and the marginal personal income tax rate on labour income, i.e.,

$$t^w = s + (1-s)t^{wp}, \tag{57}$$

where *s* is the sum of the employer's and the employee's marginal social security tax rate, expressed as a fraction of the employer's pre-tax labour cost *W*, and t^{wp} is the marginal personal labour income tax rate imposed on the employee's wage after deduction for social security tax. If additional social security tax payments generate additional benefit rights, the estimate of *s* should be adjusted correspondingly so that it only includes the element of genuine tax paid at the margin. Sørensen (2010a, ch. 6) discusses how such an adjustment can be made in the context of the Swedish system of social security. The estimate of the marginal personal tax rate t^{wp} should include the impact of the possible phase-out of any in-work benefits (such as an earned income tax credit) and of other income-tested social transfers. The estimated aggregate values of *s* and t^{wp} should be weighted averages across the various income groups, with weights reflecting the relative size of total pre-tax labour income earned by each group. Obtaining such estimates is likely to require access to a tax-benefit calculator based on micro data for the relevant country. The estimate of t^w for Sweden reported below has been produced in this way.

The effective indirect tax rate on consumption

When estimating the effective indirect tax rate on consumption (t^c), the challenge is to calculate how the various rates of VAT and excise taxes on individual consumption items contribute to the overall effective tax rate on aggregate consumption. To address this problem Sørensen (2010b) assumes that the aggregate consumption variables C_1 and C_2 in the utility function (1) are given by subutility functions of the form $C = C\left(C_H\left(H,h\right), C_o\left(x_1,...,x_N\right)\right)$, where C_H is consumption of housing services, assumed to be a CES aggregate of the consumption of services from owner-occupied housing, *H*, and the consumption of rental housing services, *h*, and where C_o is a CES aggregate of the consumption of all other goods and services, with x_i denoting consumption of good *i*. On this basis Sørensen (2010b) shows that when households maximise utility, the effective indirect tax rate on aggregate consumption can be written as the following weighted average of the effective tax rate on housing services, t_H^c, and the effective tax rate on other goods and services, t_o^c:

$$t^c = \beta_H t_H^c + \left(1-\beta_H\right) t_o^c, \qquad \beta_H \equiv \frac{\mu P_H^{1-\sigma}}{\mu P_H^{1-\sigma} + \left(1-\mu\right) P_o^{1-\sigma}}, \tag{58}$$

$$\mu = s_H P_H^{\sigma-1} Z, \qquad s_H \equiv \frac{P_H C_H}{PC}, \qquad Z \equiv \frac{1}{s_H P_H^{\sigma-1} + \left(1-s_H\right) P_o^{\sigma-1}}.$$

The variable P_H in (58) is an index of the user cost of housing services, consisting of a CES aggregate of the user costs of owner-occupied and rental housing, and P_o is a consumer price index for all other goods and services, calculated as a CES aggregate of the tax-inclusive prices of individual goods. With producer prices normalized at unity, the consumer price of good *i* is $p_i = 1 + t_i^e + t_i^v\left(1+t_i^e\right)$, where t_i^e and t_i^v are the excise tax rate and the VAT rate for good *i*, and where we assume that the country considered follows the normal practice of levying VAT on the value of sales *including* excise taxes. The parameter σ in (58) is the elasticity of substitution between housing services and other consumption. Sørensen (2010b) shows in detail how the user costs of the two types of housing and the housing service price index P_H can be calculated, given the tax treatment of owner-occupied and rental housing and assuming that consumers optimise their choice between them.

Note from (58) that the value of the share parameter μ can be estimated from data on the share of housing expenditure in total consumption spending (s_H). In a similar way, the calculation of P_H and P_o requires data on the budget shares of the consumption items entering the respective consumption aggregates. These data can be taken from the national accounts.

The effective marginal tax rate on savings income

Our variable t^r is the average value of the marginal effective tax rate on the return to household financial saving. We disaggregate total saving into "free" financial saving subject to the ordinary (marginal) personal capital income tax rate t^F, and "institutional" saving defined as saving channelled through institutional investors such as pension funds and life insurance companies where the rate of return is taxed at the effective marginal rate t^I (which is typically lower than t^F). Since the two forms of saving normally differ in terms of their liquidity and risk characteristics, it is reasonable to assume that they are imperfect substitutes. Sørensen (2010b) therefore specifies total financial saving as a CES aggregate of "free" and "institutional" saving, with a finite substitution elasticity ϕ between them. On this basis he shows that optimal portfolio behaviour implies that the overall effective marginal tax rate on saving satisfies

$$1-t^r=\left[\gamma\left(1-t^I\right)^{\phi+1}+\left(1-\gamma\right)\left(1-t^F\right)^{\phi+1}\right]^{\frac{1}{\phi+1}}, \tag{59}$$

$$\gamma=\frac{s^I\left(1-t^I\right)^{-(\phi+1)}}{s^I\left(1-t^I\right)^{-(\phi+1)}+\left(1-s^I\right)\left(1-t^F\right)^{-(\phi+1)}}, \qquad s^I\equiv\frac{1}{1+\left(\frac{1-t^F}{1-t^I}\right)\left(\frac{S^F}{S^I}\right)},$$

where S^F and S^I are the accumulated stocks of the two forms of saving. Equation (59) accounts for the fact that a rise in any of the two tax rates t^I or t^F will induce substitution away from the form of saving subjected to heavier taxation. Note that t^I and t^F are effective tax rates on the *real* rate of return. Since the corresponding statutory tax rates t^{Is} and t^{Fs} are levied on the *nominal* rate of return, we therefore have

$$t^I=\frac{t^{Is}\left(r+\pi\right)}{r}, \qquad t^F=\frac{t^{Fs}\left(r+\pi\right)}{r}, \tag{60}$$

where π is the rate of inflation.

The effective marginal tax rate on business income

The marginal effective tax rate on business income is $m^k = t^k / (\rho - \delta) = (\rho - \delta - r)/(\rho - \delta)$, since the investment tax wedge t^k is defined as the difference between the marginal pre-tax return to business investment (net of depreciation), $\rho - \delta$, and the world real interest rate, *r*. To estimate m^k we therefore need an estimate of the average user cost for domestic business capital, ρ.

The user cost will depend on the cost of finance, i.e., the discount rate applied when discounting the future cash flows from business investments. For *widely held corporations* with access to the international capital market, including the international equity market, the cost of finance is simply the world real interest rate *r* which is assumed to determine the risk-adjusted rate of return on shares required by international investors. Building on the approach of King and Fullerton (1984), Sørensen (2010b) shows that the user cost of capital associated with an *equity-financed* investment undertaken by a widely held company is given by

$$\rho^e - \delta = \left(\frac{r}{1-\tau}\right)(1-\tau a), \tag{61}$$

where τ is the statutory corporate income tax rate, and *a* is the present value of the capital allowances generated by an additional unit of investment *in excess of* allowances for true economic depreciation. Formula (61) thus accounts for the fact that tax systems typically allow accelerated depreciation and sometimes also offer additional investment incentives such as investment tax credits etc. Sørensen (2010b) explains how one may estimate the size of *a*, given the actual depreciation rules and an estimate of the true rate of economic depreciation of the asset considered.

In the case of *debt finance* where interest payments are deductible from the corporate income tax base, Sørensen (2010b) shows that the cost of capital for widely held corporations becomes

$$\rho^d - \delta = r - \left(\frac{\tau}{1-\tau}\right)(\pi + ra). \tag{62}$$

Conventional tax systems involve a tax subsidy to debt finance by allowing deductibility of the full nominal interest expense, including the inflation premium that just serves to offset the erosion of the real debt burden caused by inflation. Accordingly, we see from (62) that a higher inflation rate (π) will ceteris paribus lower the cost of capital. The overall user cost for widely held companies is a weighted average of ρ^e and ρ^d, with weights reflecting ratio of equity to debt in company balance sheets.

In practice, the business sector also includes a group of *closely held firms* (typically proprietorships and small owner-managed corporations) which do not have access to the international equity market. Instead, these firms normally obtain their finance from domestic investors subject to domestic personal taxes, so their user cost of capital will be influenced by the tax rules for interest, dividends and capital gains received by household investors. Sørensen (2010b) shows how the King-Fullerton approach can be adapted to allow an estimate of the cost of capital for closely held firms and how one may derive an average user cost of capital for all firms, assuming that the different forms of business organization are imperfect substitutes in the eyes of investors. However, since the bulk of economic activity in most advanced economies is carried out by firms with access to the international capital market, we ignore this complication here and estimate the user cost of business capital by means of (61) and (62).

V. APPLYING THE METHOD: THE MARGINAL DEADWEIGHT LOSS FROM TAXATION IN SWEDEN

To illustrate how general equilibrium interactions between the main tax bases affect the marginal deadweight loss from taxation, we will now apply the apparatus set up above to the small open Swedish economy.

V.1. Parameter values for Sweden

Table 1 summarises the parameter values obtained when the calibration methods described in section IV are applied to a data set for Sweden for 2008. The analysis in section III showed that the key determinants of marginal deadweight losses are the initial effective tax and transfer rates, the (compensated) elasticities of factor supply and demand, and the share parameters θ^k and θ^s reflecting the size of the tax bases for business income and savings income. The effective tax rates were estimated from the formulae (57) through (62), using information on statutory tax rates and rates of depreciation for tax purposes as well as national accounts data, including data for the ratio of free financial saving to institutional saving (S^F / S^I) and for the consumption of housing services relative to total private consumption ($P_H C_H / PC$). The calibration of the average rate of true economic depreciation (δ) was based on the recent estimates by Hulten (2008) of depreciation rates for different asset types. The substitution elasticities σ_h and σ_o reported in Table 1 are ”guesstimates” which are needed to calculate the effective indirect tax rate on consumption (t^c), using the method described in section III. The estimate for t^c is not very sensitive to the values of these parameters.

Table 1. Calibration of model to data for Sweden, 2008.

Elasticities of factor supply and demand	*Elasticities of substitution*	*Initial effective tax and transfer rates*	*Income and consumption shares etc.*	*Other parameters (annual basis)*
$\varepsilon_w^L = 0.264$	$\phi = 1.0$	$t^w = 0.476$	$\theta^k = 0.15$	$r = 0.05$
$\varepsilon_r^L = 0.055$	$1/\gamma = 1$	$t^c = 0.249$	$\theta^s = 0.145$	$g^w = 0.02$
$\varepsilon_w^S = 0.201$	$\sigma = 1.0$	$t^r = 0.315$ [3]	$S^F / S^I = 0.931$	$g^c = 0.01$
$\varepsilon_r^S = 0.559$	$\sigma_h = 1.5$ [1]	$m^k = 0.079$ [4]	$P_H C_H / PC = 0.2$	$\pi = 0.02$
$\hat{\varepsilon}_r^S = 0.0$	$\sigma_o = 1.0$ [2]	$b_1 = b^w = 0.227$		$\delta = 0.09$
$1/\eta = 1/3$		$b_2 = b^r = 0.086$		$T = 60$
$\varepsilon_\rho^K = 1.0$				$n = 45$

1. Elasticity of substitution between rental and owner-occupied housing (input in the calculation of t^c).
2. Elasticity of substitution between other goods (input in the calculation of t^c).
3. Calculated from (61) and (62), given the statutory tax rates $t^{Is} = 0.15$ and $t^{Fs} = 0.30$.
4. Calculated as a weigthed average of (63) and (64), given the statutory corporate tax rate $\tau = 0.263$, assuming a 60 percent weight for equity finance and a 40 percent weight for debt finance (in line with evidence for Sweden). The value of a was calculated on the assumption that half of investment takes the form of machinery while the other half takes the form of business structures.

Source: Own calculations based on information on statutory tax rules and national accounts data provided by the Swedish Ministry of Finance. For more details on the calibration, see Sørensen (2010b).

The transfer rates b_1 and b_2 were calculated from data on the total after-tax transfers to individuals below and above the retirement age, respectively, measured relative to the total wage bill. The national accounts data on total business income (net of true depreciation) relative to the total wage bill provided a direct estimate of our parameter θ^k. The estimate of θ^s was based on formula (56), assuming that the annual growth rates of real wages and consumption over the consumer's life cycle are 2 percent and 1 percent, respectively. Instead of calibrating the consumption growth rate (g^c) indirectly via the Euler equation (54) and the choice of the utility discount rate d, we simply assume a plausible value for g^c. In applying formula (56), we assume that the consumer's adult life lasts for 60 years, starting with a labour market career of 45 years. This is in line with the parameterization of the life cycle model in Attanasio and Wakefield (2010).

The starting point for the calibration of factor supply elasticities is equation (51) which gives the compensated net wage elasticity of labour supply, given values for the Frisch labour supply elasticity, $1/\eta$, the elasticity of intertemporal substitution, $1/\gamma$, and the transfer rates b_1 and b_2. In calibrating the elasticity of intertemporal substitution we followed Attanasio and Wakefield (2010) in choosing $1/\gamma = 1$ as our baseline, and the Frisch elasticity was set at 1/3.

With these parameter values equation (51) implies that $\varepsilon_w^L = 0.264$. This elasticity should be interpreted broadly to capture all types of behavioural responses affecting the labour income tax base. These responses include changes in labour force participation, changes in hours worked by those already employed, changes in labour productivity stemming from changes in work effort, education and training, shifts between remuneration in taxable cash wages and untaxed fringe benefits, shifts between market work and do-it-yourself activities in the home, shifts between the untaxed "underground economy" and the formal labour market, etc. Changes in all these dimensions of labour supply will affect the labour income tax base and are captured by the elasticity of taxable labour income which measures the response of the labour income tax base to a change in the marginal after-tax rate, defined as one minus the effective marginal labour income tax rate ($1-t^w$ in our notation). Recent empirical studies of the elasticity of taxable labour income in Sweden include Hansson (2007), Holmlund and Söderström (2007), Ljunge and Ragan (2008), and Blomquist and Selin (2009). The estimated elasticities in these studies generally vary from 0.2 to 0.5 (for women, Blomquist and Selin estimate elasticities of 1.0-1.4). These estimates are uncompensated elasticities including income effects as well as substitution effects. Since the income effect on the tax base is negative, the estimates mentioned above must be seen as lower bounds on the compensated elasticity which is relevant when calculating the deadweight loss. Our calibration $\varepsilon_w^L = 0.264$ thus seems to be on the conservative side.

Given the estimate for ε_w^L, the estimates for the compensated cross-price elasticities ε_w^S and ε_r^L follow from (47) and (48). When calculating the value of the after-tax real interest rate r^a in the formulae (48) and (49), which are based on the two-period version of the life cycle model, we assume that the length of each time period is 30 years so that the length of the adult life cycle is 60 years, consistent with the assumption made when we apply the formula for θ^s.

To find the compensated interest elasticity of saving from (49), we need an estimate of the corresponding uncompensated elasticity, $\hat{\varepsilon}_r^S$. Although it varies considerably in the short and medium term, the household savings rate tends to be roughly constant in the long run, despite the fact that the after-tax real interest rate has varied over time due to changes in the level of taxation. This suggests that the long-run uncompensated interest elasticity of saving is approximately zero. As a baseline we therefore assume that $\hat{\varepsilon}_r^S = 0$. This estimate is supported by the early empirical studies on U.S. data by Blinder (1975), Howrey and Hymans (1978) and Skinner and Feenberg (1989). From (51) and (52) we then find the compensated interest elasticity of saving to be roughly 0.56.[10]

Finally, we need an estimate of the price elasticity of capital demand, ε_ρ^K. In their survey of empirical studies of the effects of tax policy on investment, Hassett and Hubbard (2002) conclude that the numerical user cost elasticity of capital demand is probably between 0.5 and 1.0. Here we follow Auerbach and Kotlikoff (1987) in assuming that $\varepsilon_\rho^K = 1$, since this is consistent with the empirical observation that the aggregate gross profit share of GDP is relatively constant over the long run.

V.2. Marginal deadweight losses in Sweden: baseline scenarios

Based on the calibration in Table 1 and the formulae (35), (39), (43) and (46), Table 2 summarizes and decomposes our estimates of the degree of self-financing in Sweden.

According to the estimate in the first row of Table 2, about one third of the initial revenue gain from a (compensated) increase in the marginal tax rate on all labour income will be lost again due to the negative response of tax bases to the higher tax rate. Equivalently, if the marginal tax rate on all labour income is cut a bit, one third of the initial revenue loss will be recouped via the positive reaction of the various tax bases. Most of the estimated deadweight loss from a higher marginal tax rate on labour income stems from the shrinking of the labour income tax base itself, but a substantial part also reflects a decrease in the consumption tax base. Losses of revenue from business income taxes and savings income taxes only account for a minor part of the deadweight

[10] This relatively high positive elasticity might seem to conflict with the claim made by Feldstein (1978) that the compensated interest elasticity of saving is negative unless the compensated price elasticity of demand for retirement consumption is numerically larger than 1. But as pointed out by Sandmo (1981), Feldstein's analysis assumes that the consumer is compensated for a rise in the price of retirement consumption through an increase in first-period income. Sandmo argues that it is more in line with conventional thinking to assume that the consumer is compensated for a higher price of retirement consumption through a rise in retirement income, as we have done here. In that case the compensated interest elasticity of saving in the two-period life cycle model is unambiguously positive.

loss. In part this reflects that these tax bases are relatively small and that the effective marginal tax rate on business income is so low that little business tax revenue is lost when business investment goes down due to the fall in labour supply. The small revenue loss from taxes on savings income also reflects that these losses are discounted since they occur relatively late in the life cycle of the representative cohort.

Table 2. Sweden: Degree of self-financing (*DSF*) associated with a tax rate cut (%)

Cut in effective marginal tax rate on	*Contribution to DSF from higher revenue from taxes on*				*Total DSF*
	Labour income	Consumption	Business income	Savings income	
Labour income (t^w)	23.9	6.6	0.6	1.7	32.8
Consumption (t^c)	16.0	4.4	0.4	1.2	22.0
Business income (t^k)	23.9	6.6	6.2	1.7	38.5
Savings income (t^r)	26.5	7.3	0.6	25.7	60.1

Source: Own calculations based on equations (35), (39), (43), (46) and the calibration in Table 1.

By eroding real wages, indirect taxes on consumption work in part like taxes on labour income, with the same negative effect on labour supply. However, consumption taxes are also paid by individuals outside the labour force such as retirees, so the consumption tax base is broader and less elastic than the labour income tax base. Hence the marginal deadweight loss from higher consumption taxes is smaller than the marginal deadweight loss generated by higher labour income taxes, as shown in the second row in Table 2. The numbers in that row are simply two thirds of the corresponding numbers in the first row, reflecting the size of the proportionality factor $\left(1-t^w\right)/\left(1-t^w+b_1+pb_2\right)$ in formula (39) which captures that the consumption tax base includes the inelastic transfer component b_1+pb_2. Note that the static revenue change in the denominator of our measure of the DSF excludes the change in consumption taxes paid by the current old generations as they consume their public pensions and their accumulated private savings. If these lump sum tax elements were included in the static revenue change, the DSF for the consumption tax would be even lower.

The third row in Table 2 reports the DSF associated with a change in the source-based tax on business income. In practice this could be implemented through a rise in the statutory corporate income tax rate or via measures to broaden the business income tax base such as a cut in depreciation allowances. As noted in section III.3, in a small open economy the burden of a source-based capital tax is shifted onto workers through the fall in real wages generated by the drop in investment. In this way the investment tax reduces labour supply just like a labour income tax. But the investment tax creates an additional capital outflow that further reduces the business income tax base. The resulting effect on business tax revenue can be calculated as the difference between the numbers in the third and the first rows of the third column in Table 2.[11] These numbers imply that, in the Swedish context, a business income tax like the corporation tax generates a marginal deadweight loss which is about 5½ percentage points higher than the marginal deadweight loss from a higher labour income tax. It is remarkable that the marginal deadweight loss from the corporation tax is considerably higher than the marginal deadweight loss from the labour income tax despite the fact that the initial marginal effective tax rate on business income is estimated to be quite low, i.e., only about 8 per cent. However, it should be stressed that the large additional efficiency loss from a source-based business income tax relates to a tax on the "normal" return to investment. A tax that is levied only on above-normal returns would have a lower efficiency cost, at least in so far as such rents are location-specific, but in the model underlying our formulae for the DSF (assuming a competitive economy with constant returns to scale) such pure profits do not arise.

The bottom row of Table 2 shows the dynamic revenue effects of a (compensated) change in the effective tax rate on the real return to financial saving. In case of an increase in t^r, the consequent drop in savings reduces the capital income tax base, thereby generating a dynamic revenue loss of more than one fourth of the static revenue gain, as reported in the last column of the bottom row in Table 2. By reducing the net return to savings made out of labour income, a lower net return to saving also makes working less attractive, causing a drop in labour supply. The resulting deadweight losses are reflected in the first three columns of the bottom row in Table 2.

The roughly 60 per cent marginal deadweight loss from a higher savings income tax is considerably higher than the marginal deadweight losses from the other taxes considered above. Part of the explanation is that, even though the uncompensated interest elasticity of saving is assumed to be zero, the *compensated* interest elasticity of saving implied by our formula (49) is higher than the compensated elasticities assumed for the other tax bases. But the high DSF for the savings income tax may also reflect that the initial effective capital income tax rate (31.5 per cent) is far above its optimal level. The theory of optimal taxation implies that if the utility function (1) takes the weakly separable form $U(u(C_1,C_2),L)$ and the subutility function $u(C_1,C_2)$ is

[11] This difference reflects the size of the term $m^k \varepsilon_\rho^K \left(\frac{\rho-\delta}{\rho}\right)/\left(1-t^w\right)$ in formula (43).

homothetic, the optimal capital income tax rate in the two-period life cycle model is zero.[12] The utility function underlying our calibration does in fact have these properties, as already mentioned.

Despite the rather low calibrated interest elasticity of labour supply, we see from Table 2 that the dynamic losses of revenue from taxes on labour income and consumption contribute substantially to the total deadweight loss from a higher savings income tax. The reason is that the deadweight loss is measured as the present value of the dynamic loss of revenue from the taxes paid by a representative cohort of taxpayers, relative to the present value of the static revenue gain from the higher tax imposed on this cohort. Because capital income is earned relatively late in life, the present value of the static revenue gain is heavily discounted. By contrast, taxes on labour income and consumption are paid already at an early stage in the life cycle. Even if the higher capital income tax only generates a small response in labour supply and consumption, the resulting revenue changes may therefore be substantial relative to the present value of the static revenue gain from the capital income tax that only accrues to the government far into the future.

Two broad messages stand out from Table 2. The first one is that one hardly commits a large quantitative error by ignoring interactions with the tax bases for business income and savings income when estimating the marginal deadweight loss from increases in taxes on labour income and consumption. This conclusion is likely to hold beyond the Swedish context considered here because of the general methodological point that the need for discounting reduces the relative size of the tax base for savings income, and because the tax law in most countries allows accelerated depreciation which tends to keep the marginal effective tax rate on business income quite low.

By contrast, when evaluating the marginal deadweight losses from higher taxes on business income and savings income, it seems crucial to account for the impact of these taxes on the bases for the taxes on labour income and consumption. If these general equilibrium interactions are left out of the calculation, one will seriously underestimate the efficiency losses from higher taxes on saving and investment. Again, this conclusion is likely to have general validity because the need for discounting increases the size of the tax bases for labour income and consumption relative to the savings income tax base, and because a source-based business income tax works mainly as a tax on labour in a small open economy.

[12] This result follows from the theory of optimal commodity taxation; see, e.g., Sandmo (1974). Indeed, in a world where consumers have different productivities, the optimality of a zero capital income tax rate does not even require homotheticity of the subutility function $u\left(C_1, C_2\right)$; it is sufficient that consumers have the same preferences, as shown by Atkinson and Stiglitz (1976). However, Saez (2002) argues that individuals with higher earnings capacities tend to have higher savings propensities. In that case the government's desire for redistribution calls for a positive tax rate on capital income. This distributional concern is not captured by the representative agent model in the present paper.

V.3. Sensitivity analysis

Table 3 illustrates the sensitivity of the total degrees of self-financing to changes in some of the key parameters of our model. The baseline scenarios above assumed a Frisch elasticity of labour supply ($1/\eta$) of one third which implied a compensated elasticity of taxable labour income equal to 0.26, given the link reported in (51) between these two elasticities. The second and third columns in Table 3 assume Frisch elasticities of 0.2 and 0.5, respectively, corresponding to compensated elasticities of taxable labour income equal to 0.17 and 0.36. These variations in ε_w^L imply proportional variations in the compensated cross-price elasticities ε_w^S and ε_r^L, given the restrictions (47) and (48) that follow from the life cycle model. This explains why the total DSF associated with a change in the labour income tax rate varies roughly in proportion to ε_w^L, as indicated in Table 3.

Table 3. Sweden: Degree of self-financing (%) under alternative assumptions on parameter values (rounded numbers)

Change in effective marginal tax rate on	*Base line*[1]	$1/\eta = 0.2$ ($\varepsilon_w^L = 0.17$)	$1/\eta = 0.5$ ($\varepsilon_w^L = 0.36$)	$\hat{\varepsilon}_r^S = -0.3$ ($\varepsilon_r^S = 0.26$)	$\hat{\varepsilon}_r^S = 0.3$ ($\varepsilon_r^S = 0.86$)	$g^c = 0.005$ (θ^s=0.098)	$g^c = 0.015$ (θ^s=0.191)
Labour income	33	21	45	33	33	32	33
Consumption	22	14	30	22	22	21	22
Business income	38	27	50	38	38	38	39
Savings income	60	47	73	46	74	73	53

1. Based on the parameter values in Table 1, including $\varepsilon_w^L = 0.26$, $1/\eta = 1/3$, $\hat{\varepsilon}_r^S = 0$, and $g^c = 0.01$.

Source: Own calculations based on equations (37), (41), (45), (48).

The magnitude of the uncompensated interest elasticity of saving ($\hat{\varepsilon}_r^S$) does not affect the DSF associated with changes in the tax rates on labour income, consumption and business income, since these taxes have no impact on the after-tax return to household saving in a small open economy. Not surprisingly, the size of $\hat{\varepsilon}_r^S$ does influence the degree to which a cut in the tax rate on savings income pays for itself. However, we see from the fourth column in Table 3 that even when the uncompensated interest elasticity of saving is considerably below zero (instead of being zero, as assumed in our baseline), the compensated elasticity implied by our formula (49) remains significantly above zero, and the DSF therefore remains substantial.

The present value of capital income relative to labour income (θ^s) is important for the DSF associated with a change in the tax rate on savings income. The value of θ^s depends inter alia on the desired growth rate of consumption over the life cycle, g^c. The higher this growth rate, the more the consumer will save during his working career, so the higher is the present value of his capital income relative to his labour income. When the savings tax rate is cut, a part of the dynamic revenue gain stems from the rise in labour supply spurred by a compensated rise in the after-tax return to saving. With a higher value of θ^s , the present value of the dynamic revenue gain from increased labour supply will be smaller relative to the present value of the static loss of savings tax revenue. This explains the result shown in Table 3 that the DSF associated with a cut in the savings income tax decreases as the value of θ^s goes up.

Overall, the impression left by Table 3 is that the estimated marginal deadweight losses are not highly sensitive to changes in the key elasticities and parameters.

VI. SUMMARY AND CONCLUSIONS

This paper has presented a simple general equilibrium method of estimating the marginal deadweight loss from taxation in a small open economy. The framework accounts for interactions among the major tax bases and allows a decomposition of the deadweight loss into the losses arising from the shrinkage of each individual tax base. It also incorporates the crucial distinction between taxes on saving and taxes on investment and allows for the fact that a source-based corporation tax works mainly like a tax on labour when the economy is small and open. To apply the framework for the purpose of quantitative analysis, one needs information on certain compensated cross price elasticities of labour supply and savings about which relatively little is known. However, the paper shows how one can exploit some cross-restrictions implied by the standard life cycle model to calibrate these elasticities, given information on the compensated wage elasticity of labour supply about which much more is known.

To illustrate the workings of our methodology, we applied it to estimate the marginal deadweight loss from the main tax instruments in Sweden. Given the relatively high marginal tax rates in that country, it was not surprising to find that the deadweight loss from a rise in the labour income tax could be around one third of the initial static revenue gain, and that the efficiency loss from a higher savings income tax could amount to more than half of the static revenue gain. Our quantitative analysis also confirmed the theoretical expectation that the corporate income tax has a higher marginal efficiency cost than the labour income tax.

Our analysis produced two main insights which we believe to have general validity. First, one does not seem to commit a major error if one ignores the interaction with the tax bases for business income and savings income when estimating the marginal deadweight loss from taxes on labour income and consumption. Second, one *does* commit a major error if one neglects the impact on the tax bases for labour income and consumption when calculating the marginal deadweight loss from taxes on business income and savings income. Indeed, our quantitative analysis suggested that tax interaction effects may be responsible for more than a doubling of the marginal efficiency cost associated with these capital income taxes.

The analysis in this paper applies to broad-based taxes and does not capture the full efficiency cost of increases in selective taxes that only fall on a part of some tax base. For example, an excise tax will have an additional marginal efficiency cost by distorting the pattern of consumption, and selective taxes on particular types of savings income or business income will cause additional deadweight losses by distorting the pattern of saving or investment. Sørensen (2010a, 2010b) shows how the total marginal deadweight losses from selective tax increases can be estimated in a manner consistent with the analysis in the present paper.

REFERENCES

Atkinson, A. B. and J. E. Stiglitz (1976). "The design of tax structure: direct versus indirect taxation." *Journal of Public Economics* 6, 55-75.

Attanasio, O.P. and M. Wakefield (2010). The Effects on Consumption and Saving of Taxing Asset Returns. Chapter 7 in S. Adam, T. Besley, R. Blundell, S. Bond, R. Chote, M. Gammie, P. Johnson, G. Myles and J. Poterba (eds.), *Dimensions of Tax Design – The Mirrlees Review*, Institute for Fiscal Studies, London, and Oxford University Press.

Auerbach, A. J. and L. Kotlikoff (1987). *Dynamic Fiscal Policy*. Harvard University Press, Cambridge, Massachusetts.

Ballard, C., D. Fullerton, J. Shoven and J. Whalley (1985). *A General Equilibrium Method for Tax Policy Evaluation*. Chicago: University of Chicago Press.

Benge, M. (1999). “Marginal Excess Burdens of Taxes on Capital and Labour Income in A Small Open Economy”. Working Papers in Economics and Econometrics, The Australian National University, Working Paper No. 364, February 1999.

Blinder, A.S. (1975). Distribution Effects and the Aggregate Consumption Function. *Journal of Political Economy* 83, 447-475.

Blomquist, S. and H. Selin (2009). Hourly Wage Rate and Taxable Labour Income Responsiveness to Changes in Marginal Tax Rates. UCSF Working Paper 2009:1, Department of Economics, Uppsala University.

Diewert, W.E. (1983). The Measurement of Waste within the Production Sector of an Open Economy. *Scandinavian Journal of Economics* 85, 159-179.

Feldstein, M.S. (1978). The Rate of Return, Taxation and Personal Savings. *Economic Journal* 88, 482-487.

Fullerton, D. and Y.K. Henderson (1987). The Marginal Excess Burden of Different Capital Tax Instruments. National Bureau of Economic Research Working Paper No. 2353, August 1987.

Gordon, R.H. (1986). Taxation of Investment and Savings in a World Economy. *American Economic Review* 76, 1086-1102.

Goulder, L.H. and R.C. Williams (2003). The Substantial Bias from Ignoring General Equilibrium Effects in Estimating Excess Burden. *Journal of Political Economy* 111, 898-927.

Hansson, I. (1984). Marginal cost of public funds for different tax instruments and government expenditures. *Scandinavian Journal of Economics* 86, 115-130.

Hansson, Å. (2007): Taxpayer Responses to Tax Rate Changes and Implications for the Cost of Taxation in Sweden. *International Tax and Public Finance,* vol. 14, 563-582.

Harberger, A.C. (1964). The Measurement of Waste. *American Economic Review* 54, No. 3, 58-76.

Hassett, K. and R.G. Hubbard (2002). Tax Policy and Business Investment. In: A.J. Auerbach and M. Feldstein (eds.), *Handbook of Public Economics,* vol. 3, North-Holland, Amsterdam, pp. 1293-1343.

Hines, J.R. (1999). Three Sides of Harberger Triangles. *Journal of Economic Perspectives* 13, 167-188.

Holmlund, B. and M. Söderström (2008). Estimating Dynamic Income Responses to Tax Reforms: Swedish Experience. Working Paper 2008:28, Institute for Labour Market Policy Evaluation (IFAU).

Howrey, E.P. and S.H. Hymans (1978). The Measurement and Determination of Loanable Funds Saving. *Brookings Papers on Economic Activity* 2, 655-685.

Hulten, C. R. (2008). Getting Depreciation (Almost) Right. Revised version of paper presented at the meeting of the Canberra II Group in Paris, April 23-27, 2007.

Jensen, S.E.H., S.B. Nielsen, L.H. Pedersen and P.B. Sørensen (1994). Labour Tax Reform, Employment and Intergenerational Distribution. *Scandinavian Journal of Economics* 96, 381-401.

Jensen, S.E.H., S.B. Nielsen, L.H. Pedersen and P.B. Sørensen (1996). Tax Policy, Housing and the Labour Market – An Intertemporal Simulation Approach. *Economic Modelling* 13, 355-382.

King, M. A. and D. Fullerton (1984). *The Taxation of Income from Capital: A Comparative Study of the United States, the United Kingdom, Sweden and West Germany*. Cambridge: Cambridge University Press.

Ljunge, M. and K. Ragan (2008). Labor Supply and the Tax Reform of the Century. Working Paper, University of Copenhagen and Stockholm School of Economics, April 21, 2008.

Razin, A. and E. Sadka (1991). International Tax Competition and Gains from Tax Harmonization. *Economics Lettters* 37, 69-76.

Saez, E. (2002). The Desirability of Commodity Taxation under Non-linear Income Taxation and Heterogeneous Tastes. *Journal of Public Economics* 83, 217-230.

Sandmo, Agnar (1974). A note on the structure of optimal taxation. *American Economic Review* 64, 701-706.

Sandmo, A. (1981). The Rate of Return and Personal Savings. *Economic Journal* 91, 536-540.

Shoven, J. and J. Whalley (1972). A General Equilibrium Calculation of the Effects of Differential Taxation of Income from Capital in the U.S. *Journal of Public Economics* 1, 281-321.

Skinner, J. and D.R. Feenberg (1989). The Impact of the 1986 Tax Reform on Personal Saving. In: J. Slemrod (ed.), *Do Taxes Matter? The Effect of the 1986 Tax Reform Act on the U.S. Economy*. MIT Press, Cambridge.

Sørensen, P. B. (2004). Measuring Taxes on Capital and Labor: An Overview of Methods and Issues. Chapter 1 in P.B. Sørensen (ed.): *Measuring the Tax Burden on Capital and Labor*. Cambridge, MA: MIT Press.

Sørensen, P. B. (2010a). *Swedish Tax Policy: Recent Trends and Future Challenges.* Report to the Expert Group on Public Economics (ESO) 2010:4. Regeringskansliet, Finansdepartementet, Stockholm.

Sørensen, P. B. (2010b). *Calculating the Deadweight Loss from Taxation in a Small Open Economy.* Report to the Expert Group on Public Economics (ESO) 2010:4 Supplement. Regeringskansliet, Finansdepartementet, Stockholm.

Sørensen, P. B. (2011). Calibrating the life cycle model. Technical appendix to paper on *Measuring the Deadweight Loss from Taxation in a Small Open Economy,* Department of Economics, University of Copenhagen, October 2011.