CHAPTER 16
CONSUMPTION, INCOME AND WEALTH

Themes of the chapter

- Why study consumption?
- The simple Keynesian consumption function
- The consumer’s intertemporal budget constraint
- Optimal allocation of consumption over time
- The relationship between consumption, income, interest and wealth
- Taxation, public debt and private consumption

THREE GOOD REASONS FOR STUDYING PRIVATE CONSUMPTION

- Private consumption is by far the largest component of aggregate demand
- Economic welfare depends in large part on consumption
- The counterpart to consumption is saving which is a precondition for capital formation and long run growth

Focus of the chapter

- Focus on total consumption
- Focus on the allocation of consumption over time
THE SIMPLE KEYNESIAN CONSUMPTION FUNCTION

\[ C_t = a + b \cdot Y^d_t, \quad a > 0, \quad 0 < b < 1 \] \hspace{1cm} (1)

\[ b \equiv dC/dY^d = \text{marginal propensity to consume} \]
\[ C/Y^d = b + a/Y^d = \text{average propensity to consume} \]

Properties of the Keynesian consumption function:

1) Current consumption depends only on current income

2) The marginal propensity to consume is positive, but less than 1

3) The average propensity to consume falls as income goes up

PROBLEMS WITH THE KEYNESIAN CONSUMPTION FUNCTION

Theoretical problem: Why should consumption depend only on current income?

Empirical problem:

*Microeconomic cross section data* suggest that the average propensity to consume falls as income goes up

but

*Macroeconomic time series data* show that the average propensity to consume does not systematically fall with rising income, but that it is roughly constant
STYLIZED FACTS ABOUT THE RELATIONSHIP BETWEEN CONSUMPTION AND INCOME:
MICROECONOMIC CROSS SECTION DATA

Stylized relationship between income and consumption in microeconomic cross-section data

STYLIZED FACTS ABOUT THE RELATIONSHIP BETWEEN CONSUMPTION AND INCOME:
MACROECONOMIC TIME SERIES DATA

Stylized relationship between income and consumption in macroeconomic time series data
CRITERIA FOR A SATISFACTORY
THEORY OF CONSUMPTION

• The theory must be consistent with optimizing behaviour at the micro level

• The theory must be consistent with the microeconomic cross section
data as well as with the macroeconomic time series data

In the following we will try to develop such a theory of consumption

CONSUMER PREFERENCES

The representative consumer’s planning horizon is divided into two periods:
‘the present’ (period 1) and ‘the future’ (period 2)

Lifetime utility:

\[ U = u(C_1) + \frac{u(C_2)}{1 + \rho}, \quad u' > 0, \quad u'' < 0, \quad \rho > 0 \]  \hspace{1cm} (2)

Properties of the utility function:

• the marginal utility of consumption in each period is positive, but diminishing
  (provides an incentive for consumption smoothing)

• the consumer is impatient: the rate of time preference \( \rho \) is positive

The consumer wishes to trade off present against future consumption with the
purpose of maximizing \( U \). The terms of this trade-off are determined by
THE CONSUMER’S BUDGET CONSTRAINT

Notation

\( V = \) real financial wealth
\( r = \) real rate of interest
\( Y_L = \) real labour income
\( T = \) real net tax payment (taxes minus transfers)
\( C = \) real consumption

Assumptions

- all payments take place at the start of each period (unimportant assumption)
- the capital market is perfect (no borrowing constraints)

The budget constraint for period 1

\[ V_2 = (1 + r) \left( V_1 + Y_1^L - T_1 - C_1 \right) \]  \( (3) \)

The budget constraint for period 2

\[ C_2 = V_2 + Y_2^L - T_2 \]  \( (4) \)
THE CONSUMER’S BUDGET CONSTRAINT

Insert (3) into (4) to get

The consumer’s intertemporal budget constraint

\[ C_1 + \frac{C_2}{1 + r} = V_1 + Y_1^L - T_1 + \frac{Y_2^L - T_2}{1 + r} \]  

(5)

Implication: the present value of total consumption over the life cycle cannot exceed the present value of disposable lifetime income plus the initial stock of wealth

THE CONSUMER’S BUDGET CONSTRAINT

We can simplify the budget constraint by introducing the concept of

Human wealth (human capital)

\[ H_1 \equiv Y_1^L - T_1 + \frac{Y_2^L - T_2}{1 + r} \]  

(9)

Human wealth is given by the present value of disposable labour income over the remaining part of the life cycle

Substitution of (9) into (8) yields a simplified expression for

The intertemporal budget constraint

\[ C_1 + \frac{C_2}{1 + r} = V_1 + H_1 \]  

(10)
OPTIMAL ALLOCATION OF CONSUMPTION OVER TIME

Substitution of the budget constraint (10) into the utility function (2) yields

\[ U = u(C_1) + \frac{u((1+r)(V_1 + H_1 - C_1))}{1 + \rho} \]  

(11)

Maximization of (11) with respect to \( C_1 \) yields the first-order condition

\[ \frac{dU}{dC_1} = 0 \Rightarrow u'(C_1) - \left( \frac{1+r}{1+\rho} \right) u' \left( \frac{C_1}{(1+r)(V_1 + H_1 - C_1)} \right) \]

\[ \upharpoonright \]

\[ u'(C_1) = \left( \frac{1+r}{1+\rho} \right) u'(C_2) \]  

(12)

*Interpretation:* In the optimum the consumer is indifferent between consuming an extra unit today and saving an extra unit today.

OPTIMAL ALLOCATION OF CONSUMPTION OVER TIME

By rearranging (12) we get

**The Keynes-Ramsey rule**

\[ \frac{u'(C_1)}{u'(C_2)/(1 + \rho)} = 1 + r \]  

(13)

*Interpretation:* the marginal rate of substitution between present and future consumption (the left-hand side) must equal the relative price of present consumption (the right-hand side), as shown in Figure 16.3.

*Note:* For \( r = \rho \) it follows from (12) and (13) that the consumer wishes to smooth consumption completely, that is \( C_1 = C_2 \). Figure 16.4 illustrates consumption smoothing in the case where \( V_1 = 0 \).
OPTIMAL ALLOCATION OF CONSUMPTION OVER TIME

Figure 16.3: The consumer’s optimal intertemporal allocation of consumption

OPTIMAL ALLOCATION OF CONSUMPTION OVER TIME

Figure 16.4a: Consumption smoothing for a consumer with relatively low income during period 1 \((V_1=0)\)
DERIVING THE CONSUMPTION FUNCTION

Suppose now that the consumer’s utility function takes the form

\[ u(C_t) = \frac{1}{1 - 1/\sigma} C_t^{1-1/\sigma} \quad \text{for} \quad \sigma > 0, \quad \sigma \neq 1 \]  

(15.a)

\[ u(C_t) = \ln C_t \quad \text{for} \quad \sigma = 1 \]  

(15.b)

To analyze the properties of this utility function, we introduce the *intertemporal elasticity of substitution in consumption (IES)*, defined as

\[ IES \equiv \frac{d(C_2 / C_1) / (C_2 / C_1)}{dMRS(C_2 : C_1) / MRS(C_2 : C_1)} = \frac{d \ln(C_2 / C_1)}{d \ln MRS(C_2 : C_1)} \]  

(16)

*IES* measures the degree to which the consumer is willing to substitute between current and future consumption, as illustrated in Figure 16.6.
DERIVING THE INTERTEMPORAL ELASTICITY OF SUBSTITUTION

Recall that the general lifetime utility function is

\[ U = u(C_1) + \frac{u(C_2)}{1 + \rho}, \quad u' > 0, \quad u'' < 0, \quad \rho > 0 \]  

(2)

From (2) we may find \( MRS \) in the following way:

\[ dU = 0 \Rightarrow u'(C_1) \cdot dC_1 + \left( \frac{u'(C_2)}{1 + \rho} \right) \cdot dC_2 = 0 \quad \Leftrightarrow \]

\[ \frac{dC_2}{dC_1} = -\left( \frac{u'(C_1)}{u'(C_2)/(1 + \rho)} \right)^{\text{MRS}(C_2,C_1)} \]

(14)

According to equation (15.a) we have

\[ u(C_t) = \frac{1}{1 - 1/\sigma} C_t^{-1/\sigma} \quad \text{for} \quad \sigma > 0, \quad \sigma \neq 1 \]

which implies that

\[ u'(C_t) = C_t^{-1/\sigma} \]

(15.c)
DERIVING THE IES

Using (14), (15.c) and the definition of IES given in (16), we now find that

\[
MRS(C_2 : C_1) = \frac{u'(C_1)}{u'(C_2)/(1 + \rho)} = \frac{C_1^{-1/\sigma}}{C_2^{-1/\sigma} / (1 + \rho)} = (1 + \rho)(C_2 / C_1)^{1/\sigma}
\]

\[
\ln MRS(C_2 : C_1) = \ln(1 + \rho) + (1/\sigma) \ln(C_2 / C_1)
\]

\[
d \ln MRS(C_2 : C_1) = (1/\sigma)d \ln(C_2 / C_1)
\]

\[
IES = \frac{d \ln(C_2 / C_1)}{d \ln MRS(C_2 : C_1)} = \frac{d \ln(C_2 / C_1)}{(1/\sigma)d \ln(C_2 / C_1)} = \sigma
\] (17)

THE UTILITY FUNCTION WITH A CONSTANT IES

Thus our utility function (15) has the property that IES is constant and equal to \(\sigma\). The magnitude of \(\sigma\) depends on the curvature of the indifference curves, as illustrated in Figure 16.7

Figure 16.7: The relation between the shape of the indifference curve and the intertemporal substitution elasticity
DERIVING THE CONSUMPTION FUNCTION

From the Keynes-Ramsey rule we know that

\[ MRS \left( C_2, C_1 \right) = 1 + r \]

Given the utility function (15.a), this rule implies that

\[
(1 + \rho)(C_2 / C_1)^{1/\sigma} = 1 + r \quad \Leftrightarrow \quad
\]

\[
C_2^{1/\sigma} = \left( \frac{1 + r}{1 + \rho} \right) C_1^{1/\sigma} \quad \Leftrightarrow \quad
\]

\[
C_2 = \left( \frac{1 + r}{1 + \rho} \right)^\sigma C_1
\]

(18)

DERIVING THE CONSUMPTION FUNCTION

From the intertemporal budget constraint (10) we know that

\[
C_1 + \frac{C_2}{1 + r} = V_1 + H_1 \quad \text{(10)}
\]

Inserting (18) into (10), we get

\[
C_1 + (1 + r)^{-\sigma} (1 + \rho)^{-\sigma} C_1 = V_1 + H_1
\]

\[ \Updownarrow \]

\[
C_1 = \theta \cdot (V_1 + H_1),
\]

(19)

\[
0 < \theta \equiv \frac{1}{1 + (1 + r)^{\sigma-1}(1 + \rho)^{-\sigma}} < 1
\]

Implication: Current consumption is proportional to total current wealth. The propensity to consume wealth is positive, but less than one.
CONSUMPTION AND INCOME

Using the definition of human wealth given in (9), we find from (19) that

\[ C_1 = \theta \cdot \left( Y_1^d + \frac{Y_2^d}{1+r} + V_1 \right), \quad Y_i^d \equiv Y_i^L - T_i \]  \hspace{1cm} (20)

which may be rewritten as

\[ \frac{C_1}{Y_i^d} = \hat{\theta} \]  \hspace{1cm} (21)

\[ \hat{\theta} = \theta \left( 1 + \frac{R}{1+r} + v_1 \right), \quad R \equiv \frac{Y_2^d}{Y_1^d}, \quad v_1 \equiv \frac{V_1}{Y_i^d} \] \hspace{1cm} (22)

If the expected rate of income growth is denoted by \( g \), we may write equation (20) in the following way:

\[ \frac{C_1}{Y_i^d} = \theta \left( 1 + \frac{1+g}{1+r} + v_1 \right) \]  \hspace{1cm} (23)

EXPLAINING THE RELATIONSHIP BETWEEN CONSUMPTION AND INCOME

**Cross section data:** In a cross section of consumers in a given year many persons with a low current income may expect to earn a higher future income. Hence they will have a high propensity to consume, since \( R \) in (22) will be high. Other individuals with high current incomes will expect a lower future income (a low value of \( R \)) and will therefore have a low propensity to consume.

The difference between current and future income may be due to the fact that income varies systematically over the life cycle (the life cycle theory, Modigliani), or may be due to temporary random fluctuations in income (the permanent income theory, Friedman).

**Time series data:** In the long run the variables \( g, r \) and \( v_1 \) are roughly constant. Hence the average propensity to consume will also be roughly constant, according to (23).
CONSUMPTION AND INTEREST

From (19) we recall that

\[ C_1 = \theta \cdot (V_1 + H_1), \]

\[ 0 < \theta \equiv \frac{1}{1 + (1 + r)^{\sigma - 1} (1 + \rho)^{-\sigma}} < 1 \]

From the definition of \( \theta \) we find

\[ \frac{d\theta}{dr} = \frac{(1 - \sigma)(1 + r)^{\sigma - 2} (1 + \rho)^{-\sigma}}{\left[1 + (1 + r)^{\sigma - 1} (1 + \rho)^{-\sigma}\right]^2} \quad (24) \]

The effect of a change in the rate of interest on the propensity to consume is ambiguous because of offsetting income and substitution effects.

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CONSUMPTION AND INTEREST

However, a rise in the real rate of interest also has two negative wealth effects on consumption:

**The human wealth effect:** A rise in the rate of interest reduces the present value of lifetime labour income, since

\[ H_1 \equiv Y_1^L - T_1 + \frac{Y_2^L - T_2}{1 + r} \quad (9) \]

**The financial wealth effect:** A rise in the rate of interest reduces the market value of financial wealth, since stock prices and housing prices vary negatively with the real interest rate.
THE GENERALIZED CONSUMPTION FUNCTION

We may summarize our theory of private consumption in the generalized consumption function:

\[ C_1 = C\left( Y^d_{(+)} , g , r, V_1 \right) \]  

(36)

Expectations about the future affect consumption via \( g \) and via \( V_1 \).

IMPORTANT CONCEPTS AND POINTS IN CHAPTER 16

• The simple Keynesian consumption function

• The lifetime utility function, the rate of time preference and the intertemporal elasticity of substitution

• The consumer’s intertemporal budget constraint

• Human capital

• The optimal allocation of consumption over time: the Keynes-Ramsey rule and consumption smoothing

• Derivation of the consumption function

• Explanation of the relationship between income and consumption in cross section data and in time series data: the life cycle theory and the permanent income theory
IMPORTANT CONCEPTS AND POINTS IN CHAPTER 16

• The relationship between wealth and consumption
• The relationship between the real interest rate and consumption
• The effects of temporary and permanent tax cuts
• The intertemporal government budget constraint
• The Ricardian Equivalence Theorem
• Reasons why Ricardian Equivalence is likely to fail