Monetary Economics: Macro Aspects, 11/2 2004
Henrik Jensen

Plan for today:

1. Money in the utility function (ended)

   a. Welfare costs of inflation

   b. Potential non-superneutrality of money

   c. Dynamics and calibration

Literature: Walsh (2003, Chapter 2, pp. 59-80, but check the Appendix as well)

Welfare Costs of Inflation

- Inflation affects real money holdings by affecting nominal interest rates and thus the opportunity cost of holding money:

$$\frac{u_m(c^{ss}, m^{ss})}{u_c(c^{ss}, m^{ss})} = \frac{i^{ss}}{1+i^{ss}} = I \qquad \text{(money demand)}$$

$$i^{ss} = r^{ss} + \pi^{ss} \qquad \text{(Fisher relationship)}$$

- Within the model framework with a relevant welfare measure (household utility), what are the welfare costs of inflation?
- Is there an optimal rate of inflation in the model?

On the optimal rate of inflation

- Bailey/Friedman intuition:
  - Private marginal cost of holding money is increasing in the nominal interest rate
  - Social marginal cost of creating money is essentially zero

  $\Longrightarrow$ Equating private and social marginal cost requires *a zero nominal interest rate*

  - By the Fisher relationship it follows that $\pi^{ss} = -r^{ss} < 0$ is optimal
  - I.e., the optimal rate of change in prices involves *deflation* equal to the real interest rate ("The Friedman Rule")

- This is formally confirmed in the model when finding the utility maximizing nominal money growth rate subject to resource constraint of economy
- First-order condition for choice of steady-state nominal money growth rate:

$$u_c\left(f\left(k^{ss}\right)-\delta k^{ss},m^{ss}\right)\frac{\partial\left(f\left(k^{ss}\right)-\delta k^{ss}\right)}{\partial m^{ss}}\frac{\partial m^{ss}}{\partial \theta^{ss}}+u_m\left(c^{ss},m^{ss}\right)\frac{\partial m^{ss}}{\partial \theta^{ss}}=0$$

or,

$$u_m\left(c^{ss},m^{ss}\right)=0$$

- This implies $i^{ss}=0$, and condition determines what Friedman called the "optimal quantity of money"
  - Note that with some finite $\overline{m}$ defined as $u_m\left(c^{ss},\overline{m}\right)=0$, and $u_m\left(c^{ss},m^{ss}\right)<0$ for $m^{ss}>\overline{m}$, this $\overline{m}$ is the optimal quantity of money

What are then the welfare costs of inflation?

- Could be computed as the area under the money demand curve at a given positive nominal interest rate (Figure 2.2)
  - This is the "consumer surplus" lost by a positive nominal interest rate, and thus inflation higher than what dictated by the Friedman rule. Some estimates indicate that inflation of 10% corresponds to 1-3% of GDP per year
- Could, as Lucas suggests, be computed as the percentage increase in steady-state consumption needed to compensate for a suboptimal low real money stock caused by too high $i/(1+i)\equiv\Upsilon$
  - Normalizing $c^{ss}=1$, this implies that the cost of inflation $w(\Upsilon)$ is implicitly given as

    $$u\left(1+w\left(\Upsilon\right),m\left(\Upsilon\right)\right)=u\left(1,m^{*}\right)$$

    where m^* is the optimal quantity of real money balances, and $m(\Upsilon)$ is the money demand function, $m'<0$.
  - With specific form of the utility function, and using numbers from estimated money demand functions, Lucas finds a 10% nominal interest rate is equivalent of around 1.3% lost steady-state consumption
  - A large number or not? In present value terms it is much higher (as it is 1.3% each and every year.....)
- Note that MIU model ignores other cost of inflation (its variability, its influence on an unindexed tax system, ...)

Potential non-superneutrality of money

- Is superneutrality of money a robust feature of the MIU model?
- In the model of previous lecture, endogenous savings behavior uniquely defines steady-state capital
  - Capital is accumulated or decumulated until its net marginal product (real interest rate) equals households' subjective real interest rate:

$$f_k(k^{ss}) + 1 - \delta = \frac{1}{\beta}$$

  - Hence, long-run superneutrality can only fail if the marginal product of capital is affected by inflation
- Possible if production function contains another endogenous input factor, which is affected by inflation
- "Candiates"? The obvious one: *Money* in the production function: $y_t = f(k_{t-1}, m_t)$, $f_m > 0$
  - E.g., if $f_{km} > 0$ (more money makes capital more productive) higher inftlation leads to lower real money balances and lower steady-state capital

- More natural possibility is endogenous *labor input* in production
- Arises in MIU model if it is amended by a labor supply choice by households
- This is achieved by having *leisure* enter in utility function:

$$u_t = u(c_t, m_t, l_t)$$

- Assuming that l is the fraction of time spent on leisure, the production function is

$$y_t = f(k_{t-1}, 1 - l_t),$$

or,

$$y_t = f(k_{t-1}, n_t)$$

with $n_t = 1 - l_t$ being fraction of time spent on work (NB: n_t IS NOT THE POPULATION GROWTH RATE!)

- Households now face **an additional decision**: How much time should be devoted to work; how much to leisure?
- The relevant optimality condition is

$$u_l(c_t, m_t, l_t) = u_c(c_t, m_t, l_t) f_n(k_{t-1}, 1 - l_t) \qquad (2.34')$$

Marginal gain of leisure is equated to the marginal cost, which is the utility loss from lower consumption times the marginal product of labor (the real wage)

- In steady state one have then the following three relationships:

$$u_l\left(c^{ss}, m^{ss}, l^{ss}\right) = u_c\left(c^{ss}, m^{ss}, l^{ss}\right) f_n\left(k^{ss}, 1 - l^{ss}\right) \qquad (l \text{ versus } c \text{ choice})$$

$$f_k\left(k^{ss}, 1 - l^{ss}\right) + 1 - \delta = \frac{1}{\beta} \qquad \text{(constant capital)}$$

$$c^{ss} = f\left(k^{ss}, 1 - l^{ss}\right) - \delta k^{ss} \qquad \text{(national account)}$$

- Note: If $u_l\left(c^{ss}, m^{ss}, l^{ss}\right)/u_c\left(c^{ss}, m^{ss}, l^{ss}\right)$ is independent of m^{ss}, these equations determine k^{ss}, l^{ss} and c^{ss}. Long-run superneutrality holds!
  - This will be the case if utility is *separable* in money; e.g. $u = v(c, l)\, g(m)$ (u_l and u_c are affected by m in the same way)
  - Also, of course, it will be the case if u_l and u_c are not affected by m at all
- But if $u_l\left(c^{ss}, m^{ss}, l^{ss}\right)/u_c\left(c^{ss}, m^{ss}, l^{ss}\right)$ *depends* on m^{ss}, long-run superneutrality will not hold
- Why would u_l or u_c depend on m? And in which direction?
- Note that if u_l and u_c are independent of m, then superneutrality holds in the short run as well — dynamics "collapse" into a real Ramsey-style model (the "Keynes-Ramsey rule" depicting the evolution of marginal utility of consumption will no longer be affected by money)

- Specific functional form of utility function:

$$u\left(c_t, m_t, l_t\right) = \frac{\left(a c_t^{1-b} + (1-a)\, m_t^{1-b}\right)^{\frac{1-\Phi}{1-b}}}{1-\Phi} + \Psi \frac{l_t^{1-\eta}}{1-\eta},$$

$0 < a < 1$, $b > 0$, $\eta > 0$, $\Phi > 0$, $\Psi > 0$ $(b, \eta, \Phi \neq 1)$

  - Φ is coefficient of relative risk aversion
  - b is inverse nominal interest elasticity of money demand — see (2.25), p. 57
- What is $u_l\left(c^{ss}, m^{ss}, l^{ss}\right)/u_c\left(c^{ss}, m^{ss}, l^{ss}\right)$ with this specification?

$$\frac{u_l}{u_c} = \frac{\Psi l_t^{-\eta}}{a\left(a c_t^{1-b} + (1-a)\, m_t^{1-b}\right)^{\frac{b-\Phi}{1-b}} c_t^{-b}}$$

  - Hence, if $\Phi = b$, $u_{cm} = 0$ and superneutrality holds in the short *and* the long run
  - If $\Phi < b$ (empirically plausible), then $u_{cm} > 0$.
    - Higher expected inflation will reduce real money balances and decrease marginal utility of consumption
    - Households substitute towards leisure, and labor supply decreases
    - Superneutrality fails in the short and long run
  - If $\Phi > b$ (empirically less plausible), then $u_{cm} < 0$ and superneutrality fails "in the opposite direction"
- Note that it is *anticipated* changes in inflation that causes real effects. An unanticipated, temporary, change in π_t has *no* effects, as it does not affect the nominal interest rate that ultimately determines money demand. Only when π_{t+1} is affected, is the nominal interest rate affected through the Fisher relationship, $i_t = \pi_{t+1} + r_t$

Dynamics and calibration

- Given that superneutrality fails in the short run due to endogenous labor choice, a relevant issue is whether the MIU model has short-run properties which match the data
- I.e., how is monetary shocks transmitted to the real economy, and how will monetary policy be able to play a stabilizing role?
- For this purpose a stochastic version of the model is formulated.
- Exogenous shocks bringing the economy away from steady state will be *technology shocks* and *shocks to the growth rate of nominal money supply*

Model and private sector optimization. General case (a la Appendix in Walsh)

- Production function is amended to

$$y_t = f\left(k_{t-1}, 1 - l_t, z_t\right)$$

where z_t is a technology shock

- Assumption:

$$z_t = \rho z_{t-1} + e_t, \qquad |\rho| < 1,$$

with e_t being a mean-zero, white-noise shock

- Nominal money growth is assumed to be

$$\theta_t = \theta^{ss} + u_t$$

where u_t is a shock to the growth rate

- Assumption:

$$u_t = \gamma u_{t-1} + \phi z_{t-1} + \varphi_t, \qquad 0 \leq \gamma < 1, \quad \phi \lesseqgtr 0$$

with φ_t being a mean-zero, white-noise shock.

  – Note that there may or may not be serial correlation in the shocks to nominal money growth
  – Note that money growth may or may not respond toward past technology shocks, and may be either *procyclical* ($\phi > 0$) or *countercyclical* ($\phi < 0$).

- Per-period utility function and budget constraint are

$$u\left(c_t, m_t, l_t\right)$$

and

$$y_t + \tau_t + (1 - \delta)\, k_{t-1} + \frac{1}{1 + \pi_t} m_{t-1} = c_t + k_t + m_t.$$

(ignoring financial assets b_t as in last lecture....)

- As in MIU model without endogenous labor, households maximizes discounted lifetime utility subject to the budget constraint
- Again, dynamic programming method is used
- Note, however, that since l_t is a choice variable, it inappropriate to treat available resources as the state variable at period t. Instead, state variables will therefore be k_{t-1} and $a_t \equiv \tau_t + m_{t-1}/\left(1 + \pi_t\right)$.

- The optimization is then characterized by the value function

$$V(a_t, k_{t-1}) = \max \mathrm{E}_t \{u(c_t, m_t, l_t) + \beta V(a_{t+1}, k_t)\}$$

where the maximization is over c, m, k, and l subject to the budget constraint and the definition of the state variable a_t. E_t is the rational expectations operator.

- One substitutes the constraint and definition so as to eliminate k_t and a_{t+1} and get an unconstrained maximization problem

- First-order condition with respect to c_t:

$$u_c(c_t, m_t, l_t) = \mathrm{E}_t \beta V_k(a_{t+1}, k_t) \qquad (2.51')$$

(as $\partial a_{t+1}/\partial c_t = 0$ by the definition of a). Usual interpretation: Marginal gain of consumption must equal the expected marginal loss in terms of lower capital in next period

- First-order condition with respect to m_t:

$$u_m(c_t, m_t, l_t) + \beta \mathrm{E}_t V_a(a_{t+1}, k_t) \frac{1}{1+\pi_{t+1}} = \beta \mathrm{E}_t V_k(a_{t+1}, k_t) \qquad (2.53')$$

Usual interpretation: Marginal gain in terms of current utility and expected next period monetary wealth must equal the expected marginal loss in terms of lower capital in next period

- First-order condition with respect to l_t:

$$u_l(c_t, m_t, l_t) = \mathrm{E}_t \beta V_k(a_{t+1}, k_t) f_n(k_{t-1}, 1-l_t, z_t) \qquad (2.54')$$

Marginal gain of leisure is equated to the marginal cost, which is the value loss from less next-period capital, times the marginal product of labor (the real wage)

Mathematical digression "not for lecturing", <u>but for reading:</u> Elimination of the value function

- We know that optimum will be characterized by optimal values of c_t, m_t, and l_t as functions of the state variables. Call these functions

$$c_t = c(a_t, k_{t-1}), \quad m_t = m(a_t, k_{t-1}), \quad l_t = l(a_t, k_{t-1}).$$

- The value function is thus by definition given as

$$V(a_t, k_{t-1}) = u(c(a_t, k_{t-1}), m(a_t, k_{t-1}), l(a_t, k_{t-1})) + \beta \mathrm{E}_t V(a_{t+1}, k_t)$$

This holds for all a_t, k_{t-1} so we have

$$\begin{aligned} V_a(a_t, k_{t-1}) &= u_c(c(a_t, k_{t-1}), m(a_t, k_{t-1}), l(a_t, k_{t-1}))\, c_a(a_t, k_{t-1}) \\ &\quad + u_m(c(a_t, k_{t-1}), m(a_t, k_{t-1}), l(a_t, k_{t-1}))\, m_a(a_t, k_{t-1}) \\ &\quad + u_l(c(a_t, k_{t-1}), m(a_t, k_{t-1}), l(a_t, k_{t-1}))\, l_a(a_t, k_{t-1}) \\ &\quad + \beta \mathrm{E}_t V_a(a_{t+1}, k_t) \frac{\partial a_{t+1}}{\partial a_t} + \beta \mathrm{E}_t V_k(a_{t+1}, k_t) \frac{\partial k_t}{\partial a_t} \\ &= \beta \mathrm{E}_t V_k(a_{t+1}, k_t) \end{aligned}$$

where last equality follows from the fact that when $c_t = c(a_t, k_{t-1})$, $m_t = m(a_t, k_{t-1})$, $l_t = l(a_t, k_{t-1})$, it follows that

$$\begin{aligned} \frac{\partial a_{t+1}}{\partial a_t} &= \frac{1}{1+\pi_{t+1}} m_a(a_t, k_{t-1}), \\ \frac{\partial k_t}{\partial a_t} &= 1 - c_a(a_t, k_{t-1}) - m_a(a_t, k_{t-1}) - f_n(k_{t-1}, 1-l_t, z_t)\, l_a(a_t, k_{t-1} \end{aligned}$$

such that all the terms in front of $c_a(a_t, k_{t-1})$, $m_a(a_t, k_{t-1})$ and $l_a(a_t, k_{t-1})$ are zero. I.e., at an optimum, the marginal value of changing c, m, or l must be zero.

- Likewise we get

$$V_k(a_t, k_{t-1}) = \beta \mathrm{E}_t V_k(a_{t+1}, k_t) [f_k(k_{t-1}, 1-l_t, z_t) + 1 - \delta]$$

- So, by the first-order condition guiding c:

$$u_c(c_t, m_t, l_t) = V_a(a_t, k_{t-1})$$

- We then get the corresponding relationship for guiding money choice similar to simple MIU model:

$$\begin{aligned} u_m(c_t, m_t, l_t) + \beta \mathrm{E}_t u_c(c_{t+1}, m_{t+1}, l_{t+1}) \frac{1}{1+\pi_{t+1}} &= \beta \mathrm{E}_t V_k(a_{t+1}, k_t) \\ &= u_c(c_t, m_t, l_t) \ (*) \end{aligned}$$

- Also, the condition guiding consumption can be modified by use of $V_k(a_t, k_{t-1}) = \beta \mathrm{E}_t V_k(a_{t+1}, k_t)[f_k(k_{t-1}, n_t, z_t) + 1 - \delta]$:

$$\begin{aligned} u_c(c_t, m_t, l_t) &= \mathrm{E}_t \beta V_k(a_{t+1}, k_t) \\ &= \beta \mathrm{E}_t \beta \mathrm{E}_{t+1} V_k(a_{t+2}, k_{t+1})[f_k(k_t, 1-l_{t+1}, z_{t+1}) + 1 - \delta] \end{aligned}$$

and using that $V_a(a_t, k_{t-1}) = \beta \mathrm{E}_t V_k(a_{t+1}, k_t)$, gives $V_a(a_{t+1}, k_t) = \beta \mathrm{E}_{t+1} V_k(a_{t+2}, k_{t+2})$ and thus

$$\begin{aligned} u_c(c_t, m_t, l_t) &= \beta \mathrm{E}_t \mathrm{E}_{t+1} V_a(a_{t+1}, k_t)[f_k(k_t, 1-l_{t+1}, z_{t+1}) + 1 - \delta] \\ &= \beta \mathrm{E}_t R_t u_c(c_{t+1}, m_{t+1}, l_{t+1}) \qquad (**) \end{aligned}$$

with $R_t \equiv f_k(k_t, 1-l_{t+1}, z_{t+1}) + 1 - \delta$. I.e., (**) is the "modified Keynes-Ramsey rule"

- Finally, we get the condition for the choice of l_t, which becomes

$$u_l(c_t, m_t, l_t) = u_c(c_t, m_t, l_t) f_n(k_{t-1}, 1-l_t, z_t), \qquad (***)$$

- Hence, the equations (*), (**) and (***), together with the budget constraint, provide solutions for the paths of c, m, l, and k.

End of mathematical digression "not for lecturing"

Particular functional forms of utility and production functions

- Model is solved by numerical methods under assumptions about particular functional forms for utility and production function
- Utility (as before):

$$u(c_t, m_t, l_t) = \frac{\left(a c_t^{1-b} + (1-a) m_t^{1-b}\right)^{\frac{1-\Phi}{1-b}}}{1-\Phi} + \Psi \frac{l_t^{1-\eta}}{1-\eta}$$

- Production function, Cobb-Douglas:

$$y_t = k_{t-1}^{\alpha} n_t^{1-\alpha} \mathrm{e}^{z_t}, \qquad 0 < \alpha < 1$$

- Steady-state solution: Note that the real interest rate,

$$R_t = f_k(k_t, 1-l_{t+1}, z_{t+1}) + 1 - \delta$$

then becomes

$$\begin{aligned} R_t &= \alpha \mathrm{E}_t k_t^{\alpha-1} (1-l_{t+1})^{1-\alpha} \mathrm{e}^{z_{t+1}} + 1 - \delta \\ &= \alpha \frac{\mathrm{E}_t y_{t+1}}{k_t} + 1 - \delta. \end{aligned}$$

By the steady-state condition $R^{ss} = 1/\beta$, this only determines the *ratio* y^{ss}/k^{ss}

- This *ratio* will be independent of monetary factors, but the *levels* y^{ss}, k^{ss}, c^{ss} may not, if superneutrality fails so that l^{ss} will be affected

Dynamic effects of money and technology shocks

- To assess the quantitative effects of money and technology shocks, the model is calibrated and simulated
- Calibration: Assign empirically plausible values the parameters of the model
- Simulation:
  - Perform a linearization of the model's dynamic equations (everything is expressed as percentage deviations from steady state);
  - solve this system by numerical methods (various simulation programs are available on the internet);
  - create artificial time series data from the system
- From the artificial data one evaluates the properties of the model in terms of:
  - Standard deviations of various relevant variables, and their s.d. relative to output
  - Correlation coefficients of various variables with output
  - Impulse response patterns of variables when shocks hit

- **Main results** (when $b > \Phi$; implying $u_{cm} > 0$)
  - Steady-state non-superneutrality is of the form of: Higher θ $\rightarrow$lower output
  - If money shocks, φ_t-shocks, shall pay a role, persistence in money growth is necessary ($\gamma > 0$ is needed). If not, the shock will *not* affect expected next-period inflation, and thus — through the Fisher equation — period t nominal interest rate. Real money holdings are unaltered and the consumption-leisure trade-off unaltered
  - The effects of money shocks on labor and output are stronger the more persistence in money growth, but the effects are quantitatively **very** small
  - If technology shocks are met with procyclical money, output is more stable. The magnitude, however, is modest
  - Main effects of money shocks are on inflation and nominal interest rates
  - Positive money shocks lead to *higher* nominal interest rates. In contrast with usual IS/LM story (where a *liquidity effect* is present: nominal rates fall to increase money demand). Reason is flexible prices in the MIU model (contrary to the sticky price IS/LM model).
    - ∗ Prices adjust instantaneously so as to *reduce* real money supply, matching the fall in demand resulting from higher nominal interest rates.

Summary

- The MIU framework provides a setting in which the welfare costs of inflation can be assessed, and where the optimal inflation rate can be determined
- This, in turn, is equivalent of determining the "optimal quantity of money"
- Some aspects of inflation for public finance and taxation, however, are neglected

- The stochastic, dynamic model without the superneutrality property can be used to assess the importance of monetary shocks for economic fluctuations
- In the calibrated, MIU model with endogenous labor, money matters for business cycle fluctuations, but **not very much**
- This is one indication that flexible-price models may be ill-suited for analysis of monetary phenomena in the short run

Plan for next lectures

Monday, February 16

1. Shopping-time models
2. Cash-in-Advance Models (certainty)

Literature: Walsh (2003, Chapter 3, pp. 95-111)

Wednesday, February 18

1. Cash-in-Advance models (stochastic)
2. Money and real costs of transactions

Literature: Walsh (2003, Chapter 3, pp.126-131)