Monetary Economics: Macro Aspects, 25/2 2004
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Plan for today:

1. Public budget accounting, inflation and debt
2. Equilibrium seigniorage

Literature: Walsh (Chapter 4, pp. 135-164)

3. Plan for next lecture

Introductory remarks

- Inflation or the nominal interest rate have been viewed as a **tax** on household's resources in the previous lectures.
  - In particular through the **erosion of real money balances**
- The flip-side of the coin have been ignored: The proceedings from the tax have not been modelled; i.e., the government budget has been (almost) neglected
- This is an omission, as one ignores important questions like:
  - Can inflation be effectively be used as a means of financing public expenditures, deficits and debt?
  - Will monetary and fiscal policy interact in ways that qualify some of the conclusions reached so far from the flex-price models?
  - Will debt and deficit policy have implications for monetary policy and thus, e.g., inflation?
  - If inflation is used as a tax, what will its optimal value be?
    - (will the Friedman rule robust to public finance considerations?)

Public budget accounting, inflation and debt

- The core element linking monetary and fiscal policy is the **public budget constraint**
- First consider the fiscal branch of government (the Treasury). It faces the following budget identity in nominal terms

$$G_t + i_{t-1}B^T_{t-1} = T_t + \left(B^T_t - B^T_{t-1}\right) + RCB_t \qquad (4.1)$$

Superscript T denotes total public debt; RCB_t is receipts from the central bank

- Then consider the central bank's budget identity

$$\left(B^M_t - B^M_{t-1}\right) + RCB_t = i_{t-1}B^M_{t-1} + (H_t - H_{t-1}) \qquad (4.2)$$

Superscript M denotes public debt held by the central bank; H_t is "high-powered money" — the monetary base; the central bank's own liabilities

- Let $B_t = B^T_t - B^M_t$ be public debt held by the private sector. The the two budget identities are combined to the **consolidated budget identity**

$$G_t + i_{t-1}B_{t-1} = T_t + (B_t - B_{t-1}) + (H_t - H_{t-1}) \qquad (4.3)$$

- Express things in real terms and in relation to total nominal income $P_t y_t$, and ignore population and output growth:

$$g_t + \overline{r}_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + \frac{H_t - H_{t-1}}{P_t y_t} \qquad (4.4')$$

(lower-case letters are variables relative to total nominal income)

- The term

$$\overline{r}_{t-1} = \frac{1 + i_{t-1}}{1 + \pi_t} - 1$$

is the **ex post** real interest rate

- Define

$$r_{t-1} = \frac{1 + i_{t-1}}{1 + \pi^e_t} - 1$$

as the **ex ante** real interest rate, and the budget identity becomes

$$g_t + r_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + \frac{(1 + r_{t-1})(\pi_t - \pi^e_t)}{1 + \pi_t}b_{t-1} + \frac{H_t - H_{t-1}}{P_t y_t} \qquad (4.5')$$

- Note the revenue caused by $\pi_t > \pi^e_t$ as higher-than-expected inflation **erodes** the real servicing costs of debt; but inflation in itself, when anticipated, does not reduce the real debt burden.
- The last term of (4.5') is **seigniorage**, the real income generated by "issuing non-interest bearing debt": money
- Seigniorage relative to total nominal income is

$$s_t \equiv h_t - \frac{1}{1 + \pi_t}h_{t-1} = h_t - h_{t-1} + \frac{\pi_t}{1 + \pi_t}h_{t-1} \qquad (4.6')$$

- Importantly, even in steady state with $h_t - h_{t-1} = 0$, one has

$$s = \frac{\pi}{1 + \pi}h,$$

which is **positive** for **positive inflation**

- Alternative intuition: As h is a real liability for the government, which doesn't pay interest, inflation reduces the real return, and thereby reduces "real interest" payments on money
- What if inflation is zero? Will there be no revenue from seigniorage? Yes, by issuing non-interest bearing debt (=money) instead of interest bearing debt (bonds), **the government saves interest payments**
- To see this, rewrite budget identity, defining $d_t = b_t + h_t$ as **total** liabilities, as

$$\begin{aligned} & g_t + r_{t-1}\left(d_{t-1} - h_{t-1}\right) \\ = & \ t_t + \left(d_t - d_{t-1}\right) + \frac{\left(1 + r_{t-1}\right)\left(\pi_t - \pi_t^e\right)}{1 + \pi_t}\left(d_{t-1} - h_{t-1}\right) + \frac{\pi_t}{1 + \pi_t} h_{t-1} \end{aligned}$$

and thus

$$\begin{aligned} g_t + r_{t-1} d_{t-1} \ = & \ t_t + \left(d_t - d_{t-1}\right) + \frac{\left(1 + r_{t-1}\right)\left(\pi_t - \pi_t^e\right)}{1 + \pi_t} d_{t-1} \\ & + \left[\frac{\pi_t - \left(1 + r_{t-1}\right)\left(\pi_t - \pi_t^e\right)}{1 + \pi_t} + r_{t-1}\right] h_{t-1} \end{aligned}$$

$$g_t + r_{t-1} d_{t-1} = t_t + \left(d_t - d_{t-1}\right) + \frac{\left(1 + r_{t-1}\right)\left(\pi_t - \pi_t^e\right)}{1 + \pi_t} d_{t-1} + \frac{i_{t-1}}{1 + \pi_t} h_{t-1}$$

- So, when formulated in terms of total liabilities, the steady-state seigniorage is

$$\overline{s} = \frac{i}{1 + \pi} h \tag{4.9}$$

- So, a Friedman rule eliminates seigniorage, and requires offsetting fiscal policy changes to maintain budget identity

- In general, irrespective of definition of seigniorage, a change in monetary financing requires offsetting changes in either taxes, spending or debt
- In which manner monetary changes affect fiscal policy depends on the definition of fiscal policy
  - If it is in terms of fixed spending and interest rate bearing debt, changes in s and the offsetting changes in taxes are monetary policy
  - If it is in terms of fixed spending and total liabilities, changes in $\overline{s}$ and offsetting taxes and composition of liabilities are monetary policy
- So which definition is relevant, depends on how fiscal policy is conducted

- Note implication of simple version of budget identity (ignoring unanticipated inflation)

$$g_t + r b_{t-1} = t_t + b_t - b_{t-1} + s_t,$$

the "solvency requirement":

$$\left(1 + r\right) b_{t-1} + \sum_{i=0}^{\infty} \frac{g_{t+i}}{\left(1 + r\right)^i} = \sum_{i=0}^{\infty} \frac{t_{t+i} + s_{t+i}}{\left(1 + r\right)^i}$$

which holds when $\lim_{i \to \infty} \left(1 + r\right)^{-i} b_{t+i} = 0$ (no "Ponzi games")

- So, if government has initial debt, it must at some point run surpluses, $t_{t+i} + s_{t+i} > g_{t+i}$, generated through taxes or seigniorage

- This raises the issue of whether government debt or deficits will ultimately create seigniorage and thus inflation
- This will generally depend on the **fiscal-monetary regime**
  - If fiscal policy is "dominant" (or "active"), monetary policy must be passive, and secure solvency
  - If monetary policy is "dominant", fiscal policy must secure solvency
- Hence, in regimes of fiscal dominance, it may be the case that debt and deficits will be inflationary
- Also, it may be the case that **monetary contractions** (e.g., aimed at reducing inflation) will reduce seigniorage revenues, increasing debt, which ultimately requires increased seigniorage, and thus, **inflation in the future** (Sargent and Wallace's "Unpleasant Monetarist Arithmetic")
- This emphasizes that treating money as independent of fiscal policy, could be **very misleading**, as monetary policy changes could very well be the result of changes in fiscal policy
  - . . . the effects on the macroeconomy could be very different depending upon the source of the change in monetary policy

A simple model to prove the point

- Assume government spending is zero. The budget constraint then becomes

$$(1 + r_{t-1})\, b_{t-1} = t_t + b_t + s_t \qquad (4.15)$$

- Let the present value of taxes cover **a fraction** of existing government liabilities $(1 + r_{t-1})\, b_{t-1}$:

$$T_t \equiv \sum_{s=t}^{\infty} \left(\frac{1}{1 + r_s} \right)^{s-t} t_s = \psi\, (1 + r_{t-1})\, b_{t-1}, \qquad 0 < \psi \leq 1$$

- For $\psi = 1$, any debt is fully **backed** by taxes (this is sometimes referred to as a Ricardian fiscal policy — or, non-dominance in fiscal policy)
- For $\psi < 1$ only a fraction is backed, and to secure solvency some seigniorage is required. I.e., some fiscal dominance is present
- One can write

$$T_t = t_t + \frac{1}{1 + r_t} T_{t+1}$$

(forward it successively, and one gets the present value expression). Hence,

$$T_t = t_t + \frac{1}{1 + r_t} \psi\, (1 + r_t)\, b_t = t_t + \psi b_t$$

- From the assumption about T_t one gets

$$\psi\, (1 + r_{t-1})\, b_{t-1} = t_t + \psi b_t$$

Note that with $\psi = 1$ one indeed gets the government budget constraint with $s_t = 0$. So, for $\psi < 1$, seigniorage is required

- Now consider the households' budget constraint (y_t is endowment)

$$y_t + (1 + r_{t-1})\, b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} - t_t = c_t + m_t + b_t \qquad (4.16)$$

- Inserting the expression for t_t from the government budget constraint one gets

$$y_t + (1 - \psi)\,(1 + r_{t-1})\, b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} = c_t + m_t + (1 - \psi)\, b_t$$

- For $\psi = 1$, one sees that debt disappears. In equilibrium it will play no role for determination of the price level; only the money stock matters
- For $\psi < 1$, debt will matter for price level determination
- To exemplify the role of debt for prices and inflation, consider a MIU preference specification for households, where the per-period utility function is given by

$$u\,(c_t, m_t) = \ln c_t + \delta \ln m_t, \qquad \delta > 0,$$

- The households then maximize life-time discounted utility subject to the budget constraint

Deriving the money demand function. Material not for lecturing, but for reading

- Maximization characterized by

$$V\,(b_{t-1}, m_{t-1}) = \max \{\ln c_t + \delta \ln m_t + \beta V\,(b_t, m_t)\}$$

where maximization is over c, m, and b and subject to the budget constraint. As usual, use the budget constraint to eliminate b_t to obtain an unconstrained maximization problem over c and m

- First-order condition w.r.t. c:

$$\frac{1}{c_t} = \frac{\beta}{1 - \psi} V_b\,(b_t, m_t)\,. \qquad \text{(i)}$$

- First-order condition w.r.t. m:

$$\frac{\delta}{m_t} + \beta V_m\,(b_t, m_t) = \frac{\beta}{1 - \psi} V_b\,(b_t, m_t)\,. \qquad \text{(ii)}$$

- Relationships between partial derivatives of the value function by the envelope theorem:

$$V_b\,(b_{t-1}, m_{t-1}) = \beta\,(1 + r_{t-1})\, V_b\,(b_t, m_t)\,, \qquad \text{(iii)}$$

$$V_m\,(b_{t-1}, m_{t-1}) = \frac{\beta}{(1 + \pi_t)\,(1 - \psi)} V_b\,(b_t, m_t)\,. \qquad \text{(iv)}$$

- Forward (iii) and multiply by $\beta/\,(1 - \psi)$ on both sides to get:

$$\frac{\beta}{1 - \psi} V_b\,(b_t, m_t) = \frac{\beta^2}{1 - \psi}\,(1 + r_t)\, V_b\,(b_{t+1}, m_{t+1})\,.$$

- Then use (i) to obtain the consumption Euler equation (the Keynes-Ramsey rule):

$$\frac{1}{c_t} = \beta\,(1 + r_t)\,\frac{1}{c_{t+1}}$$

$$c_{t+1} = \beta\,(1 + r_t)\, c_t \qquad \text{(v)}$$

- Then use (iv) on (ii) to get

$$\begin{aligned}\frac{\delta}{m_t}+\frac{\beta^2}{(1+\pi_{t+1})(1-\psi)}V_b(b_{t+1},m_{t+1}) &= \frac{\beta}{1-\psi}V_b(b_t,m_t)\\ \frac{\delta}{m_t}+\frac{\beta}{(1+\pi_{t+1})}\frac{1}{c_{t+1}} &= \frac{1}{c_t}\\ \frac{\delta}{m_t}+\frac{1}{(1+\pi_{t+1})(1+r_t)}\frac{1}{c_t} &= \frac{1}{c_t}\end{aligned}$$

  where the last two equations follows from applying (iv) and (i), and finally (v).

- From this the money demand relationship follows as:

$$\begin{aligned}m_t &= \delta\left[1-\frac{1}{(1+\pi_{t+1})(1+r_t)}\right]^{-1}c_t\\ m_t &= \delta\left[1-\frac{1}{(1+i_t)}\right]^{-1}c_t\end{aligned}$$

  and thus

$$m_t=\delta\left(\frac{1+i_t}{i_t}\right)c_t \qquad \text{(vi)}$$

- Note that **this follows immediately from the general characterization of optimal money demand from the MIU approach in Chapter 2**

$$\frac{u_m(c_t,m_t)}{u_c(c_t,m_t)}=\frac{i_t}{1+i_t}$$

- This indeed gives

$$\frac{\delta/m_t}{1/c_t}=\frac{i_t}{1+i_t},$$

  and thus the money demand function (vi)

End of material not for lecturing

- Now define

$$w_t\equiv m_t+(1-\psi)\,b_t$$

  as the "total" real wealth net of taxes: Real money balances plus non-tax-backed government debt

- Budget constraint then becomes

$$y_t+(1+r_{t-1})\,w_{t-1}-\frac{i_{t-1}}{1+\pi_t}m_{t-1}=c_t+w_t$$

- Note the term

$$\frac{i_{t-1}}{1+\pi_t}m_{t-1}$$

  which is the deduction from total net wealth arising from the fact that real money pays no interest

- Remark that this term **corresponds exactly** to the seigniorage definition $\overline{s}$, (4.9), relevant when a fixed fiscal policy is one with a fixed path for **total** liabilities: money and debt

- Use the lagged money demand function and the consumption Euler equation to get

$$\begin{aligned} y_t + (1+r_{t-1})\, w_{t-1} - \frac{i_{t-1}}{1+\pi_t}\delta\frac{1+i_{t-1}}{i_{t-1}}c_{t-1} &= c_t + w_t \\ y_t + (1+r_{t-1})\, w_{t-1} - \delta\,(1+r_{t-1})\, c_{t-1} &= c_t + w_t \\ y_t + (1+r_{t-1})\, w_{t-1} - \frac{\delta}{\beta}c_t &= c_t + w_t \end{aligned}$$

- Use that $y_t = c_t$ in equilibrium and examine the steady state:

$$w^{ss} = \frac{\delta}{\beta r^{ss}} y^{ss} = \frac{M^{ss} + (1-\psi)\, B^{ss}}{P^{ss}}$$

- The price level is thus determined as

$$P^{ss} = \frac{\beta r^{ss}}{\delta y^{ss}}\left[M^{ss} + (1-\psi)\, B^{ss}\right]$$

- Hence, **government debt matters for the price level** when $\psi < 1$.
- Only when all government debt is backed by taxes, $\psi = 1$, will there be the usual proportionality between nominal money and prices. Otherwise, higher B^{ss} will, for $\psi < 1$, require more reliance on seigniorage, leading to an increase in the price level

Equilibrium seigniorage

- Before considering (next lecture) how inflation is *optimally* used as a tax, and before (re)considering the optimality of the Friedman rule, we assess:
  - What can seigniorage achieve in terms of financing given deficits? Anything? Or are there limits?
  - What are the inflationary implications of relying on seigniorage? Can hyperinflations result from seigniorage collection?

Are there limits to collection of seigniorage?

- Yes!
  - On the one hand, higher inflation and nominal interest rates, increase seigniorage for given real money balances
  - On the other hand, real money balances will fall as inflation and nominal interest rates increase (a money demand response)
  - Hence, as the inflation tax goes up, the **tax base** is going down
- This is shown formally in a MIU model
- Here, it suffices to find the money demand function from the usual first-order condition:

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i_t}{1+i_t} \equiv I_t$$

- Specific form of per-period utility function:

$$u\left(c_t, m_t\right) = \ln c_t + m_t\left(B - D\ln m_t\right), \quad B > D > 0$$

- Resulting money demand relation

$$\frac{B - D\ln m_t - D}{1/c_t} = I_t$$

leading to

$$m_t = A\mathrm{e}^{-I_t/Dc_t}, \quad A \equiv \mathrm{e}^{(B-D)/D} \qquad (4.24')$$

A standard specification of money demand

- Often replaced by

$$m_t = K\mathrm{e}^{-\alpha\pi_t^e}, \quad \alpha > 0,$$

in studies of hyperinflations (where assumption is that real interest rate and consumption/output fluctuates relatively little), or for steady-state/long-run analyses, where by the Fisher relationship inflation and nominal interest rate moves one-for-one

- Steady state seigniorage is (from (4.9))

$$\begin{aligned}\overline{s} &= \frac{i}{1+\pi}m \\ &= \left(1+r\right)\frac{i}{1+i}m \\ &= \left(1+r\right)I\,m\end{aligned}$$

- Using the money demand function one gets

$$\overline{s} = \left(1+r\right)I\,A\exp\left[-\frac{I}{Dc}\right]$$

- For I close to zero, seigniorage is close to zero; this will thus be a characteristic of relatively **low inflation**
- For I very high, seigniorage is also close to zero; this will thus be a characteristic of relatively **high inflation**
- Hence, with inflation increasing from a low level, seigniorage is increasing, but eventually the falling money demand reduces seigniorage. A maximal amount of seigniorage thus exist.
- I other words an inflation rate π^* exists for which seigniorage is at a maximum.
  - For $\pi > \pi^*$ equilibrium seigniorage is decreasing in π
  - For $\pi < \pi^*$ equilibrium seigniorage is increasing in π
  - A seigniorage “Laffer curve” is faced by the government

Inflationary implications of relying on seigniorage

- Remark that the Laffer curve property implies that **two** steady-state inflation rates can finance the same deficit; a **high** and a **low** inflation rate
- Also, there are limits as to how much one can finance by seigniorage
- Both aspects can potentially lead to hyperinflation (often defined as monthly inflation rates of +50%)

- Stability properties of the two steady states, both accomplishing the same financing target?
- Assume sluggish adjustment of money demand due to, e.g., adjustment costs or "slow" reactions in inflation expectations (as in Walsh).
- Consider the high-inflation steady-state
  - If inflation temporarily goes up, seigniorage goes up as money demand reacts little
  - Households start gradually to reduce real money balances; this causes seigniorage to fall
  - If inflation remains permanently at the higher level, seigniorage will eventually become lower than target
  - Hence inflation must increase further to maintain required seigniorage revenues
  - ...and the process continues with ever increasing inflation

- Consider the low-inflation steady-state
  - If inflation temporarily goes up, seigniorage goes up as money demand reacts little
  - Households start gradually to reduce real money balances; this causes seigniorage to fall
  - If inflation remains permanently at the higher level, seigniorage will eventually become higher than target
  - Hence, inflation must decrease to maintain required seigniorage revenues
  - ...and the process continues back to the low-inflation steady-state
- Hence, low-inflation steady state is stable, while high-inflation steady state is unstable

- Therefore, if some shock brings inflation above the high-inflation steady state, the result will be hyperinflation
- Also, **if financing requirement suddenly increases** above what is feasible to finance by seigniorage, the government may engage in futile financing attempts by printing money at a faster rate, thereby driving money balances down, leading to attempts to raise revenue by increasing money growth even further, etc.
- In both cases, hyperinflations can only be stopped by fiscal reform

- Note that in both "stories" of hyperinflations, inflation was caused by money growth; i.e., inflation was based on fundamentals
- As an aside, note that hyperinflations can be **non-fundamental**; i.e., occur in isolation of money growth. These are labelled **speculative** hyperinflations (or "bubble paths")
- As an example, let money demand be (now variables are logarithms):

$$m_t - p_t = -\alpha\left(\mathrm{E}_t p_{t+1} - p_t\right), \quad \alpha > 0,$$

- This is rearranged as an expression for the (log of) price level:

$$p_t = \frac{1}{1+\alpha} m_t + \frac{\alpha}{1+\alpha}\mathrm{E}_t p_{t+1}$$

This is a first-order expectational difference equation in p_t (as it depends on its expected future value)

- Money is for simplicity assumed to be constant, $m_t = \theta_0$
- The variable θ_0 is the model's **fundamental**, and a solution of p_t depending only on the fundamental and parameters is a fundamental solution
- To find this solution, use the **method of undetermined coefficients:**
  - Conjecture a form of the solution by undetermined coefficients
  - Forward the solution, take expectations and insert into the expectational difference equation in p_t
  - Identify the coefficients

- Here: Conjecture the solution $p_t = X\theta_0$, where X is the undetermined coefficient
- Forward the conjecture and take expectations: $\mathrm{E}_t p_{t+1} = X\theta_0$
- Insert into difference equation:

$$\begin{aligned} p_t &= \frac{1}{1+\alpha}\theta_0 + \frac{\alpha}{1+\alpha}X\theta_0 \\ &= \frac{1+\alpha X}{1+\alpha}\theta_0 \end{aligned}$$

- The undetermined coefficient is now identified as it must be the case that

$$X = \frac{1+\alpha X}{1+\alpha}$$

and thus $X = 1$ Hence, $p_t = \theta_0$ is the fundamental solution.

- However, it is easy to see that infinitely many solutions of the form

$$p_t = \theta_0 + bub_t, \quad bub_t \lesseqgtr 0$$

exists when

$$\mathrm{E}_t bub_{t+1} = \frac{1+\alpha}{\alpha} bub_t$$

- To see this, note that is it consistent with the difference equation:

$$\begin{aligned} \theta_0 + bub_t &= \frac{1}{1+\alpha}\theta_0 + \frac{\alpha}{1+\alpha}\mathrm{E}_t\left[\theta_0 + bub_{t+1}\right] \\ \theta_0 + bub_t &= \frac{1}{1+\alpha}\theta_0 + \frac{\alpha}{1+\alpha}\left[\theta_0 + \frac{1+\alpha}{\alpha}bub_t\right] \\ \theta_0 &= \theta_0 \end{aligned}$$

- Hence, if $bub_t > 0$, we have ever rising prices, even though the money supply is constant

Summary

- Monetary and fiscal policy are linked through the public budget constraint
- Ignoring this may be valid if governments have access to lump-sum taxation and follow policies that fully back interest-bearing debt with taxes
- Otherwise, important channels from monetary policy to fiscal policy and vice versa may be overlooked, as the financing properties of inflation is ignored
- Also, it is important to stress that an observed change in monetary policy may or may not be due to fiscal considerations, and therefore have different implications for the real economy depending on the source of the change
- While a potential financing tool, one must be aware of the dangers of hyperinflation associated with reliance on seigniorage as a means of financing public expenditures

Plan for next lecture

1. Optimal taxation and seigniorage
2. Robustness of the Friedman rule?

Literature: Walsh (Chapter 4, pp. 172-187; pp. 192-195)