

**Plan for today:**

Open-economy Aspects (I)

1. The Obstfeld and Rogoff two-country model
2. Solution under flexible prices

Literature: Walsh (2003, Chap. 6, pp. 269-282).

## Introductory remarks

- All analyses so far, have been confined to closed-economy model
- This misses out important transmission mechanisms in most real-life economies:
  - The exchange rate channel
  - The impact of external shocks (policy- and non-policy shocks)
- Purpose today is to develop a tractable, general equilibrium model for open economies
- It builds closely on the famous paper by Obstfeld and Rogoff (1995, *Journal of Political Economy*): “Exchange Rate Dynamics Redux”
- Model initiated a new generation of micro-founded, open-economy models with incomplete nominal adjustment
  - A literature now known as the “New Open Economy Macroeconomics”
- While inherently a sticky-price model, the foundations of the model are easiest seen in a flexible price version
  - Sticky prices are introduced next time

# The Obstfeld-Rogoff Two-Country Model:

The flexible price version

## Basics

- World of two countries: Domestic/Home and Foreign
  - Continuum  $z \in [0, n]$  of agents live in Home country
  - Continuum  $z \in (n, 1]$  of agents live in Foreign country
- Each agent is monopolist supplier of distinct good, which is imperfect substitute with all other goods
  - Monopolistic competition market structure

- Utility of a representative domestic agent  $j$ :

$$U^j = \sum_{t=0}^{\infty} \beta^t \left[ \log C_t^j + b \log \frac{M_t^j}{P_t} - \frac{k}{2} y_t(j)^2 \right], \quad b, k > 0 \quad (6.1)$$

- Definition of consumption index:

$$C^j = \left[ \int_0^1 c^j(z)^q dz \right]^{\frac{1}{q}}, \quad 0 < q < 1. \quad (6.2)$$

Aggregate of **all goods** produced in the world. Note with  $q < 1$ , goods are imperfect substitutes

- Corresponding domestic **consumer price** index (see notes for derivation)

$$P = \left[ \int_0^1 p(z)^{\frac{q}{q-1}} dz \right]^{\frac{q-1}{q}} \quad (6.3)$$

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- Foreign agents' utility functions are similar
- Nominal budget constraint of representative domestic agent  $j$ :

$$P_t C_t^j + M_t^j + P_t T_t + P_t B_t^j \leq p_t(j) y_t(j) + R_{t-1} P_t B_{t-1}^j + M_{t-1}^j$$

- $B_t^j$  is internationally freely tradable bond with real (gross) return  $R_t$
- $T_t$  are real government taxes
- Balanced government budget and no government spending:

$$0 = P_t T_t + (M_t - M_{t-1})$$

(note mistake in Walsh, p. 271: “ $P_t T_t = (M_t - M_{t-1})$ ”)

- **Money supplies are the policy instruments** in the model

- Budget constraint in real terms:

$$C_t^j + \frac{M_t^j}{P_t} + T_t + B_t^j \leq \frac{p_t(j)}{P_t} y_t(j) + R_{t-1} B_{t-1}^j + \frac{1}{1 + \pi_t} \frac{M_{t-1}^j}{P_{t-1}}$$

$\pi_t$  is domestic consumer-price inflation

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- No barriers to trade in goods

- The price of a good  $z$  is the same Home and abroad measured in common currency:

$$p(z) = S_t p^*(z)$$

- $S$  is **nominal exchange rate** (price of foreign currency in terms of domestic currency  $\Rightarrow S \uparrow$  is a domestic depreciation)
- $p^*(z)$  is foreign currency price of good  $z$  (stars indicate foreign variables)
- This law-of-one-price leads to PPP:

$$P_t = S_t P_t^* \quad (6.8)$$

\* Note! Empirically problematic assumption in short run....

### Optimal behavior

- Each agent must determine optimally:

- Aggregate consumption dynamics (and thus asset holdings)
- Relative demand for different goods (“composition” of aggregate demand)
- Production effort of “own” good

#### Optimal production decision

- Optimal demand for good  $z$  by individual  $j$  (see notes)

$$c^j(z) = \left[ \frac{p(z)}{P} \right]^{-\frac{1}{1-q}} C^j.$$

- Demand for good  $z$  is increasing in total consumption
- Demand for good  $z$  is decreasing in the real price of the good,  $p(z)/P$
- Note that  $1/(1-q)$  is the elasticity of substitution between goods.
- Hence, the limiting case of perfect competition applies when  $q \rightarrow 1$  (horizontal demand curve and no monopoly power)

- **Total demand** in the two countries of a good  $z$  is:

- $n$  times domestic demand plus  $1 - n$  times foreign demand.

- A representative foreign consumer's demand function is

$$c^{*j}(z) = \left[ \frac{p^*(z)}{P^*} \right]^{-\frac{1}{1-q}} C^{*j}$$

- As the nominal exchange rate is  $S$ , this is the same as

$$c^{*j}(z) = \left[ \frac{Sp^*(z)}{SP^*} \right]^{-\frac{1}{1-q}} C^{*j},$$

and thus, by the law of one price,

$$c^{*j}(z) = \left[ \frac{p(z)}{P} \right]^{-\frac{1}{1-q}} C^{*j}$$

- **Total demand**,  $y^d(z)$ , is therefore

$$\begin{aligned} y^d(z) &= n \left[ \frac{p(z)}{P} \right]^{-\frac{1}{1-q}} C + (1-n) \left[ \frac{p(z)}{P} \right]^{-\frac{1}{1-q}} C^* \\ &= \left[ \frac{p(z)}{P} \right]^{-\frac{1}{1-q}} [nC + (1-n)C^*] \\ &= \left[ \frac{p(z)}{P} \right]^{-\frac{1}{1-q}} C^w, \end{aligned}$$

- $C^w \equiv nC + (1-n)C^*$  is world consumption
- Optimal production given by standard consumption-leisure trade-off:

$$\frac{MUTILITY_{leisure}}{MUTILITY_{consumption}} = MRETURN OF WORK$$

OR,

$$\frac{MLOSS_{work}}{MUTILITY_{consumption}} = MRETURN OF WORK$$

- We have from utility function:

$$\frac{MLOSS_{work}}{MUTILITY_{consumption}} = \frac{k y_t^j}{\left(1/C_t^j\right)}$$

- What is the real return from work? It is

$$\frac{p_t(z) y_t(z)}{P_t}$$

so marginal return is found as

$$\frac{p_t(z)}{P_t} + \left[ \partial \left( \frac{p_t(z)}{P_t} \right) / \partial y_t(z) \right] y_t(z)$$

- The real price is found by “inverting” the demand curve:

$$\frac{p_t(z)}{P_t} = \left[ \frac{y_t^d(z)}{C_t^w} \right]^{q-1}$$

– Note again, that with  $q < 1$  the return for work effort is lower, as it reduces the relative price; only with  $q \rightarrow 1$  is the price taken as given, and perfect competition mimicked

- The marginal product of work effort is therefore

$$q \left[ \frac{y_t^d(z)}{C_t^w} \right]^{q-1}$$

- Optimal consumption-leisure choice is therefore characterized by

$$\frac{k y_t^j}{\left(1/C_t^j\right)} = q \left[ \frac{y_t^j}{C_t^w} \right]^{q-1} \quad (6.6')$$

### Optimal money demand decision

- Derived from standard, general condition:

$$\frac{MUTILITY_{money}}{MUTILITY_{consumption}} = \frac{i_t}{1 + i_t}$$

with  $i_t = R_t (1 + \pi_{t+1}) - 1$  being the nominal interest rate

- With the assumed utility function:

$$\frac{b \left[ 1 / \left( M_t^i / P_t \right) \right]}{1 / C_t^i} = \frac{i_t}{1 + i_t}$$

and therefore simple money demand function

$$\frac{M_t^i}{P_t} = b C_t^i \frac{1 + i_t}{i_t} \quad (6.7)$$

### Optimal intertemporal consumption allocation

- Simple version of Keynes-Ramsey rule:

$$MUTILITY_{consumption}(t) = \beta R_t MUTILITY_{consumption}(t + 1)$$

$$\begin{aligned} \frac{1}{C_t^j} &= \beta R_t \frac{1}{C_{t+1}^j} \\ C_{t+1}^j &= \beta R_t C_t^j \end{aligned} \quad (6.5)$$

### Equilibrium conditions and flex-price implications

- Equilibrium satisfies the optimality conditions (Home and Foreign); budget constraints, PPP, and two additional market clearing conditions:

- Goods market clearing:

$$C_t^w = n \frac{P_t(h)}{P_t} y_t(h) + (1 - n) \frac{P_t^*(f)}{P_t^*} y_t^*(f) \equiv Y_t^w$$

with  $p_t(i)$  and  $y_t(i)$  being representative price and production in country  $i$

- Bond-market clearing:

$$n B_t + (1 - n) B_t^* = 0$$

- As well known, monopolistic competition gives no real effects of money (but is natural modelling framework when introducing sticky prices)

- This is confirmed in this model:

- E.g., an increase in  $M_t$  leads to a proportional increase in  $p_t(h)$  and  $P_t$  and a proportional increase in  $S_t$  (a nominal depreciation)
- All real variables and foreign prices are unchanged

- Model also exhibits superneutrality

- An increase in the growth rate of  $M_t$  will increase Home CPI inflation and home nominal interest rate, and create a higher increase in  $S_t$
- All real variables and foreign prices are unchanged (this follows from the separability of the utility function)

- Emphasized in steady-state solution for domestic and foreign consumption (from the budget constraints):

$$C = \frac{p(h)}{P} y(h) + (R - 1) B \quad (6.9)$$

$$C^* = \frac{p^*(f)}{P^*} y^*(f) - (R - 1) \frac{n}{1 - n} B \quad (6.10)$$

Real steady-state consumption equals real steady-state income (real value of output plus/minus bond income)

## A linear approximation

- As indicated, the nominal exchange rate plays a role under flexible prices to neutralize real effects of monetary shocks
- A log-linearized version of model serves to highlight this (lower-case letters are log deviations)

- Domestic optimality conditions:

$$y_t = \frac{1}{1 - q} [p_t - p_t(h)] + c_t^w \quad (6.13)$$

$$(2 - q) y_t = (1 - q) c_t^w - c_t \quad (6.18)$$

$$n_t - p_t = c_t - \delta (r_t + \pi_{t+1}), \quad \delta = \beta / (\bar{\Pi} - \beta) \quad (6.20)$$

$$c_{t+1} = c_t + r_t \quad (6.16)$$

- Domestic price index written in terms of foreign producer prices and the nominal exchange rate:

$$p_t = n p_t(h) + (1 - n) [s_t + p_t^*(f)] \quad (6.11)$$

- These five equations plus the five Foreign analogues, and the definition of  $c_t^w$ , will determine the eleven endogenous variables:

- Home and foreign production ( $y_t, y_t^*$ )
- Home, foreign and world consumption ( $c_t, c_t^*, c_t^w$ )
- Prices and the nominal exchange rate ( $p_t(h), p_t, p_t^*(f), p_t^*, s_t$ )
- The real interest rate ( $r_t$ )

- Money neutrality is seen immediately from linearized model
- The two price indices, imply the PPP relationship:

$$s_t = p_t - p_t^*$$

- Used in domestic price index:

$$p_t = np_t(h) + (1 - n)(p_t - p_t^* + p_t^*(f))$$

$$0 = n[p_t(h) - p_t] + (1 - n)[p_t^*(f) - p_t^*]$$

or,

$$0 = n\chi_t + (1 - n)\chi_t^*$$

- Demand functions are correspondingly re-written as

$$y_t = \frac{1}{1 - q}\chi_t + c_t^w,$$

$$y_t^* = \frac{1}{1 - q}\chi_t^* + c_t^w$$

- These three equations will along with...

– ...the two consumption Euler equations

– ...the two production decision equations

– ...the definition of  $c_t^w$

- ... determine the **real allocation**:  $\chi_t, \chi_t^*, y_t, y_t^*, c_t, c_t^*, c_t^w, r_t$

- Hence, changes in  $m_t$  or  $m_t^*$  have no real effects
- Only effects on prices and the nominal exchange rate (as mentioned previously, an increase in  $m_t$  increase  $p_t, p_t(h)$  and  $s_t$  proportionally)

- To see how the nominal exchange rate is determined, subtract the money market equilibrium conditions:

$$m_t - m_t^* - (p_t - p_t^*) = (c_t - c_t^*) - \delta(\pi_{t+1} - \pi_{t+1}^*)$$

(note that the inflation differential measures the nominal interest differential, as Home and Foreign face same real interest rate)

- Then use the PPP relationship in level and change form:

$$m_t - m_t^* - s_t = (c_t - c_t^*) - \delta(s_{t+1} - s_t) \quad (6.23)$$

- First-order (expectational) difference equation in  $s_t$  :

$$s_t = \frac{\delta}{1 + \delta}s_{t+1} + \frac{1}{1 + \delta}[(m_t - m_t^*) - (c_t - c_t^*)]$$

- Solving (6.23) successively forward and imposing

$\lim_{i \rightarrow \infty} (\delta / (1 + \delta))^i s_{t+i} = 0$  yields

$$s_t = \frac{1}{1 + \delta} \sum_{i=0}^{\infty} \left( \frac{\delta}{1 + \delta} \right)^i [(m_{t+i} - m_{t+i}^*) - (c_{t+i} - c_{t+i}^*)]$$

Nominal exchange rate equals present value of current and future relative money supplies and demands (the latter quantified by the consumption differential)

- Expression can be simplified as any consumption differential is **permanent** since

$$c_{t+1} = r_t + c_t$$

and

$$c_{t+1}^* = r_t + c_t^*$$

imply

$$c_{t+1} - c_t = c_{t+1}^* - c_t^*$$

and

$$c_{t+1} - c_{t+1}^* = c_t - c_t^*$$

- Hence,

$$s_t = - (c_t - c_t^*) + \frac{1}{1+\delta} \sum_{i=0}^{\infty} \left( \frac{\delta}{1+\delta} \right)^i (m_{t+i} - m_{t+i}^*)$$

(since  $\frac{1}{1+\delta} \sum_{i=0}^{\infty} \left( \frac{\delta}{1+\delta} \right)^i = 1$ )

- Hence,

–  $m_t > m_t^*$ : relative supply of home currency increases, and the relative value of the currency falls ( $s_t$  increases; home currency depreciates)

–  $m_{t+i} > m_{t+i}^*$ : future relative supply of home currency increases; future inflation differential increases, and reduces relative demand for home currency; relative value of the currency falls ( $s_t$  increases; home currency depreciates)

–  $c_t > c_t^*$ : current or future money demand shifts towards home currency; relative value of the currency increases ( $s_t$  decreases; home currency appreciates)

- Alternative exposition featuring the relationship between nominal interest rates and the nominal exchange rates; **uncovered interest parity**

- Real interest rate is identical across countries:

$$i_t - \pi_{t+1} = i_t^* - \pi_{t+1}^*$$

and thus

$$i_t - i_t^* = \pi_{t+1} - \pi_{t+1}^*$$

- From the PPP relationship in first differences:

$$i_t - i_t^* = s_{t+1} - s_t$$

- Hence, in a world with perfect capital mobility and floating exchange rates, the nominal interest rate differential equals (expected) depreciation rate of home currency

- Only then is holding home and foreign bonds equally attractive

- Empirically, tests of the “UIP” relationship have been unsuccessful

– Not necessarily reason for abandoning model

– Note, as with estimations of term-structure relationships covered in Chapter 10, **how monetary policy is conducted**, will matter for regression results

\* If the nominal interest rate was instrument, and was responding to the exchange rate, a regression of the type

$$s_{t+1} - s_t = a + b(i_t - i_t^*) + \varphi_{t+1}$$

should not necessarily give  $b = 1$  (as  $i_t - i_t^*$  would be a positive function of  $s_t$ )



## Concluding remarks

- The flex-price version of the Obstfeld-Rogoff model only gives preliminaries for the analysis of monetary policy in open economies
- The model is therefore amended with one period price rigidity. The role of the nominal exchange rate for real allocation is then evident:
$$p_t = np_t(h) + (1 - n)[s_t + p_t^*(f)];$$
with  $p_t(h)$  and  $p_t^*(f)$  fixed, fluctuations in the nominal exchange rates cause fluctuations in the aggregate price index
  - New transmission channel of monetary policy
  - Due to the openness of economies, monetary policy in one country will have effects on the other
  - Good or bad effects? Scope for policy coordination?
- Next time.....

## Plan for next lecture

Monday, May 10, 12-14

Open-economy Aspects (II)

1. The Obstfeld and Rogoff two-country model with sticky prices
2. An example of international monetary policy coordination

Literature: Walsh (2003, Chap. 6, pp. 282-297). Read also (small)

Section 6.4 on the small open economy.

As supplementary recent readings on policy coordination, I recommend Benigno (2002, *Journal of International Economics*) and Clarida et al. (2002, *Journal of Monetary Economics*)