Monetary Economics: Macro Aspects, 9/2 2004
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Plan for today:

1. Money in the utility function (start)
   a. Short on the Tobin effect mentioned in Chapter 2's introduction
   b. The basic money in the utility function model
   c. Optimal behavior and steady-state equilibrium properties: Long-run superneutrality of money

Literature: Walsh (2003, Chapter 2, pp. 43-59)

Introductory remarks

- The standard model for (exogenous) economic growth is the simple Solow model featuring a fixed savings rate and a law of motion for physical capital accumulation
- When extended with optimizing savings behavior, we get the Ramsey model: The foundation for the Real Business Cycle research program
- Models have no role for money and monetary policy
- Purpose of models/analyses in coming lectures is to introduce a role for money in these type of models
- Money is introduced in various ways: Often short-cuts
  - Short-cuts are helpful for understanding simple features, and the more robust results are to particular short cut, of course, the better
- The so-called Tobin model extends the Solow model by *postulating* a demand for money (the short cut)
  - Highlights the implications of inflation for the choice between investment in physical and financial assets
- The Money-in-the-Utility function model extends the Ramsey model by *postulating that money gives utility* (the short cut)
  - Highlights the importance of microfoundations and optimizing private-sector behavior (absent in the Tobin model)

The Tobin Effect

- Like Solow model, output is produced by a CRS production function:
$$Y_t = F\left(K_{t-1}, N_t\right),$$
which in "intensive" form is:
$$y_t = f\left(k_{t-1}\right), \quad y_t \equiv Y_t/N_t, \quad k_{t-1} \equiv K_{t-1}/N_{t-1}$$
(population grows at a constant rate $N_t/N_{t-1} = 1 + n$)
- Households invest in capital or money, M_t (of which one unit buys $1/P_t$ goods). Real per capita wealth:
$$a_t \equiv k_t + m_t, \quad m_t \equiv \left(M_t/P_t\right)/N_t$$
- Government makes real lump-sum transfers (or withdrawals) to households in the form of money supply changes:
$$\tau_t = \frac{\Delta M_t}{P_t N_t}$$
- Economy-wide household budget constraint is (no depreciation of capital)
$$Y_t + \tau_t N_t + \frac{M_{t-1}}{P_t} = C_t + \Delta K_t + \frac{M_t}{P_t}$$
or
$$Y_t + \tau_t N_t + \frac{M_{t-1}}{\left(1+\pi_t\right) P_{t-1}} = C_t + \Delta K_t + \frac{M_t}{P_t}$$
Note how inflation $\pi_t \equiv \left(P_t/P_{t-1}\right) - 1$ *erodes* initial resources available for consumption, investment in capital and future real money holdings

- Rewrite budget constraint so it depicts resources available for consumption, for investment in capital and accumulation of real money holdings (subtract M_{t-1}/P_{t-1} from both sides):
$$\begin{aligned} Y_t + \tau_t N_t + \frac{M_{t-1}}{\left(1+\pi_t\right) P_{t-1}} - \frac{M_{t-1}}{P_{t-1}} &= C_t + \Delta K_t + \Delta \frac{M_t}{P_t} \\ Y_t + \tau_t N_t - \frac{\pi_t M_{t-1}}{\left(1+\pi_t\right) P_{t-1}} &= C_t + \Delta K_t + \Delta \frac{M_t}{P_t} \end{aligned}$$
Note that inflation *erodes* available resources
- As in Solow model, $0 < s < 1$ is saved, $1 - s$ is consumed. $\Longrightarrow$
$$\Delta K_t + \Delta \frac{M_t}{P_t} = s\left(Y_t + \tau_t N_t - \frac{\pi_t M_{t-1}}{\left(1+\pi_t\right) P_{t-1}}\right)$$
- In per capita terms:
$$\Delta k_t + \Delta\left(\frac{M_t}{P_t}\right)\frac{1}{N_t} = s\left(y_t + \tau_t - \frac{\pi_t}{\left(1+\pi_t\right)\left(1+n\right)} m_{t-1}\right) - \frac{n}{1+n} k_{t-1}$$
- Expressed exclusively as a physical capital accumulation expression:
$$\begin{aligned} \Delta k_t &= s\left(y_t + \tau_t - \frac{\pi_t}{\left(1+\pi_t\right)\left(1+n\right)} m_{t-1}\right) - \frac{n}{1+n} k_{t-1} \\ &\quad - \frac{\theta_t - \pi_t}{\left(1+\pi_t\right)\left(1+n\right)} m_{t-1} \end{aligned}$$
  - Per capita physical capital changes with *total* savings, net of "depreciation" due to population growth, and net of changes in real per capital money
  - $\theta_t \equiv \Delta M_t/M_{t-1}$ is the growth rate of the nominal money supply

- Use the definition of transfers

$$\tau_t = \frac{\Delta M_t}{P_t N_t} = \frac{\theta_t}{(1+\pi_t)(1+n)} m_{t-1},$$

and we get

$$\Delta k_t = sf(k_{t-1}) - (1-s)\frac{\theta_t - \pi_t}{(1+\pi_t)(1+n)} m_{t-1} - \frac{n}{1+n} k_{t-1}$$

- Increases in real money per capita

$$\Delta\left(\frac{M_t}{P_t}\right)/N_t = \frac{\theta_t - \pi_t}{(1+\pi_t)(1+n)} m_{t-1}$$

have two effects:

a) It crowds out Δk_t *completely* for *given total* savings

b) It increases savings by a fraction $s < 1$

- The net effect is *negative*: Increases in real money per capita divert resources away from physical capital accumulation

- **Steady state** ($\Delta m_t = \Delta k_t = 0$)

  – $\Delta m_t = 0$ implies $\Delta m_t / m_{t-1} = 0$ and therefore

$$\begin{aligned}(1+\theta) &= (1+\pi)(1+n) \\ \pi &\approx \theta - n\end{aligned}$$

  – $\Delta k_t = 0$ implies

$$\begin{aligned}sf(k^{ss}) &= (1-s)\frac{\theta-\pi}{(1+\pi)(1+n)} m^{ss} + \frac{n}{1+n} k^{ss} \\ sf(k^{ss}) &= (1-s)\frac{1+\theta-(1+\pi)}{(1+\theta)} m^{ss} + \frac{n}{1+n} k^{ss} \\ sf(k^{ss}) &= (1-s)\frac{n}{(1+n)} m^{ss} + \frac{n}{1+n} k^{ss},\end{aligned}$$

condensed as

$$\begin{aligned}sf(k^{ss}) &= [(1-s)\phi^{ss}+1]\,\overline{n}k^{ss} \\ \phi^{ss} &\equiv \frac{m^{ss}}{k^{ss}}, \qquad \overline{n} \equiv \frac{n}{1+n}\end{aligned}$$

- Note that ϕ^{ss} will affect the steady-state value of physical capital and thus per-capita output

- In a standard Solow diagram, ϕ^{ss} is an additional part in investment needed to maintain a constant value of capital per labor

- Hence, *higher* ϕ^{ss} *decreases* k^{ss}.

- But what determines the steady-state real money to physical capital-ratio?

  – Assumption: Capital and money's relative real yields

    * Physical capital yields $f_k(k)$ (the real interest rate, r)
    * Real money "yields" $-\pi/(1+\pi)$ (as seen before; inflation erodes the value)

  – I.e., $\phi \equiv \phi(r,\pi) = \phi(f_k(k),\pi)$; both partial derivatives negative

- Steady-state relationship then becomes

$$sf\left(k^{ss}\right) = \left[\left(1-s\right)\phi\left(f_k\left(k^{ss}\right), \pi^{ss}\right) + 1\right]\overline{n}k^{ss}$$

- Total differentiation yields

$$dk^{ss} = \frac{\left(1-s\right)\phi_{\pi}\overline{n}k^{ss}}{sf_k - \overline{n}\left[1 + \left(1-s\right)\phi + \left(1-s\right)\phi_r f_{kk}\right]}d\pi$$

- One can show

$$\frac{dk^{ss}}{d\pi} > 0$$

- The (Mundell-) Tobin Effect: Higher inflation decreases m^{ss}/k^{ss} implying a substitution away from real money towards physical capital
- Result: Higher money growth and inflation causes higher output

Discussion

- Neutrality of money holds in model (different *levels* of M_t has no real effects — only effects on P_t)
- One-for-one long run relationship between nominal money growth and inflation as in data
- Positive long-run relationship between money growth and output — monetary superneutrality of money fails
  - Particular relationship not in data
  - Hence, *if* effect is important, other effects must neutralize it in real life
- Why is money in the model? It hurts! (is the short cut a good one?)
- All behavior is postulated; is this reasonable? What would optimizing agents do?

Money in the utility function (start)

- This model amends the standard Ramsey model (optimizing households) with money
  - I.e., a qualification of the Tobin model like the Ramsey model qualifies the Solow model
- The short-cut here for introducing a role for money is that real money *provides utility* per se
  - One interpretation: Money facilitates transactions on the market and reduces "shopping time" (money as such is an otherwise useless commodity....)
- Hence, it is assumed that the per-period utility function of households is

$$U_t = u\left(c_t, m_t\right)$$

with u being increasing and concave in both arguments. (It is, of course, *real* money that enters in u, i.e., M_t's value relative to what it can buy at price P_t.)

- Often, to ensure existence of an equilibrium where money is held (a "monetary equilibrium"), it is assumed that for some $\overline{m}$, $u_m\left(c_t, \overline{m}\right) = 0$ and $u_m\left(c_t, m_t\right) < 0$ for $m_t > \overline{m}$
- Aim of representative household is to maximize:

$$W = \sum_{t=0}^{\infty} \beta^t u\left(c_t, m_t\right), \qquad 0 < \beta < 1. \tag{2.1}$$

- What is the relevant constraint?
- Assuming the physical capital depreciates at rate $0 < \delta < 1$, the economy-wide budget constraint is
(*ignoring* for simplicity financial asset holdings B_t used in Walsh)

$$Y_t + \tau_t N_t + (1 - \delta) K_{t-1} + \frac{M_{t-1}}{P_t} = C_t + K_t + \frac{M_t}{P_t}, \tag{2.2'}$$

The left-hand side is available resources at t for consumption, the capital stock and real money

- In per-capita version, *assuming no population growth as in Walsh* (set $n = 0$), we get

$$y_t + \tau_t + (1 - \delta) k_{t-1} + \frac{1}{1 + \pi_t} m_{t-1} = c_t + k_t + m_t$$

- Assuming again $y_t = f\left(k_{t-1}\right)$, we get

$$\omega_t = f\left(k_{t-1}\right) + \tau_t + (1 - \delta) k_{t-1} + \frac{1}{1 + \pi_t} m_{t-1} = c_t + k_t + m_t \tag{2.4'}$$

- Hence, ω_t is the total available resources, treated *as given* at t by the households.
  - It is the relevant *state variable* when choosing the optimal paths of c, k, and m at date t

- Household's optimization problem is solved by dynamic *programming* (could be done with Lagrangian methods as well)
- Involves use of the *value function* — the maximal value of W given optimal behavior of the household, and given the current state (ω)
- The value function V must therefore satisfy

$$V(\omega_t) = \max\{u(c_t, m_t) + \beta V(\omega_{t+1})\}$$

Maximization is over c_t, m_t, k_t subject to the budget constraint and the definition of ω_{t+1} (available resources one period ahead)

- To make it simple, one substitutes out ω_{t+1} and substitutes out k_t as $k_t = \omega_t - c_t - m_t$.One then maximizes (unconstrained) over c_t and m_t:

$$\max\left\{\begin{array}{c} u(c_t, m_t) \\ +\beta V\left(\underbrace{f(k_t) + \tau_t + (1-\delta)k_t + \frac{1}{1+\pi_{t+1}}m_t}_{\omega_{t+1}}\right)\end{array}\right\}$$

and thus

$$\max\left\{\begin{array}{c} u(c_t, m_t) \\ +\beta V\left(\begin{array}{c} f(\omega_t - c_t - m_t) + \tau_t \\ +(1-\delta)(\omega_t - c_t - m_t) + \frac{1}{1+\pi_{t+1}}m_t\end{array}\right)\end{array}\right\}$$

- First-order condition concerning choice of c_t:

$$\begin{aligned} u_c(c_t, m_t) + \beta V_\omega(\omega_{t+1})\frac{\partial \omega_{t+1}}{\partial c_t} &= 0 \\ u_c(c_t, m_t) - \beta V_\omega(\omega_{t+1})[f_k(k_t) + 1 - \delta] &= 0 \qquad (2.6') \end{aligned}$$

Marginal utility of period t consumption equals its marginal loss (in terms of the marginal value of less period $t+1$ capital)

- First-order condition concerning choice of m_t:

$$u_m(c_t, m_t) + \beta V_\omega(\omega_{t+1})\frac{\partial \omega_{t+1}}{\partial m_t} = 0$$

$$u_m(c_t, m_t) + \beta V_\omega(\omega_{t+1})\left[\frac{1}{1+\pi_{t+1}} - (f_k(k_t) + 1 - \delta)\right] = 0 \qquad (2.8')$$

Marginal utility period t real money (in terms of direct utility plus marginal value of more real money resources in period $t+1$) equals marginal loss (in terms of the marginal value of less period $t+1$ capital)

- Furthermore, transversality conditions must be satisfied:

$$\begin{aligned} \lim_{t\to\infty} \beta^t u_c(c_t, m_t) k_t &= 0 \\ \lim_{t\to\infty} \beta^t u_c(c_t, m_t) m_t &= 0 \end{aligned}$$

(otherwise over-accumulation of k and m is taking place — lifetime utility could be improved through higher consumption by accumulating less)

- In the first-order conditions one can eliminate the value function V by use of the so-called *Envelope Theorem:*
- Note: optimal consumption and money holding choices will be functions of ω_t
- Define these as $c_t = c(\omega_t)$ and $m_t = m(\omega_t)$, respectively
- The value function is therefore *by definition* characterized as

$$V(\omega_t) = u(c(\omega_t), m(\omega_t)) + \beta V(\omega_{t+1}). \qquad (*)$$

As (*) holds for *any* value of ω_t it follows that

$$\begin{aligned} V_\omega(\omega_t) &= u_c(c(\omega_t), m(\omega_t))\, c'(\omega_t) + u_m(c(\omega_t), m(\omega_t))\, m'(\omega_t) \\ &\quad + \beta V_\omega(\omega_{t+1}) \frac{\partial \omega_{t+1}}{\partial \omega_t}. \qquad (**) \end{aligned}$$

- Now, find $\partial\omega_{t+1}/\partial\omega_t$ when $c_t = c(\omega_t)$ and $m_t = m(\omega_t)$ applies. One gets

$$\begin{aligned} \frac{\partial \omega_{t+1}}{\partial \omega_t} &= [f_k(k_t) + 1 - \delta](1 - c'(\omega_t) - m'(\omega_t)) \\ &\quad + \frac{1}{1+\pi_{t+1}} m'(\omega_t) \end{aligned}$$

- Combining this with (**):

$$\begin{aligned} V_\omega(\omega_t) &= u_c(c(\omega_t), m(\omega_t))\, c'(\omega_t) + u_m(c(\omega_t), m(\omega_t))\, m'(\omega_t) \\ &\quad + \beta V_\omega(\omega_{t+1}) \left\{ \begin{array}{c} [f_k(k_t) + 1 - \delta](1 - c'(\omega_t) - m'(\omega_t)) \\ + \frac{1}{1+\pi_{t+1}} m'(\omega_t) \end{array} \right\}. \end{aligned}$$

- Collect the $c'(\omega_t)$ and $m'(\omega_t)$ terms:

$$\begin{aligned} V_\omega(\omega_t) &= [u_c(c(\omega_t), m(\omega_t)) - \beta V_\omega(\omega_{t+1})(f_k(k_t) + 1 - \delta)]\, c'(\omega_t) \\ &\quad + \left[\begin{array}{c} u_m(c(\omega_t), m(\omega_t)) + \beta V_\omega(\omega_{t+1}) \frac{1}{1+\pi_{t+1}} \\ - \beta V_\omega(\omega_{t+1})[f_k(k_t) + 1 - \delta] \end{array} \right] m'(\omega_t) \\ &\quad + \beta V_\omega(\omega_{t+1})[f_k(k_t) + 1 - \delta] \end{aligned}$$

- NOTE:

$$\begin{aligned} V_\omega(\omega_t) &= \Bigg[\underbrace{u_c(c(\omega_t), m(\omega_t)) - \beta V_\omega(\omega_{t+1})(f_k(k_t) + 1 - \delta)}_{=0 \text{ by } (2.6')} \Bigg] c'(\omega_t) \\ &\quad + \left[\underbrace{\begin{array}{c} u_m(c(\omega_t), m(\omega_t)) + \beta V_\omega(\omega_{t+1}) \frac{1}{1+\pi_{t+1}} \\ - \beta V_\omega(\omega_{t+1})[f_k(k_t) + 1 - \delta] \end{array}}_{=0 \text{ by } (2.8')} \right] m'(\omega_t) \\ &\quad + \beta V_\omega(\omega_{t+1})[f_k(k_t) + 1 - \delta] \end{aligned}$$

- All terms in front of $c'(\omega_t)$ and $m'(\omega_t)$ are zero by the first-order conditions.
  - This makes sense, as these terms indeed capture the marginal values of c and m . . . . **in an optimum these must, of course, be . . . zero**

- Therefore, (**) reduces immediately to

$$V_\omega(\omega_t) = \beta V_\omega(\omega_{t+1})\left[f_k(k_t) + 1 - \delta\right]$$

- Then use the first-order condition for consumption choice,

$$u_c(c_t, m_t) - \beta V_\omega(\omega_{t+1})\left[f_k(k_t) + 1 - \delta\right] = 0,$$

to obtain Walsh's expression (which **is** technically the envelope theorem):

$$V_\omega(\omega_t) = u_c(c_t, m_t). \tag{2.10}$$

  - Marginal utility of consumption equals marginal value of wealth
  - A familiar restatement of optimal intratemporal consumption choice!

- With this expression, the first-order conditions can be rewritten as

$$u_c(c_t, m_t) = \beta u_c(c_{t+1}, m_{t+1})\left[f_k(k_t) + 1 - \delta\right]$$

(a discrete-time, money version of the standard Keynes-Ramsey rule), and

$$u_m(c_t, m_t) + \beta \frac{u_c(c_{t+1}, m_{t+1})}{1 + \pi_{t+1}} = u_c(c_t, m_t)$$

(marginal gain of m_t equal to the marginal loss in terms of lower capital in period $t+1$ — equal to the marginal utility of c_t by the "Keynes-Ramsey rule")

- These conditions, together with the budget constraint characterizes the optimal paths of c, k, and m
- We will, for now, concentrate on the long-run properties of the model; i.e., a steady state with $\Delta c_t = \Delta k_t = \Delta m_t = 0$.
- First, from "Keynes-Ramsey rule" it follows that in steady state

$$1 = \beta\left[f_k(k^{ss}) + 1 - \delta\right],$$

or,

$$f_k(k^{ss}) + 1 - \delta = \frac{1}{\beta} \tag{2.18'}$$

- This — **independently of any monetary factors** — defines the steady-state capital per capita (and thus output per capita).

- Strong contrast with Tobin model
- Difference is because this model envisions optimal behavior.
  - If, e.g., $k_t < k^{ss}$ the current marginal product of capital is relatively high (as $f_{kk} < 0$) $\Longrightarrow$ optimal to postpone consumption to later $\Longrightarrow$ capital is accumulated until $f_k(k^{ss})+1-\delta = 1/\beta$ holds again
  - If one imagined that a Tobin effect was there; one would be self-contradictory:
    * Assume higher inflation increases capital above steady state
    * Then the marginal product decreases, and households would want to consume now rather than later $\Longrightarrow$ they **endogenously** save less and capital decreases until k^{ss} is reached again!
    * Inflation has no steady-state effect on capital. Possible in Tobin model, where individuals are modelled as having an **exogenous** savings rate

- What about consumption in steady state?
- The budget constraint in steady state is

$$f(k^{ss}) + \tau^{ss} + (1-\delta)k^{ss} + \frac{1}{1+\pi}m^{ss} = c^{ss} + k^{ss} + m^{ss}$$

- Transfers are

$$\begin{aligned}\tau_t &= (M_t - M_{t-1})/P_t,\\ &= \theta_t M_{t-1}/P_t,\\ &= \frac{\theta_t}{1+\pi_t}m_{t-1},\end{aligned}$$

so in steady state:

$$\tau^{ss} = \frac{\theta}{1+\pi}m^{ss}$$

- Since a constant m implies $\pi = \theta$, as in the Tobin model, one gets

$$f(k^{ss}) - \delta k^{ss} = c^{ss}$$

This is simply the economy's resource constraint (national account): Output less gross investment equals consumption

- Implication is that c^{ss} is determined exclusively by k^{ss}; and thus also independent of money growth
- Model exhibits *superneutrality of money*

- Does money growth and inflation not affect anything?
- Yes, the opportunity cost of holding real money, and thus the steady-state value of real money
- To see this, note relative demand for consumption versus real money is given by [use first-order cond. for money holdings and divide through by $u_c(c_t, m_t)$]

$$\begin{aligned}\frac{u_m(c_t,m_t)}{u_c(c_t,m_t)} &= \frac{u_c(c_t,m_t)}{u_c(c_t,m_t)} - \frac{1}{1+\pi_{t+1}}\frac{\beta u_c(c_{t+1},m_{t+1})}{u_c(c_t,m_t)} \\ &= 1 - \frac{1}{1+\pi_{t+1}}\frac{1}{(f_k(k_t)+1-\delta)} \\ &= 1 - \frac{1}{(1+\pi_{t+1})}\frac{1}{(1+r_t)}\end{aligned}$$

with $r_t \equiv f_k(k_t) - \delta$ being the real interest rate

- Note that the real interest rate is the nominal rate net of expected inflation:

$$1 + r_t = (1+i_t)/(1+\pi_{t+1}), \qquad (r_t \approx i_t - \pi_{t+1})$$

- Hence,

$$\frac{u_m(c_t,m_t)}{u_c(c_t,m_t)} = \frac{i_t}{1+i_t} \equiv I_t \tag{2.12}$$

So, as nominal interest rate is determined by the *Fisher relationship*, $i_t \approx r_t + \pi_{t+1}$, higher inflation leads to a higher nominal interest rate, and for given c_t, m_t is likely to fall (as $u_{mm} < 0$).

- Will a unique steady-state value for m exist? Must solve $u_m(c^{ss}, m^{ss}) = I^{ss}u_c(c^{ss}, m^{ss})$. Not necessarily unique.....
- Stability properties? For separable utility, $u(c_t, m_t) = v(c_t) + \gamma\phi(m_t)$, resulting difference equation (from first-order condition) will imply a saddle-point stable $m^{ss} > 0$ (m' in Figure 2.1, page 56)
- Problem: One cannot necessarily rule out the paths with falling m below steady-state ("speculative" hyperinflations), leading to $m^{ss} = 0$. (Typically, however, we will just do it.....)

Summary

- Tobin model and MIU models both have one for one relationships between inflation and money growth
- Main difference is that the Tobin model does not have long-run superneutrality
- Reason is the postulated and policy invariant private-sector behavior.
  This difference highlights importance of microfoundations to avoid Lucas critique
- Still, the MIU approach *is* a short-cut

Plan for next lecture

Wednesday, Feb. 11

1. Money in the utility function (continued)
   a. Welfare costs of inflation
   b. Potential non-superneutrality of money
   c. Dynamics and calibration

Literature: Walsh (2003, Chapter 2, pp. 59-80, but check the Appendix as well)