

May 10, 2004

On linearizations in chapter 6

Q How does the linearization of equation (6.7) (page 272 in Walsh, 2003) yield equation (6.20) on page 274?

A This is a boring, albeit straightforward exercise, where the trick is to keep track of the steady-state values of the involved variables. Let's jog.....

We start out by the equation (6.7) (skipping the j superscripts):

$$\frac{M_t}{P_t} = bC_t \frac{1 + i_t}{i_t}$$

A first-order Taylor approximation of both sides around steady state (upper bars denote steady states) yields

$$\begin{aligned} & \frac{\bar{M}}{\bar{P}} + \frac{1}{\bar{P}} (M_t - \bar{M}) - \frac{\bar{M}}{\bar{P}^2} (P_t - \bar{P}) \\ &= b\bar{C} \frac{1 + \bar{i}}{\bar{i}} + b \frac{1 + \bar{i}}{\bar{i}} (C_t - \bar{C}) - \frac{b\bar{C}}{\bar{i}^2} (i_t - \bar{i}) \end{aligned}$$

or,

$$\frac{1}{\bar{P}} (M_t - \bar{M}) - \frac{\bar{M}}{\bar{P}^2} (P_t - \bar{P}) = b \frac{1 + \bar{i}}{\bar{i}} (C_t - \bar{C}) - \frac{b\bar{C}}{\bar{i}^2} (i_t - \bar{i})$$

This is re-written as

$$\begin{aligned} & \frac{\bar{M}}{\bar{P}} \frac{M_t - \bar{M}}{\bar{M}} - \frac{\bar{M}}{\bar{P}} \frac{P_t - \bar{P}}{\bar{P}} \\ &= b\bar{C} \frac{1 + \bar{i}}{\bar{i}} \frac{C_t - \bar{C}}{\bar{C}} - \frac{b\bar{C}}{\bar{i}} \frac{(i_t - \bar{i})}{\bar{i}} \end{aligned}$$

Letting lower-case letters denote percentage deviations from steady state, which under a first-order approximation equal log deviations, we get

$$\frac{\bar{M}}{\bar{P}} m_t - \frac{\bar{M}}{\bar{P}} p_t = b\bar{C} \frac{1 + \bar{i}}{\bar{i}} c_t - \frac{b\bar{C}}{\bar{i}} \hat{i}_t$$

where \hat{i}_t denotes the percentage deviation of the nominal interest rate from steady state. Dividing the left-hand side by $\frac{\bar{M}}{\bar{P}}$ and the right-hand-side by $b\bar{C} \frac{1 + \bar{i}}{\bar{i}}$, we get

$$m_t - p_t = c_t - \frac{1}{1 + \bar{i}} \hat{i}_t \quad (*)$$

Equation (*) is the linearized money demand relation expressed as a function of the *nominal* interest rate.

To express this in terms of inflation and the real interest rate, we use the Fisher equation

$$R_t = \frac{1 + i_t}{1 + \pi_{t+1}}$$

This is approximated by

$$R_t - \bar{R} = \frac{1}{1 + \bar{\pi}} (i_t - \bar{i}) - \frac{1 + \bar{i}}{(1 + \bar{\pi})^2} (\pi_{t+1} - \bar{\pi})$$

Divide the left-hand side by \bar{R} and the right-hand-side by $(1 + \bar{i}) / (1 + \bar{\pi})$ to get

$$\frac{R_t - \bar{R}}{\bar{R}} = \frac{\bar{i}}{1 + \bar{i}} \frac{i_t - \bar{i}}{\bar{i}} - \frac{\pi_{t+1} - \bar{\pi}}{1 + \bar{\pi}}$$

Letting $\bar{\Pi} \equiv 1 + \bar{\pi}$, we can rewrite this as

$$r_t = \frac{\bar{i}}{1 + \bar{i}} \hat{i}_t - \hat{\pi}_{t+1}$$

where $\hat{\pi}_{t+1}$ is the deviation of (gross) inflation from steady state. Hence,

$$\hat{i}_t = \frac{1 + \bar{i}}{\bar{i}} (r_t + \hat{\pi}_{t+1})$$

Insert this into (*) to get:

$$m_t - p_t = c_t - \frac{1}{\bar{i}} (r_t + \hat{\pi}_{t+1}) \quad (**)$$

We are almost finished now. We know from the consumption Euler equation that the steady-state gross real interest rate is $1/\beta$. Hence, by the steady-state version of the Fisher equation we get:

$$\frac{1}{\beta} = \frac{1 + \bar{i}}{\bar{\Pi}}$$

From this we get the steady state nominal interest rate as

$$\bar{i} = \frac{\bar{\Pi}}{\beta} - 1$$

Using this in (**) we get

$$m_t - p_t = c_t - \frac{\beta}{\bar{\Pi} - \beta} (r_t + \hat{\pi}_{t+1}),$$

or in Walsh's notation:

$$m_t - p_t = c_t - \delta (r_t + \hat{\pi}_{t+1}), \quad (***)$$

where

$$\delta \equiv \frac{\hat{\beta}}{\hat{\Pi} - \beta}.$$

Equation (***) is Walsh's equation (6.20), with the only difference being that I have used a "hat notation" to distinguish log deviations in gross inflation from non-linearized inflation (Walsh does not make that distinction explicit).