Exercise, March 10

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Abstract

This note present the solution to the small exercise that we didn't have time to do in class on March 10: The derivation of the first-order conditions on page 7 on the slides from the lectures of March 3.

The agent on island i has the per-period utility function

$$u\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right),$$

and wishes to maximize the expected discounted sum of this. As seen on page 6 of the slides, the relevant budget constraint, when transfers are viewed as being proportional to real money holdings, is (where $Y_t^i = \left(N_t^i\right)^{1-\alpha}$ has been substituted in):

$$C_t^i + \frac{M_t^i}{P_t^i} = \left(N_t^i\right)^{1-\alpha} + \frac{T_t}{1+\pi_t^i}\frac{M_{t-1}^i}{P_{t-1}^i}.$$

Here T_t represents the (gross) return on real money holdings. Let λ^i denote the Lagrange multiplier on the budget constraint. We then get the following necessary first-order conditions for optimal choices of C_t^i, M_t^i/P_t^i and N_t^i:

$$u_C\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) - \lambda_t^i = 0, \qquad (*)$$

$$u_{M/P}\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) - \lambda_t^i + \beta \mathrm{E}_t^i \lambda_{t+1} \frac{T_{t+1}}{1+\pi_{t+1}} = 0, \qquad (**)$$

$$-u_{1-N}\left(C_t^i, M_t^i/P_t^i, N_t^i\right) + \lambda_t^i (1-\alpha)\left(N_t^i\right)^{-\alpha} = 0, \qquad (***)$$

Now note that (*) and (***) can immediately be combined into

$$u_{1-N}\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) = \left[(1-\alpha)\frac{Y_t^i}{N_t^i}\right] u_C\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right),$$

which is the first of the two conditions on page 7 of the slides. This is the intratemporal consumption-leisure optimality condition. The left-hand side is the utility loss from supplying more labor (in terms of the lost utility from leisure). The right-hand side is the gain measured by the extra utility from more consumption multiplied with the relative price of labour to consumption, the real wage. At an optimum, the marginal loss must equal the marginal gain.

Then note that equations (*) and (***) can be combined into

$$\begin{aligned} u_C\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) &= u_{M/P}\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) \\ &\quad +\beta \mathrm{E}_t^i \frac{T_{t+1}}{1+\pi_{t+1}} u_C\left(C_{t+1}, M_{t+1}/P_{t+1}, 1-N_{t+1}\right), \end{aligned}$$

which is the second of the two conditions on page 7 of the slides. This is an intertemporal expression, which can be interpreted as a condition for optimal money demand: The left-hand side is the utility loss of holding less real money in terms of the lost utility from consumption. The right-hand side is the gain from extra money in terms of current utility, as well as the future expanded consumption possibilities (discounted back to period t utility by β and multiplied by the relative price of money relative to consumption, $T_{t+1}/(1+\pi_{t+1})$). At an optimum, the marginal loss must equal the marginal gain(s).