Written exam for the M.Sc in Economics
Institute of Economics, University of Copenhagen

Monetary Economics: Macro Aspects

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4-hour exam

Answers and comments

This document provides an example of an answer for the exam. Note that the curriculum and questions are on a very high level; hence, one can attain a decent grade even when some of the mathematics are not fully correct, but are substituted by satisfactorily economic reasoning (hence, the answers here should regarded as ones that would give "more than 13"). Here and there, footnotes include comments explaining what is sufficient for "passing" a sub-question, as well as other additional information.

QUESTION 1:

(i) **False.** In the Tobin (1956) model, agents are assumed to divide their holding of real wealth between real money and physical capital based on the assets' relative returns. A higher inflation rate in steady state, implies that the return to real money (which approximately equals minus the inflation rate) decreases, and this makes agents accumulate more physical capital relative to real money. Hence, higher inflation implies a higher steady-state capital stock and, in consequence, a higher per capita output level.

(ii) **False.** To explain this, it suffices to look at the steady state. From the consumption Euler equation $u_c(c_t, m_t, l_t) = \beta(1 + r_t) u_c(c_{t+1}, m_{t+1}, l_{t+1})$ we have the steady-state relationship $1 + r = 1/\beta$. As the real interest rate equals the marginal product of capital, we have the condition $1 + f_k(k^{ss}, 1 - l^{ss}) = 1/\beta$. In the case of exogenous labour supply, we would have long run superneutrality. However, as labour supply is endogenous, and characterized by the optimality condition guiding the consumption-leisure choice, $u_l(c^{ss}, m^{ss}, l^{ss}) = u_c(c^{ss}, m^{ss}, l^{ss}) f_l(k^{ss}, 1 - l^{ss})$, superneutrality only holds in the steady state, if both $u_l(c^{ss}, m^{ss}, l^{ss})$ and $u_c(c^{ss}, m^{ss}, l^{ss})$ are independent of m^{ss} (e.g., if utility is separable in all arguments). Otherwise superneutrality fails in directions depending upon the cross derivatives of the utility function.[1]

(iii) **False.** In this setting, money is superneutral, as there is only a CIA constraint on consumption goods. Hence, capital accumulation is unaffected, and the steady-state capital stock is independent of inflation (as labour is exogenous). By the national accounts, consumption is also unaffected. As utility only depends on consumption, any inflation rate is as good as the other. Hence, one cannot determine the optimal rate of inflation in such a simplified model.

(iv) **False.** In the Lucas "islands" model, it is *un*anticipated aggregate money shocks, which have real effects. When these arrive, agents do not know — due to imperfect information — whether the observed money shock is caused by local factors (in which case they should react) or by aggregate factors (in which case all prices will move proportionally, and no reaction is optimal). In performing the signal extraction, they optimally react somewhat as they assign some probability to the fact that the shock is local. How much they react will depend upon the relative variances of local and aggregate shocks (e.g., a high variance of local shocks makes it relatively likely that a shock is indeed local, and agents therefore respond relatively strong to the observed shock). If an aggregate shock is anticipated, agents know by assumption that the observed shock an aggregate shock (not affecting relative prices), so they optimally do nothing.

(v) **False.** In the Poole (1970) model, an interest rate operating procedure insulates output completely from money market shocks. Fluctuations in output is therefore driven by, e.g., demand shocks. Under a money supply operating procedure, the interest rate responds endogenously to money market shocks; these shocks thus cause movements in output, while, e.g., demand shocks will have relatively smaller effects on output, as the interest rate works as an "automatic stabilizer." Hence, choosing between the two operating procedures before shocks are known, involves a comparison of the variances of money market disturbances and the variance of, e.g., demand shocks. So, if there is high degree of stability on the money market (relative to the goods market), one should opt for a money supply operating procedure, and *not* on an interest rate operating procedure.

[1] A description of the various possibilities is not necessary.

QUESTION 2:[2]

(i) Equation (1) is an IS curve, where aggregate demand is seen to exhibit inertia in the sense that lagged demand is a determinant of current demand. One also observes that the real interest rate has a one-period transmission lag; i.e., it affects demand with a one period delay. Equation (2) is the economy's AS curve, or Phillips curve, where from it is seen that higher output immediately leads to inflationary pressures. Also, the specification in one of high inflation persistence (alternatively, one may label it an "accelerationist" Phillips curve; or an expectations augmented Phillips curve, where expectations are entirely backward looking).

(ii) (Here the hints are followed.)

a) Forward (1) one period to get:

$$y_{t+1} = a_1 y_t - a_3 \left(i_t - \mathrm{E}_t \pi_{t+1}\right) + u_{t+1}.$$

Forward (2) one period to get:

$$\pi_{t+1} = \pi_t + \gamma y_{t+1} + \eta_{t+1},$$

and take period t expectations on both sides:

$$\mathrm{E}_t \pi_{t+1} = \pi_t + \gamma \mathrm{E}_t y_{t+1}.$$

Insert this expression for $\mathrm{E}_t \pi_{t+1}$ into the expression for y_{t+1}:

$$y_{t+1} = a_1 y_t - a_3 \left(i_t - \pi_t - \gamma \mathrm{E}_t y_{t+1}\right) + u_{t+1}.$$

Take period t expectations on both sides to get

$$\mathrm{E}_t y_{t+1} = a_1 y_t - a_3 \left(i_t - \pi_t - \gamma \mathrm{E}_t y_{t+1}\right),$$

from which $\mathrm{E}_t y_{t+1}$ follows as

$$\mathrm{E}_t y_{t+1} = \frac{a_1}{1 - a_3\gamma} y_t - \frac{a_3}{1 - a_3\gamma} \left(i_t - \pi_t\right).$$

This is inserted back into the expression for y_{t+1} so as to get

$$\begin{aligned} y_{t+1} &= a_1 y_t - a_3 \left(i_t - \pi_t - \gamma \frac{a_1}{1 - a_3\gamma} y_t + \gamma \frac{a_3}{1 - a_3\gamma} \left(i_t - \pi_t\right)\right) + u_{t+1} \\ &= \frac{a_1}{1 - a_3\gamma} y_t - \frac{a_3}{1 - a_3\gamma} \left(i_t - \pi_t\right) + u_{t+1} \\ &= \theta_t + u_{t+1} \end{aligned}$$

[2] *All* the computations in this question follow slavishly Walsh (1998, Section 10.5.1.3) for the special case of $a_2 = 0$.

with
$$\theta_t \equiv \frac{a_1}{1-a_3\gamma} y_t - \frac{a_3}{1-a_3\gamma}\left(i_t - \pi_t\right)$$
The expression $y_{t+1} = \theta_t + u_{t+1}$ is then inserted into the one-period forwarded Phillips curve, so as to get:

$$\pi_{t+1} = \pi_t + \gamma\theta_t + v_{t+1}, \quad v_{t+1} \equiv \gamma u_{t+1} + \eta_{t+1}$$

b) As policy in period t can only affect variables in period $t+1$ and onwards, we treat θ_t as the policy instrument and π_t as the state variable, and set up the dynamic programing problem as

$$V\left(\pi_t\right) = \min_{\theta_t} \mathrm{E}_t \left\{ \frac{1}{2}\left(\lambda y_{t+1}^2 + \pi_{t+1}^2 + \beta V\left(\pi_{t+1}\right)\right) \right\},$$

where the minimization is subject to $y_{t+1} = \theta_t + u_{t+1}$ and $\pi_{t+1} = \pi_t + \gamma\theta_t + v_{t+1}$. These constraints, however, are inserted into the value function so as to give an unconstrained problem:

$$\begin{aligned} V\left(\pi_t\right) &= \min_{\theta_t} \mathrm{E}_t \left\{ \frac{1}{2}\left(\lambda\left(\theta_t + u_{t+1}\right)^2 + \left(\pi_t + \gamma\theta_t + v_{t+1}\right)^2\right) \right. \\ &\quad \left. + \beta V\left(\pi_t + \gamma\theta_t + v_{t+1}\right) \right\} \end{aligned}$$

The relevant first-order condition is then readily found as

$$\begin{aligned} &\mathrm{E}_t \left\{ \lambda\left(\theta_t + u_{t+1}\right) + \gamma\left(\pi_t + \gamma\theta_t + v_{t+1}\right) \right. \\ &\left. + \beta\gamma V_\pi\left(\pi_t + \gamma\theta_t + v_{t+1}\right) \right\} \\ = \; & 0, \end{aligned}$$

or,
$$\left(\lambda + \gamma^2\right)\theta_t + \gamma\pi_t + \beta\gamma \mathrm{E}_t V_\pi\left(\pi_{t+1}\right) = 0.$$

c) We have that the value function in optimum is given by

$$\begin{aligned} V\left(\pi_t\right) &= \mathrm{E}_t \left\{ \frac{1}{2}\left(\lambda\left(\theta_t + u_{t+1}\right)^2 + \left(\pi_t + \gamma\theta_t + v_{t+1}\right)^2\right) \right. \\ &\quad \left. + \beta V\left(\pi_t + \gamma\theta_t + v_{t+1}\right) \right\}, \end{aligned}$$

where θ_t is an optimal function of the state. Differentiating the value function then gives

$$\begin{aligned} V_\pi\left(\pi_t\right) &= \mathrm{E}_t \left\{ \lambda\left(\theta_t + u_{t+1}\right)\frac{\partial\theta_t}{\partial\pi_t} + \left(\pi_t + \gamma\theta_t + v_{t+1}\right)\left(1 + \gamma\frac{\partial\theta_t}{\partial\pi_t}\right) \right. \\ &\quad \left. + \beta V_\pi\left(\pi_t + \gamma\theta_t + v_{t+1}\right)\left(1 + \gamma\frac{\partial\theta_t}{\partial\pi_t}\right) \right\}. \end{aligned}$$

By the envelope theorem, all coefficients to $\partial\theta_t/\partial\pi_t$ can be ignored (as they are zero by the first-order condition for optimal choice of θ_t; cf. above); hence:

$$V_\pi(\pi_t) = \pi_t + \gamma\theta_t + \beta \mathrm{E}_t V_\pi(\pi_{t+1}).$$

d) Multiply this equation by γ on both sides:

$$\gamma V_\pi(\pi_t) = \gamma\pi_t + \gamma^2\theta_t + \beta\gamma \mathrm{E}_t V_\pi(\pi_{t+1}),$$

and eliminate $\beta\gamma\mathrm{E}_t V_\pi(\pi_{t+1})$ from the first-order condition,

$$\left(\lambda + \gamma^2\right)\theta_t + \gamma\pi_t + \gamma V_\pi(\pi_t) - \gamma\pi_t - \gamma^2\theta_t = 0,$$

such that

$$\lambda\theta_t = -\gamma V_\pi(\pi_t).$$

Forward this one period, multiply by β, and take period t expectations:

$$\lambda\beta\mathrm{E}_t\theta_{t+1} = -\beta\gamma\mathrm{E}_t V_\pi(\pi_{t+1}),$$

so as to finally get from the first-order condition

$$\left(\lambda + \gamma^2\right)\theta_t + \gamma\pi_t - \lambda\beta\mathrm{E}_t\theta_{t+1} = 0.$$

This is solved for θ_t by conjecturing $\theta_t = B\pi_t$, which implies $\mathrm{E}_t\theta_{t+1} = B\mathrm{E}_t\pi_{t+1} = B(\pi_t + \gamma\theta_t)$, where the last equality follows from the Phillips curve. Hence, we have:

$$\left(\lambda + \gamma^2\right)\theta_t + \gamma\pi_t - \lambda\beta B(\pi_t + \gamma\theta_t) = 0,$$

which by use of the conjecture becomes

$$\left(\lambda + \gamma^2\right)B\pi_t + \gamma\pi_t - \lambda\beta B(\pi_t + \gamma B\pi_t) = 0.$$

This validates the conjecture, and identifies B from

$$-\left(\lambda(1-\beta) + \gamma^2\right)B - \gamma + \lambda\beta\gamma B^2 = 0.$$

The solutions are

$$B = \frac{\lambda(1-\beta) + \gamma^2 \pm \sqrt{\left(\lambda(1-\beta) + \gamma^2\right)^2 + 4\lambda\beta\gamma^2}}{2\lambda\beta\gamma}.$$

I.e., one is negative and one is positive. As the inflation equation becomes $\pi_{t+1} = (1 + \gamma B)\pi_t + v_{t+1}$, only the positive solution to B is relevant (the positive value cannot be optimal as it implies explosive inflation and output).

One can then, from the definition of θ_t compute the associated optimal interest rule[3]:

$$B\pi_t = \frac{a_1}{1 - a_3\gamma} y_t - \frac{a_3}{1 - a_3\gamma} (i_t - \pi_t),$$

and thus

$$i_t = \left[1 - \frac{(1 - a_3\gamma) B}{a_3}\right] \pi_t + \frac{a_1}{a_3} y_t.$$

(iii) The essential result is that the coefficient on π_t is greater than one. This implies that upward inflationary pressures are met by an increase in the nominal interest rate implying an increase in the *real* interest rate. In this model, this is stabilizing, as such policy behavior depresses demand, and push inflation back towards steady-state. (In contrast, if the coefficient was less that one, inflationary pressures would be met by an increase in the nominal interest rate that is insufficient to prevent the real interest rate from falling; as a result, demand will increase, and inflation will be driven further up, and the economy would be on an explosive path.)

(iv) The magnitude is a function of several parameters of the model; e.g., the slope of the Phillips-curve (γ), the relative preference for output stabilization (λ), and the real interest rate sensitivity of demand (a_3). While the magnitude decreases with λ, one cannot conclude that a high value is necessarily synonymous with a high preference for inflation stabilization (i.e., a low value of λ). For example, it could just as well be a consequence of a low value of a_3, which — for given preferences — implies that to obtain a desired degree of inflation stabilization the central bank must move the interest rate more. It is excellent (but not necessary) to add that what appears in the optimal interest rate rule does generally not say much about preferences, as is evident from the fact that the coefficient on output is strictly positive even in the case of $\lambda = 0$. In that case the central bank cares exclusively about inflation, but nevertheless responds to output because it helps predict future inflation.

QUESTION 3:

(i) Equation (1) is a conventional expectations augmented Phillips curve, which can be rationalized by e.g., presence of one-period nominal wage contracts or Lucas-style asymmetric information about local versus aggregate shocks. Equation (2)

[3]Note that, with the provided hints, it is possible to compute this *without* at all describing how one solves for the *optimal* value of θ_t. Credit, however, is, of course, given to students demonstrating that they can handle the optimization problem appropriately.

shows that the central bank dislikes fluctuations in the output gap around a target k, and fluctuations in inflation around a target value of zero. A positive target value for the output gap can be rationalized by the fact that the natural rate of output is considered inefficiently low (say, due to imperfect competition).

(ii) The relevant first order condition is [where (1) has been substituted into (2)]:

$$-\lambda\left(x_t - k\right) - \kappa\left(\mathrm{E}_{t-1}\pi_t + \kappa x_t + \varepsilon_t\right) = 0.$$

Taking period $t-1$ expectations one gets

$$-\lambda\left(\mathrm{E}_{t-1}x_t - k\right) - \kappa\left(\mathrm{E}_{t-1}\pi_t + \kappa \mathrm{E}_{t-1}x_t + \mathrm{E}_{t-1}\varepsilon_t\right) = 0.$$

Using that (since ε_t is mean zero) $\mathrm{E}_{t-1}x_t = 0$ by (1), one gets inflation expectations from this expression as

$$\mathrm{E}_{t-1}\pi_t = \frac{\lambda}{\kappa}k.$$

Inserting this back into the first order condition yields

$$-\lambda\left(x_t - k\right) - \kappa\left(\frac{\lambda}{\kappa}k + \kappa x_t + \varepsilon_t\right) = 0,$$

and thus

$$x_t = -\frac{\kappa}{\lambda + \kappa^2}\varepsilon_t.$$

This is inserted into (1) to get inflation as

$$\pi_t = \frac{\lambda}{\kappa}k + \frac{\lambda}{\lambda + \kappa^2}\varepsilon_t.$$

These solutions demonstrate that the economy will suffer from an inflation bias when $k > 0$. Since, the central bank is expected to aim at pushing output above the natural rate, inflation expectations will go up to a point where the inflationary consequences of expansive monetary policy is too costly. In equilibrium inflation is excessive and average output is at the natural rate. The shock ε_t, however, is efficiently "spread out" between inflation and output to a degree determined by the relative preference for output versus inflation stability (λ)

(iii) Appointing a conservative central banker implies that less emphasis will be put on attaining the output gap target of $k > 0$. Inflation expectations will thus go down, and the economy will end up in a situation with a lower inflation bias. This is the benefit of the conservative central banker. The loss, however, is that the stabilization of the ε-shock becomes inefficient, as output will become too volatile, and inflation too stable. Some degree of "conservativeness" is, however, always beneficial. The reason is that starting from a situation of $\lambda^c = \lambda$, lowering λ^c will cause a gain in terms of a lower inflation bias which is of first order, while the loss in terms of distorted shock stabilization is of second order. Hence $0 < \lambda^c < \lambda$ is always optimal.

(iv) The relevant first-order condition is [where (4) has been substituted into (2)]:

$$-\lambda\left(x_t - k\right) - \kappa\pi_t = 0.$$

This is inserted into (4):

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \kappa\left[\left(\kappa/\lambda\right)\pi_t - k\right] + \varepsilon_t,$$

which becomes

$$\pi_t = \frac{\lambda\beta}{\lambda + \kappa^2}\mathrm{E}_t\pi_{t+1} + \frac{\lambda\kappa}{\lambda + \kappa^2}k + \frac{\lambda}{\lambda + \kappa^2}\varepsilon_t.$$

This is a first-order expectational difference equation with one unstable root; so there is a unique non-explosive solution to π_t. Conjecture a solution of the following format:[4]

$$\pi_t = Y + X\varepsilon_t,$$

where Y and X are the undetermined coefficients to be determined. Forward the conjecture one period, and take period t expectations:

$$\pi_{t+1} = Y + X\varepsilon_{t+1},$$

$$\mathrm{E}_t\pi_{t+1} = Y + X\mathrm{E}_t\varepsilon_{t+1},$$
$$\mathrm{E}_t\pi_{t+1} = Y$$

(as $\mathrm{E}_t\varepsilon_{t+1} = 0$ since ε is serially uncorrelated). Insert this into the difference equation:

$$\pi_t = \frac{\lambda\beta}{\lambda + \kappa^2}Y + \frac{\lambda\kappa}{\lambda + \kappa^2}k + \frac{\lambda}{\lambda + \kappa^2}\varepsilon_t$$
$$\pi_t = \frac{\lambda\beta Y + \lambda\kappa k}{\lambda + \kappa^2} + \frac{\lambda}{\lambda + \kappa^2}\varepsilon_t$$

This verifies the form of the conjecture, and identifies the coefficients by the equations

$$Y = \frac{\lambda\beta Y + \lambda\kappa k}{\lambda + \kappa^2}$$
$$X = \frac{\lambda}{\lambda + \kappa^2}$$

So, Y follows as

$$Y = \frac{\lambda\kappa}{\lambda\left(1 - \beta\right) + \kappa^2}k,$$

[4]It is sufficient to derive the solution for the case of $k = 0$, and verbally argue that there will be an inflation bias (i.e., $Y > 0$) when $k > 0$ applies.

Hence,

$$\pi_t = \frac{\lambda\kappa}{\lambda(1-\beta)+\kappa^2}k + \frac{\lambda}{\lambda+\kappa^2}\varepsilon_t.$$

The social inefficiencies of this solution are two-fold. First, there is an inflation bias when $k > 0$, and for the same reasons when (1) applies. In addition, there is inefficient stabilization of the ε-shock. The central bank could improve inflation stabilization, if it could commit to a continuation of contractive policies following a temporary positive realization of ε. The reason is that this will dampen inflation expectations and thus current inflation; i.e., it would improve the inflation-output gap trade off. However, such a commitment is not time consistent, as the central banker — when the shock has passed — has no incentives to contract.

(v) [5]A conservative central banker is beneficial for the same reasons as when (1) applies, as it reduces the inflation bias. It will, as long as the ε-shock is serially uncorrelated not be beneficial in terms of stabilization policy, as it will not improve the inflation-output gap trade off. In the case of $k = 0$, the argument for a conservative central banker vanishes, as there is no inflation bias. Only in the case where the shock has serial correlation, will conservativeness also improve when $k = 0$, as the private sector then knows that the future aftermath of the shock will be met by more contractive monetary policy, which dampens inflation expectations and thus current inflation. I.e., conservativeness improves the inflation-output gap trade off for the central bank. With serially uncorrelated shocks, however, this argument does not apply. A brilliant answer will note that other delegation mechanisms can be superior if they induce inertia into monetary policy making (e.g., nominal income growth targeting).

[5]This is a very hard question, and one can obtain a high total grade without even answering it.