Written exam for the M.Sc in Economics
Institute of Economics, University of Copenhagen

Monetary Economics: Macro Aspects

Semester: Spring 2004

June 18, 2004

4-hour exam

This set contains four pages. All questions must be answered.
In the evaluation, the three main questions will be weighted equally

No auxiliary material is allowed

To be answered in Danish or English

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) Consider the model of Barro and Gordon, where output, y, is given by $y = \pi - \pi^e + \varepsilon$, where π is inflation, π^e is inflation expectations and ε is a supply shock. Social welfare is given by $U = -(y-k)^2 - \pi^2$, $k > 0$. Delegating monetary policy conduct to a "conservative" central banker with utility function $U^c = -(y-k)^2 - (1+\delta)\pi^2$, $\delta > 0$, is disadvantageous if the variance of ε is sufficiently high.

(ii) According to the Friedman rule, the optimal rate of inflation is zero.

(iii) Consider a money-in-the-utility-function model, where agents' per-period utility functions are given by $u = a \ln c_t + (1-a) \ln m_t + l_t^{1-\eta}/(1-\eta)$, $0 < a < 1$, $\eta > 0$, where c_t is consumption, m_t are real money balances, and l_t is leisure. Labour, $n_t = 1 - l_t$, is used for production. In this setting, money is superneutral in steady state.

(iv) In a cash-in-advance model where superneutrality fails, unanticipated money shocks have short-run real effects.

(v) When a central bank follows the rule for the nominal interest rate originally described by John Taylor, an increase in inflation will lead to an increase in the real interest rate.

QUESTION 2:

An explicit inflation target as a means of solving credibility problems

Consider the following model of a closed economy:

$$y = a(\pi - \pi^e) + \varepsilon, \qquad a > 0, \tag{1}$$

where y is output, π is inflation, π^e denotes inflation expectations, and ε is a mean-zero shock with variance σ^2. The economy's policy authority affects aggregate demand through monetary policy, and for simplicity π is assumed to be the policy instrument. The sequence of events are: First, inflation expectations are formed (rationally), then the shock ε is realized, then π is chosen, and finally y is determined. The aim of monetary policy is to maximize social welfare given by:

$$U = -\frac{1}{2}\left[\lambda (y-k)^2 + (\pi - \pi^*)^2\right], \qquad k > 0, \quad \lambda > 0, \tag{2}$$

where π^* is the socially optimal inflation rate.

(i) Describe briefly the economic mechanisms and assumptions underlying (1) and (2).

(ii) Derive the optimal time-consistent outcomes for output and inflation. What is the social inefficiency of this solution? Explain, and state the solutions for output and inflation under commitment.

(iii) Assume now that society delegates monetary policymaking to a central banker with a utility function given by

$$U^{CB} = -\frac{1}{2}\left[\lambda (y-k)^2 + \left(\pi - \pi^T\right)^2\right], \tag{3}$$

where π^T is the inflation target of the central banker. How will the optimal time-consistent outcomes change (relative to those derived in (ii)) when this central banker takes office?

(iv) What is the societal optimal value of π^T? Explain the result. How and why does this value potentially differ from π^*?

QUESTION 3:

Monetary policy with a "cash-in-advance" constraint

Consider an economy formulated in discrete time and under certainty, where the utility of a representative agent is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1, \tag{1}$$

where c_t is real consumption and $u' > 0$, $u'' < 0$. The agent faces the budget constraint

$$\begin{aligned} \omega_t &\equiv f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + \frac{m_{t-1} + (1+i_{t-1}) b_{t-1}}{1+\pi_t} \\ &= c_t + k_t + m_t + b_t, \end{aligned} \tag{2}$$

where k_{t-1} is real capital at the end of period $t-1$, f is a production function where $f' > 0$, $f'' < 0$, τ_t denotes real monetary transfers from the government, $0 < \delta < 1$ is the rate of depreciation of capital, m_{t-1} denotes real money holdings at the end of period $t-1$, i_{t-1} is the nominal interest rate on bonds (denoted b_{t-1} in real terms), and π_t is the rate of inflation.

The agent also faces the following cash-in-advance constraint on consumption:

$$c_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau_t. \tag{3}$$

(i) Comment briefly on equations (1), (2) and (3).

(ii) Examine the optimal choices of consumption, capital and real money holdings. For that purpose show first that the budget constraint (2) can be rewritten as

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + (1-\delta) k_t + \frac{m_t}{1+\pi_{t+1}} + R_t (\omega_t - c_t - k_t - m_t),$$

with $R_t \equiv (1+i_t)/(1+\pi_{t+1})$ being the real interest rate, thereby eliminating bond holdings. Use that the agent's optimization problem can be characterized by

$$V(\omega_t, m_{t-1}) = \max \left\{ u(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t \left(c_t - \frac{m_{t-1}}{1+\pi_t} - \tau_t \right) \right\},$$

where maximization is over c, k, and m, and where μ_t is the multiplier on (3). Then derive and interpret these necessary optimality conditions:

$$u_c(c_t) = \beta R_t V_\omega(\omega_{t+1}, m_t) + \mu_t,$$

$$\beta V_\omega(\omega_{t+1}, m_t) [f_k(k_t) + 1 - \delta] = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta R_t V_\omega(\omega_{t+1}, m_t).$$

(iii) Show that by use of the Envelope theorem one finds

$$V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

$$V_m(\omega_t, m_{t-1}) = \mu_t \frac{1}{1+\pi_t}.$$

(iv) Define $\lambda_t \equiv V_\omega(\omega_t, m_{t-1})$, and use the result from (iii) together with the money demand relation from (ii), to obtain an expression for the nominal interest rate, i_t, as a function of μ_{t+1} and λ_{t+1}. Explain this relationship with particular focus on the role of a binding or non-binding cash-in-advance constraint.

(v) Show formally that monetary policy — here different rates of nominal money growth — has no real affects in the steady state of this economy. Explain the result. Which variables will, on the other hand, be affected by different long-run nominal money growth rates?

(vi) Discuss briefly which extensions of this basic “cash-in-advance” framework could generate long-run real effects of monetary policy.