Written exam for the M.Sc in Economics
Institute of Economics, University of Copenhagen

Monetary Economics: Macro Aspects

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4-hour exam

Answers and comments

This document provides an example of an answer for the exam. Note that the curriculum and questions are on a very high level; hence, one can attain a decent grade even when some of the mathematics are not fully correct, but are substituted by satisfactorily economic reasoning (hence, the answers here should regarded as ones that would give "more than 13").

QUESTION 1:

(i) **False.** In the Barro and Gordon model, the discretionary equilibrium is characterized by an inefficiently high inflation rate, the so-called inflation bias. The policy reaction to the supply shock, on the other hand, is efficient. By delegation to a "conservative" central banker, the average inflation rate will be brought down, but shock stabilization is distorted (inflation becomes suboptimally stable, and output too unstable). This could at first glance suggest that if shock stabilization is sufficiently important, that if the variance of ε is sufficiently high, then a conservative central banker is disadvantageous. This reasoning is wrong, however, as the beneficial effects of $\delta > 0$ are of first order (it mitigates an inefficiency), while the detrimental effects are of second order as shock stabilization is efficient at $\delta = 0$. Hence, some $\delta > 0$ is optimal. The very good answer will note that the optimal value of δ will decrease with the variance of the supply shock.

(ii) **False.** The Friedman rule stipulates that the private opportunity costs of holding real money balances should be zero. I.e., the nominal interest rate should be zero. According to the Fisher relationship, this implies an inflation rate equal to the negative of the real rate of return (i.e., deflation equal to the real interest rate).

(iii) **True.** In steady state, different rates of money growth and inflation will affect the real money stock through the effects on the nominal interest rate and money demand. If this should have any real effects, it must affect households' labour supply decision. This can only happen if either the marginal utility of leisure depends on real money balances and/or if the marginal utility of consumption depends on real money balances (as this would then alter the consumption-leisure trade off). But with function u, consumption, real money balances and leisure enter separable, and therefor neither the marginal utility of leisure, nor the marginal utility of consumption, will be affected by different steady-state values of real money balances. Hence superneutrality prevails.

(iv) **False.** An unanticipated shock to money will only have effects on the current rate on inflation, and not expected future inflation. It will not affect the nominal interest rate, and thus not the "tax" on consumption induced by the cash-in-advance constraint. However, if the money shock is anticipated to affect expected next-period inflation, then the nominal interest rate is affected, and the consumption (and leisure) choice. It may be noted that this is why one has to have serial correlation in the process for nominal money, if money shocks should have any effects at all in the standard cash-in-advance model.

(v) **True.** The original 1993 Taylor rule was written as

$$r = p + 0.5y + 0.5\,(p - 2) + 2$$

where r is the nominal interest rate, y is the output gap, and p is the inflation rate. From this rule one sees that an increase in inflation of one percentage point, the nominal interest rate is raised by 1.5 percentage points. Hence, the nominal interest rate raises by more than inflation, implying an increase in the real interest rate. It should be mentioned that this phenomenon in many models is concluded to be a good policy principle for stabilizing inflation, as an increase in inflation is met by contractive monetary policy, which though its negative demand and output consequences will help dampen price increases.

QUESTION 2:

(i) Equation (1) describes the economy's aggregate supply curve, and it takes the form of an expectations augmented Phillips-curve relationship. Such an equation can be rationalized with one period nominal contracts, or, e.g., the Lucas misperceptions theory. Equation (2) shows that social welfare depends negatively on deviations in output from k, and deviations in inflation from π^*. It is central that $k > 0$, as this means that it is socially optimal to have an output level higher than the natural rate (which from (1) can be seen to be zero on average).

(ii) With the described move structure, substitution of (1) into (2) provides the following maximization problem:

$$\min_{\pi} U = -\frac{1}{2}\left[\lambda\left(a\left(\pi - \pi^e\right) + \varepsilon - k\right)^2 + \left(\pi - \pi^*\right)^2\right],$$

where π^e is taken as given. The first-order condition is

$$\lambda a\left(a\left(\pi - \pi^e\right) + \varepsilon - k\right) + \left(\pi - \pi^*\right) = 0$$

Solve for private sector expectations by taking expectations on this expression:

$$\mathrm{E}\left[\lambda a\left(a\left(\pi - \pi^e\right) + \varepsilon - k\right) + \left(\pi - \pi^*\right)\right] = 0$$

$$\mathrm{E}\left[-\lambda a k + \left(\pi - \pi^*\right)\right] = 0$$

and thus

$$\pi^e = \mathrm{E}\left[\pi\right] = \pi^* + \lambda a k.$$

Substitute this expression back into the first-order condition to get

$$\lambda a\left(a\left(\pi - \pi^* - \lambda a k\right) + \varepsilon - k\right) + \left(\pi - \pi^*\right) = 0,$$

from which actual inflation follows as

$$\left(1 + \lambda a^2\right)\left(\pi - \pi^*\right) + \lambda a\left(-\lambda a^2 k + \varepsilon - k\right) = 0,$$

$$\left(1 + \lambda a^2\right)\left(\pi - \pi^*\right) + \lambda a \varepsilon - \lambda a\left(\lambda a^2 + 1\right)k = 0,$$

$$\pi = \pi^* + \lambda a k - \frac{\lambda a}{1 + \lambda a^2}\varepsilon.$$

Inserting this expression along with the expression for inflation expectations into (1) then readily recovers the solution for output:

$$y = a\left(\pi^* + \lambda a k - \frac{\lambda a}{1 + \lambda a^2}\varepsilon - \left[\pi^* + \lambda a k\right]\right) + \varepsilon,$$

$$y = \frac{1}{1+\lambda a^2}\varepsilon.$$

The social inefficiency of this solution is that inflation on average exceeds the optimal inflation rate; i.e., there is an inflation bias equal to λak. The reason is that the policymaker after inflation expectations have been set has an incentive to set inflation above expectations in order to increase output towards k. The private sector, however, anticipates this policy incentive and sets expectations sufficiently high such that the policymaker refrains from setting inflation above expectations (as this will bee to costly in terms of inflation). The result is too high inflation and no output effects. Regarding the supply shock, the private sector does not know its realization, so the policymaker optimally "spreads out" the effects of the shock on output and inflation.

Had the policymaker been able to commit to a policy, it would, of course, be to one where the inflation bias is absent, i.e., where actual inflation and output were

$$\pi = \pi^* - \frac{\lambda a}{1+\lambda a^2}\varepsilon,$$

$$y = \frac{1}{1+\lambda a^2}\varepsilon.$$

(iii) The only change with the new central banker is that its explicit inflation target (potentially) differs from π^*. The solution for the time-consistent solutions for inflation and output is found in the same manner as in subquestion (ii), except for replacing π^* with π^T. I.e., one recovers

$$\pi = \pi^T + \lambda ak - \frac{\lambda a}{1+\lambda a^2}\varepsilon,$$

$$y = \frac{1}{1+\lambda a^2}\varepsilon.$$

One sees that there again is an inflation bias in the sense that inflation on average exceeds the central banker's inflation target of π^T by λak.

(iv) The optimal value of π^T is one which secures that the solutions found in (iii) replicates the commitment solution found in (ii). As the solution for output is independent of π^T, it suffices to look at the solution for inflation. A value of π^T replicating the commitment solution is thus one that satisfies

$$\pi^T + \lambda ak - \frac{\lambda a}{1+\lambda a^2}\varepsilon = \pi^* - \frac{\lambda a}{1+\lambda a^2}\varepsilon.$$

From this one obtains the optimal value of π^T as

$$\pi^T_{optimal} = \pi^* - \lambda ak.$$

We see that this is a value *below* the optimal rate of inflation π^*. The reason for this is that the appointed central banker will then aim for a combination of loose monetary policy to try boost output above the natural rate and a contractive policy aiming at a lower than optimal average inflation rate. The two opposing incentives are with the proper choice of π^T then exactly offsetting in the sense that the equilibrium average inflation rate becomes π^*.

QUESTION 3:

(i) Equation (1) is the standard intertemporal utility of an infinitely lived household, where only consumption enter as an argument. Equation (2) is the budget constraint stating that total wealth ω_t consists of output, transfers from the government, the net stock of capital, the real value of money carried over from past period, and finally the real value of bonds carried over from past period. This wealth can in period t be allocated among consumption, capital, real money holdings, and real bond holdings. Equation (3) is the CIA constraint that stipulates that consumption in period t cannot exceed the real value of money holdings carried over from the past period plus transfers.

(ii) The wealth definition in (2) can be forwarded one period:

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + (1-\delta) k_t + \frac{m_t + (1+i_t) b_t}{1+\pi_{t+1}}.$$

Then using that $b_t = \omega_t - c_t - k_t - m_t$, and $R_t \equiv (1+i_t)/(1+\pi_{t+1})$, this expression can be written as

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + (1-\delta) k_t + \frac{m_t}{1+\pi_{t+1}} + R_t (\omega_t - c_t - k_t - m_t),$$

The agent's optimization problem is characterized by

$$V(\omega_t, m_{t-1}) = \max \left\{ u(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t \left(c_t - \frac{m_{t-1}}{1+\pi_t} - \tau_t \right) \right\}.$$

The first-order condition w.r.t. c_t is

$$u_c(c_t) - \beta R_t V_\omega(\omega_{t+1}, m_t) - \mu_t = 0$$

from which one readily recovers

$$u_c(c_t) = \beta R_t V_\omega(\omega_{t+1}, m_t) + \mu_t.$$

This states that at the optimum the agent chooses consumption at t such that the marginal gain in terms of marginal utility equals the marginal loss, which takes the form of the utility loss arising from less wealth in the next period (multiplied by the real interest rate and discounted back to period t by β) as well as the loss accruing from the CIA constraint.
The first-order condition w.r.t. k_t is

$$\beta V_\omega(\omega_{t+1}, m_t) [f_k(k_t) + 1 - \delta] - \beta R_t V_\omega(\omega_{t+1}, m_t) = 0$$

which is readily rewritten as

$$\beta V_\omega (\omega_{t+1}, m_t) [f_k (k_t) + 1 - \delta] = \beta R_t V_\omega (\omega_{t+1}, m_t) .$$

This states that capital is chosen such that the associated marginal gain in terms of more wealth in next period (multiplied by the net marginal product of capital), equals the marginal loss in terms of lower wealth in form of bonds (multiplied by the real interest rate). Note that this expression delivers the familiar relationship $R_t = f_k (k_t) + 1 - \delta$.
The first-order condition with respect to m_t is

$$\beta \frac{1}{1 + \pi_{t+1}} V_\omega (\omega_{t+1}, m_t) + \beta V_m (\omega_{t+1}, m_t) - \beta R_t V_\omega (\omega_{t+1}, m_t) = 0$$

which is readily rewritten as

$$\beta \frac{1}{1 + \pi_{t+1}} V_\omega (\omega_{t+1}, m_t) + \beta V_m (\omega_{t+1}, m_t) = \beta R_t V_\omega (\omega_{t+1}, m_t) .$$

This states that real money holdings are chosen such that the marginal gains (in terms of more wealth in the next period as well as more utility of money per se due to the liquidity services it provides, if $V_m (\omega_{t+1}, m_t) > 0$, which holds when the CIA constraint binds) equal the marginal loss in terms of the utility loss of lower financial wealth.

(iii) Differentiating the value function with respect to ω_t and taking into account that c_t, k_t and m_t will be optimal functions of the states (ω_t and m_{t-1}) whereby one can ignore any effects of ω_t on those variables, one immediately obtains

$$V_\omega (\omega_t, m_{t-1}) - \beta R_t V_\omega (\omega_{t+1}, m_t) = 0,$$

or

$$V_\omega (\omega_t, m_{t-1}) = \beta R_t V_\omega (\omega_{t+1}, m_t) .$$

Likewise, differentiating the value function with respect to m_{t-1} holding constant c_t, k_t and m_t, one gets

$$V_m (\omega_t, m_{t-1}) - \mu_t \frac{1}{1 + \pi_t} = 0$$

or

$$V_m (\omega_t, m_{t-1}) = \mu_t \frac{1}{1 + \pi_t} .$$

Here one sees that the marginal value of money is only positive if the CIA constraint binds; i.e., in which case money provides liquidity services.

(iv) The money demand function, restated here,

$$\beta\frac{1}{1+\pi_{t+1}}V_{\omega}\left(\omega_{t+1},m_t\right)+\beta V_m\left(\omega_{t+1},m_t\right)=\beta R_t V_{\omega}\left(\omega_{t+1},m_t\right)$$

can now with the expression for the value function derivatives found in (iii) be expressed as

$$\frac{1}{1+\pi_{t+1}}\lambda_{t+1}+\mu_{t+1}\frac{1}{1+\pi_{t+1}}=R_t\lambda_{t+1}.$$

This yields

$$\begin{aligned}\lambda_{t+1}+\mu_{t+1}&=\left(1+i_t\right)\lambda_{t+1},\\ \frac{\lambda_{t+1}+\mu_{t+1}}{\lambda_{t+1}}&=\left(1+i_t\right),\end{aligned}$$

and thus

$$i_t=\frac{\mu_{t+1}}{\lambda_{t+1}}.$$

It is thus seen that the nominal interest rate is only positive if the CIA binds. In that case money is necessary to purchase goods, and the price of goods is increased by the opportunity cost of holding that money, and that is indeed when the nominal interest rate is positive. A brilliant answer will reconsider the first-order condition for consumption, which with the definitions of λ and μ takes the form

$$u_c\left(c_t\right)=\beta R_t\lambda_{t+1}+\mu_t$$

Since the first result of (iii) gives $\lambda_t=\beta R_t\lambda_{t+1}$ this becomes

$$\begin{aligned}u_c\left(c_t\right)&=\lambda_t+\mu_t\\&=\lambda_t\left(1+\frac{\mu_t}{\lambda_t}\right)\\&=\lambda_t\left(1+i_{t-1}\right).\end{aligned}$$

This clearly shows that with a binding CIA constraint, and thus positive nominal interest rate, consumption is being "taxed" by the constraint.

(v) From the expression

$$V_{\omega}\left(\omega_t,m_{t-1}\right)=\beta R_t V_{\omega}\left(\omega_{t+1},m_t\right).$$

one immediately recovers the steady-state relationship

$$R^{ss}=\frac{1}{\beta}$$

Combining this with the expression for the real interest rate, one gets

$$\frac{1}{\beta}=f_k\left(k^{ss}\right)+1-\delta$$

Hence, the capital stock is determined unrelated to monetary factors. The reason is that the capital accumulation process in the model is not distorted by the CIA constraint. The steady-state real interest rate is given by the households' subjective real interest rate $(1/\beta)$, and that is exclusively given by the net marginal product of capital. From the national account identity, $c^{ss} = f(k^{ss}) - \delta k^{ss}$, it thus follows that consumption is unaffected as well. Different long-run monetary growth rates will therefore only affect monetary factors. Higher money growth will lead to higher inflation, and thus, as the real interest rate is constant, to higher nominal interest rates. This in turn will lead to lower real money holdings.

(vi) One obvious extension is to introduce labour explicitly into the model. Then the consumption-leisure decision will be distorted as the CIA constraint does not apply to labour. A higher money growth rate could then make consumption relatively more expensive than leisure, and labour supply and output would go down. Another extension would be to introduce credit and cash goods. Different monetary policies would then shift demand between these types of good. Also, if one introduces a CIA constraint of investment, then the choice of k_t will be affected by different rates of money growth; e.g., a higher rate of money growth would now also "tax" investment leading to a lower steady-state capital stock.