Answer to the "contest question" posed at the lectures on Marhc 22

"Monetary Economics: Macro Aspects"
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Let us first repeat the question(s):

> Other approaches?
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> - What is the fundamental source of the inflation bias problem?
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> - What could another optimal preference structure look like?
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The general question "Other approaches?" referred to the issue of finding other ways (as opposed to, e.g., Rogoff's 'conservative' central banker or Walsh's linear contract) of changing the preferences of the central banker so as to address the Barro and Gordon inflation bias problem. The answer to the first sub-question is that the central bank has a preference for output above the natural rate.[1] The answer to the second one follows then intuitively:

[1] That is, in the case of quadratic preferences of output. In the case of linear preferences over output, the inflation bias arises because the central bank wants output to be as high as possible. In that case, you will realize after having read this note that the socially optimal preferences to let the central bank act with, are ones which ignore output altogether (because with linear preferences over output, output stabilization is irrelevant).

Another preference structure could be one where the central bank does **not** have a preference for output being higher than the natural rate.

This intuition is formally modelled by the winner of the contest whose e-mail is reproduced in the following (any technical or linguistic mistakes in the following are solely the responsibility of Henrik Jensen).[2]

In the Barro-Gordon model, the central bank (CB) has inflation, π, as the monetary policy instrument, and the sequence of events is:

1. Inflation expectations, π^e, are formed
2. Supply shock, e, is realized
3. Inflation is set (policy is determined)
4. Output is determined as $y = y^n + a(\pi - \pi^e) + e$

The fundamental source of the (in)famous inflation bias is the CB's ambitious goal that output should be above the natural rate y^n. I.e., $y(goal) = y^n + k$, $k > 0$. An optimal preference structure would be one that reduces the ambitions concerning output as optimal π (=0) is otherwise not time consistent. A sketch of this preference structure is as follows. The CB has the following utility function ("inverse" of the loss function also used in the literature) to maximize:

$$U = -\frac{v}{2}(y - y^n)^2 - \frac{1}{2}\pi^2,$$

where $v > 0$ is weight on output fluctuations relative to inflation fluctuations. Hence, the CB dislikes deviations from zero inflation and deviations in output from the natural rate. The private agents know the CB's maximization problem, has rational expectations ($\pi^e = \mathrm{E}(\pi)$). The supply shock has mean zero ($\mathrm{E}(e) = 0$).

Insert first the equation for y into U:

$$U = -\frac{v}{2}(a(\pi - \pi^e) + e)^2 - \frac{1}{2}\pi^2.$$

The relevant first-order condition (from $\partial U/\partial\pi = 0$) is

$$-va(a(\pi - \pi^e) + e) = \pi.$$

Marginal gain of inflation equals the marginal cost. (The second-order condition is $-va^2 - 1 < 0$ so we find a maximum.) Note that one can isolate inflation such that the CB's reaction function becomes

$$\pi = \frac{va(a\pi^e - e)}{1 + va^2}.$$

[2]It should be noted that this solution was acknowledged by Barro and Gordon themselves in their 1983 *Journal of Political Economy* article (see, e.g., Jensen, 2003, on this).

The first-order condition is anticipated by the private agents; however, they do not know the realization of e, but they know $\mathrm{E}(e) = 0$. We therefore take expectations of the first-order condition:

$$\mathrm{E}\left\{-va\left(a\left(\pi - \pi^{e}\right) + e\right)\right\} = \mathrm{E}\left(\pi\right),$$

$$-va\left(a\left(\mathrm{E}\left(\pi\right) - \mathrm{E}\left(\pi\right)\right) + \mathrm{E}\left(e\right)\right) = \mathrm{E}\left(\pi\right),$$

$$-va\left(a \cdot 0 + 0\right) = \mathrm{E}\left(\pi\right).$$

I.e., $\mathrm{E}(\pi) = 0$. This is inserted into the reaction function of the CB in order to get actual inflation (under discretion):

$$\begin{aligned} \pi &= \frac{va\left(a \cdot 0 - e\right)}{1 + va^{2}} \\ &= -\frac{va}{1 + va^{2}} e. \end{aligned}$$

A ((n) optimal) counter-cyclical policy reaction, and **without** and inflation bias. Output becomes

$$\begin{aligned} y &= y^{n} + a\left(\pi - \mathrm{E}\left(\pi\right)\right) + e \\ &= y^{n} + a\left(-\frac{va}{1 + va^{2}} e - 0\right) + e \\ &= y^{n} - \frac{va^{2}}{1 + va^{2}} e + \frac{1 + va^{2}}{1 + va^{2}} e \\ &= y^{n} + \frac{1}{1 + va^{2}} e; \end{aligned}$$

just as in the model with the inflation bias. With this preference structure optimal stabilization is thus attained, but the inflation bias is eliminated.