June 15, 2004

On the examples of exam questions in the recaps 6 and 11

Q Could you please provide guidance to the answers to the sample questions in these recaps?

A Regarding Sample question 3, Recap 6:

The idea is that with more indexation, i.e., higher β, the less effect has monetary policy on output (the degree of nominal rigidity in the economy goes down). Hence, for the case of no supply shocks, full indexation is optimal, i.e., $\beta = 1$, as the central bank in that case cannot affect output at all (wages get effectively flexible). In effect, it cannot create "inflation surprises," and therefore the inflation bias is removed. In the case of supply shocks and quadratic utility of output, $\beta = 1$ is no longer optimal, as output becomes too volatile in the absence of monetary policy intervention. I.e., the optimal value of β will be the result of weighting the benefits of a lower inflation bias against the cost of inefficient macroeconomic stabilization.

Regarding Sample question 4, Recap 11:

Regarding sub-question (ii), one simply inserts the policy rule, (3), into the inflation determination equation (2), and gets

$$\pi_t = g + (1 - n)\, v_t - h\varepsilon_t.$$

One then readily finds inflation expectations as

$$
\begin{aligned}
\mathrm{E}\left[\pi_t | I_{t-1}\right] &= \mathrm{E}\left[g + (1 - n)\, v_t - h\varepsilon_t | I_{t-1}\right] \\
&= g;
\end{aligned}
$$

i.e., expected — average — inflation equals the average growth rate of the nominal money supply. This is natural in a model where inflation equals nominal money growth plus a mean-zero disturbance and when nominal money growth is given by the rule (3), which includes g as the only deterministic component and then some mean-zero disturbances.

Regarding sub-question (iii), inflation is already found above. Output follows immediately as

$$y_t = (1 - n)\, v_t + (1 - h)\, \varepsilon_t.$$

I.e., the evolution of output depends on parameters in the monetary policy rule (n and h), so policy irrelevance does not hold in strong form; only in weak form as $\mathrm{E}[y_t | I_{t-1}] = 0$; i.e., monetary policy parameters cannot systematically affect the expected/average level of output.

Regarding the optimal policy rule, one inserts the solutions for y_t and π_t into U_t, and it immediately follows that $n = 1$ and $g = 0$ is optimal, as that parameter combination eliminates any effects of the v_t-shock, and secures the implicit inflation target of zero on average (this, one should actually be able to see without any computations). With these optimal values, we have

$$U_t = -\frac{1}{2}\left(\left(1 - h\right)\varepsilon_t\right)^2 - \frac{1}{2}\left(h\varepsilon_t\right)^2,$$

and it follows that $h = 1/2$ is optimal. This is not surprising, as the only shock creating a trade-off, the supply shock, should be evenly "spread out" on output and inflation (since the central bank puts equal weights on these variables). The expected value of the utility function becomes

$$\mathrm{E}\left[U_t | I_{t-1}\right] = -\frac{1}{4}\sigma_\varepsilon^2$$

Under transparency, inflation expectations are conditioned on period t information, and we have

$$\mathrm{E}\left[\pi_t | I_t\right] = \pi_t = g + \left(1 - n\right)v_t - h\varepsilon_t$$

and therefore:

$$y_t = \varepsilon_t.$$

I.e., with full information, inflation surprises are ruled out, and output fluctuates with the supply shocks. Monetary policy has no effect, and policy irrelevance of the strong form applies: Monetary policy cannot affect output at all, as that requires the ability to create unanticipated inflation. As a result, the optimal policy rule becomes one with $n = 1$ and $g = 0$ (as before and for the same reasons), and $h = 0$. The last result follows from the fact that the central bank with $h = 0$ minimizes the impact of the supply shock on inflation (its impact on output is "untouchable"). The expected utility in then

$$\mathrm{E}\left[U_t | I_{t-1}\right] = -\frac{1}{2}\sigma_\varepsilon^2.$$

This is *less* than under no transparency, as transparency removes the informational advantage of the central bank, which is necessary for creating efficient supply-shock stabilization. This model thus provides an argument *against* transparency (as it is modelled here).