3. Money's role with incomplete nominal adjustment*

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Abstract

Notes for the course "Monetary Economics: Macro Aspects," Spring 2006. The relevant literature behind these notes is:

Walsh (2003, Chapter 5, pp. 199-223).

1 Introductory remarks

- The flex-price models covered so far cannot account for observed short-run dynamics of real output following monetary shocks. Their long-run properties are by and large consistent with empirical observations, but since they are flexible-price models, their short-run properties very much resemble their long-run properties. Some "sand in the wheels" appears to be necessary if one shall explain the effects of money on short-run movements in output and inflation as observed in data.

- An obvious candidate is incomplete nominal adjustment in the short run. In flex-price models, money neutrality holds both in the short *and* long run. Clearly, if money neutrality fails in the short run due to incomplete nominal adjustment, the impact of money will be stronger. Moreover, lack of perfect nominal adjustment seems to be a realistic feature in the real world (think, e.g., of non-indexed nominal wage contracts that last for some period).

- Two types of incomplete adjustment are examined here:

  – One type highlights the role of *imperfect information*, while actually still allowing prices to be flexible. The lack of full information, however, makes economic agents act such that aggregate money and prices do not move proportionally: The real money stock and real output will then be affected in the short run.

  – Another type highlights the role of *sticky prices and wages*. Then, a change in the nominal money stock has real effects by definition — as in e.g. standard IS/LM models.

2 Flexible prices, yet incomplete nominal adjustment due to imperfect information

2.1 The general idea according to Milton Friedman

- A theoretical paradox was much debated in the mid 1900s. The issue was why is there seems to be long-run neutrality of money but not short-run neutrality. After the discovery of the Phillips curve though, it was not a paradox of much attention. Most believed in a stable long-run trade-off between inflation and unemployment (contradicting the quote by John Taylor reproduced in Walsh, 2003, Chapter 1). In the late 1960s, however, Milton Friedman and Edmund Phelps argued theoretically that the trade-off, at best, could only be a short-run phenomenon. In the long run, prices would adjust, so as to restore a "classical" equilibrium depending on **real** factors only (i.e., money neutrality would prevail). E.g., inflationary policies eroding the real wage could only have short-run effects, as nominal wage inflation eventually would catch up, restoring the real wage consistent with the "natural rate of employment."

- Friedman's general idea about why short-run effects then could occur was build upon the notion of: *informational problems* or *imperfect information.* If price and wage inflation suddenly rise, workers may, due to lack of full information, *perceive* that their real wage had gone up, even though price inflation had risen the most. Labor supply and output would then increase. When workers eventually "discover" the even higher price inflation (and discover that real wages had gone down), they reduce labor supply, eventually restoring equilibrium at the initial real wage (but with possible higher price and wage inflation).

- So, the central idea is that *unanticipated* changes in wages and prices create *"confusion" about relative prices* (here: the real wage), leading to changes in real economic behavior and real variables. When the "confusion" has been cleared up (over time), the initial equilibrium is restored; hence, only a short-run Phillips-curve trade off is present. To demonstrate that Friedman's intuitive idea holds in a firm, microfounded theoretical model, we turn to a presentation of such a model, namely a version of Robert Lucas's famous "Islands model."

2.2 A formal analysis of the idea: A MIU version of Lucas' "Islands model" (1972)

- The economy is made up of a number of isolated "islands." Nominal (money) shocks hit the islands, and due to *imperfect information*, economic agents cannot see whether the shocks are purely local or aggregate shocks.

- This "confusion" is very important:

  - If shocks are *known* to be aggregate, relative prices will be *known* to be unchanged, and economic behavior will **not** change
  - If shocks are *known* to be local, on the other hand, relative prices will be *known* to change, and economic behavior **will** change

- To create relevance of both local and aggregate prices for agents in the model, it is assumed that agents after each period are randomly relocated to another island. Then, expectations about future aggregates become relevant.

- Specifically, the model is a MIU model without capital but with endogenous labor supply. Inflation tax considerations are being suppressed letting agents view money transfers as proportional to money holdings. Then superneutrality will hold, and the analysis can focus solely on the real effects of various monetary policies due to imperfect information. Formally, on island i (superscript "i" denotes island/"local" variables), the budget constraint when transfers are treated as lump-sum (as in previous models) is

$$C_t^i + \frac{M_t^i}{P_t^i} = Y_t^i + T_t + \frac{1}{1+\pi_t^i}\frac{M_{t-1}^i}{P_{t-1}^i}.$$

Inflation erodes real money balances (as in previous models). But this effect is avoided when transfers are modelled as being proportional to money holdings. The budget constraint is then:

$$P_t^i C_t^i + M_t^i = P_t^i Y_t^i + \tau_t M_{t-1}^i + M_{t-1}^i,$$

or,

$$C_t^i + \frac{M_t^i}{P_t^i} = Y_t^i + \frac{T_t}{1+\pi_t^i}\frac{M_{t-1}^i}{P_{t-1}^i}, \qquad T_t \equiv 1 + \tau_t$$

Transfers are thus viewed as a **return** on real money holdings. The erosion of real money holdings due to inflation will be offset by the return due to transfers, and superneutrality holds in the long run.

- The per-period utility function of an inhabitant on island i is given by

$$u\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) = \left[\frac{\left(a\left(C_t^i\right)^{1-b} + (1-a)\left(M_t^i/P_t^i\right)^{1-b}\right)}{1-\Phi}\right]^{\frac{1-\Phi}{1-b}} - \frac{\Psi}{1-\eta}\left(1-N_t^i\right)^{1-\eta},$$

which is the same functional form used in Walsh (2003, Chapter 2). No superscript denotes economy-wide average variables (=aggregate variables).

- There will be four relevant equations of the model:
  - The production function:

$$Y_t^i = \left(N_t^i\right)^{1-\alpha}, \qquad 0 < \alpha < 1$$

  - The resource constraint on each island:

$$C_t^i = Y_t^i$$

  - The optimal consumption-leisure choice:

$$u_{1-N}\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) = \left[(1-\alpha)\frac{Y_t^i}{N_t^i}\right] u_C\left(C_t^i, M_t^i/P_t^i, N_t^i\right)$$

    The left-hand side is the utility loss from supplying more labor (in terms of the lost utility from leisure). The right-hand side is the gain measured by the extra utility from more consumption multiplied with the relative price of labour to consumption, the real wage. At an optimum, the marginal loss must equal the marginal gain.

– The optimal money demand choice:

$$u_C\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) = u_{M/P}\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) + \beta \mathrm{E}^i\left(\frac{T_{t+1}}{1+\pi_{t+1}}\right) u_C\left(C_{t+1}, M_{t+1}/P_{t+1}, 1-N_{t+1}\right)$$

The left-hand side is the utility loss of holding less real money in terms of the lost utility from consumption. The right-hand side is the gain from extra money in terms of current utility, as well as the future expanded consumption possibilities (discounted back to period t utility by β and multiplied by the relative price of money relative to consumption, $T_{t+1}/(1+\pi_{t+1})$). At an optimum, the marginal loss must equal the marginal gain(s). Note how this equation involves both local and aggregate variables, implying that both types of variables will be important for individuals' actions (as can be noted later, in the limiting case of $\beta = 0$, all local variables are determined without expectations about aggregate variables, and any confusion between local and aggregate variables will be non existent, and the model cannot generate real effects of money).

- For given money processes, these four equations will provide the solution for output, labor supply, consumption and prices. A log-linearized version is given by (5.1)-(5.3) in Walsh (2003) (where $C_t^i = Y_t^i$ is substituted in, and where λ is marginal utility of consumption).

- The central transmission channel of money is the one highlighted in the MIU model with endogenous labor (Walsh, 2003, Chapter 2). If real money balances increase, marginal utility of consumption changes. Assuming $b > \Phi$ (which was argued to be the empirically reasonable parameter restriction), $u_{C,M/P} > 0$, and more real money increases marginal utility of consumption. In consequence, labor supply and output goes up

- The stochastic process for log of **nominal** money on island i:[1]

$$m_t^i = \gamma m_{t-1}^i + \upsilon_t + u_t + u_t^i, \qquad 0 < \gamma < 1$$

  – u_t^i captures a "local" nominal disturbance on island i, with mean zero (it nets out across islands) and variance σ_i^2

  – u_t is an aggregate shock; u_t has mean zero and variance σ_u^2 (u_t and u_t^i are assumed independent)

  – υ_t is another aggregate shock

- As the u_t^i nets out on average, the aggregate nominal money process is

$$m_t = \gamma m_{t-1} + \upsilon_t + u_t.$$

[1] Log-linear models are often written up with lower-case letters; this is also the case with real models written in per capita terms, so be careful not to confuse log of variables with real variables in level

- The important informational assumptions are:
  - *Known variables* on island i: m_t^i, γm_{t-1}^i and υ_t
  - *Unknown variables*: u_t and u_t^i
  - Hence, agents can *infer the sum* $u_t + u_t^i$ from the process for nominal money on island i, but *not* each shock separately
  - This implies that an observed increase in m_t^i could be due to either local or aggregate shocks (or both). Agents are confused about what lies behind what they observe, so what should their economic response be?

- To answer this important question, note first some knife-edge properties of this flex-price model:
  - If an increase in $u_t + u_t^i$ is *known* to be caused by an increase in u_t *only*, agents *know* that the increase in aggregate nominal money will be associated with a proportional increase in all prices; hence, their expected real money balances are unchanged and no labor supply effects occur
  - If an increase in $u_t + u_t^i$ is *known* to be caused by an increase in u_t^i *only*, agents *know* that the increase in local nominal money will not be associated with a proportional increase in all prices; hence, their real money balances increase and labor supply increases
  - But in this imperfect information setting, they do *not* know what precisely causes $u_t + u_t^i$, and thus m_t^i, to fluctuate

- So what do they do? The make an *estimate* of u_t (or, equivalently, u_t^i), *given* the observation of $u_t + u_t^i$
  - I.e., they find $\mathrm{E}^i\left[u^i | u_t + u_t^i\right]$

- This is known as solving a "signal-extraction" problem, as agents *extract* information about u_t from the *signal* $u_t + u_t^i$

- One simple method is to assume that expectations about u_t are formed by applying a so-called "linear least squares projection." By adopting a linear least squares projection, agents find a estimate of u_t, denoted by $\widehat{u}_t$, which is a linear function of what is observed:
$$\widehat{u}_t = P\left(u_t + u_t^i\right).$$
This is a linear projection of u_t on $u_t + u_t^i$, where P is the estimation coefficient to be derived. This coefficient is found so as to minimize the expected, squared forecast error. It thus follows by solving:
$$\min_P \ \mathrm{E}\left[\widehat{u}_t - u_t\right]^2,$$

or,

$$\min_P \ \mathrm{E}\left[P\left(u_t+u_t^i\right)-u_t\right]^2$$

$$\min_P \ \mathrm{E}\left[P^2\left(u_t+u_t^i\right)^2+u_t^2-2P\left(u_t+u_t^i\right)u_t\right],$$

or, since shocks are independent and have zero means,

$$\min_P \ \left(P^2\mathrm{Var}\left[u_t+u_t^i\right]+\mathrm{Var}\left[u_t\right]-2P\mathrm{Cov}\left[\left(u_t+u_t^i\right)u_t\right]\right).$$

The solution for P is thus characterized by:

$$P\mathrm{Var}\left[u_t+u_t^i\right]-\mathrm{Cov}\left[\left(u_t+u_t^i\right)u_t\right]=0$$

or,

$$P=\frac{\mathrm{Cov}\left[\left(u_t+u_t^i\right)u_t\right]}{\mathrm{Var}\left[u_t+u_t^i\right]}=\frac{\mathrm{Var}\left[u_t\right]}{\mathrm{Var}\left[u_t\right]+\mathrm{Var}\left[u_t^i\right]}=\frac{\sigma_u^2}{\sigma_u^2+\sigma_i^2}\equiv\kappa<1.$$

Hence,

$$\mathrm{E}^i u_t=\widehat{u}_t=\kappa\left(u_t+u_t^i\right)$$

- This expectation has intuitive implications:
  - The higher is σ_u^2 relative to σ_i^2 the more likely it is that the change in $u_t+u_t^i$ are caused by changes in u_t rather than in u_t^i (and κ is higher)
  - Special case of $\sigma_u^2=\sigma_i^2$, we have that $\kappa=1/2$; i.e., there is a "fifty-fifty" chance that the shock is caused by u_t and u_t^i, respectively
  - In special case of $\sigma_i^2=0$ there are no local shocks, and there is perfect information; $\kappa=1$

- The log-linearized model is solved by method of undetermined coefficients, and central solutions are:

$$p_t=\gamma m_{t-1}+v_t+\frac{\kappa+K}{1+K}u_t \tag{5.5}$$

and

$$n_t=A\left(m_t-p_t\right)=A\frac{1-\kappa}{1+K}u_t \tag{5.6}$$

- Note that aggregate nominal money shocks, u_t, have *real effects* under imperfect information, $\kappa<1$. *Only* with $\kappa=1$, perfect information, will aggregate money shocks be transmitted proportionally onto the price level, leaving real money and employment/output unchanged (or in the case where agents do not care about aggregate variables; this is the case when $\beta=0$, and this case can be shown to lead to $K=\infty$, implying $\partial n_t/\partial u_t=0$ and $\partial p_t/\partial u_t=1$; cf. (5.5) and (5.6)).

- Intuition:
  - Assume that in period t, no local shocks hit. When, e.g., $u_t>0$, then on *all* islands, $u_t+u_t^i$ goes up

  - Due to imperfect information the observed increase will be believed to some extent to be caused by a local shock
  - On *all* islands, agents therefore believe their real money holdings will go up, and as that increases the marginal utility of consumption (by the assumption $b > \Phi$), *all* agents increase labor supply
  - *Aggregate* employment and output increase

- Note that the aggregate increase in activity are due to *misperceptions* about the shock. Even though it is an entirely aggregate shock, agents believe, rationally given the informational assumptions, that it contains some "local" component. Agents are therefore being "surprised" as their expectations turn out to be wrong *ex post*.One can see this clearly, since $u_t = m_t -$ $\mathrm{E}_t\left[m_t|\Gamma_{t-1}, \upsilon_t\right]$ (where Γ_{t-1} is the relevant information set), implies that employment is given by:

$$n_t = A\frac{1-\kappa}{1+K}\left[m_t - \mathrm{E}_t\left[m_t|\Gamma_{t-1}, \upsilon_t\right]\right]$$

  I.e., *unanticipated money* matters

- Model thus reconciles tension between long-run neutrality of money and short-run non-neutrality by assuming imperfect information

- The quantitative effect in calibrated model of a monetary surprise is small however

  - But, it should be emphasized that the transmission mechanism adopted here, is just one example of how misperceptions can lead to real effects (another example is presented in Chapter 6 of David Romer's (2002): "Advanced Macroeconomics").
  - . . . and a rather indirect one, arising from money's effects on marginal utility of consumption onto the labor supply choice
  - . . . we know from Walsh (2003, Chapter 2) that this effect is rather weak

- Other implication of model: The higher κ the weaker effects of monetary shocks

  - Hence, countries with high aggregate nominal variability (relative high σ_u^2 and thus high κ) should have steep Phillips curves: local shocks are known to be relatively unimportant, and aggregate money shocks will have little real effects. This empirical implication is supported by data

- Other important implication of model: Systematic monetary policy has no real effects. This is known as the *policy irrelevance hypothesis* The systematic parts of the money process, γm_{t-1} have υ_t no real effects; only the unsystematic part. This is a strong statement, which is generally not supported by data, where both unanticipated and anticipated money seems to matter. Typically, one cannot affect the *average* employment and output level by money shocks, but one can nevertheless affect the variability of real variables by a monetary policy rule with known (i.e., anticipated) coefficients. In that case:

  - One says that the policy irrelevance hypothesis only holds in *weak form*:
    * average employment and output are independent of systematic elements in the policy rule,
  - but not in *strong form*:
    * actual employment and output are independent of any systematic elements in the policy rule.

2.3 Summary

- Imperfect information is shown to be one candidate explanation to solve the tension between long-run neutrality of money and short-run non-neutrality. Lucas' islands model formalizes the misperceptions ideas put forth by Friedman.
- While a stylized set-up, the model highlights how imperfect information about aggregate versus disaggregate shocks implies that aggregate nominal shocks have real effects
  - . . . even in a model where prices are fully flexible
  - This insight is general and applies to many variants of the model
  - . . . as long as agents are "confused" and perceive that a relevant *real* variable has changed.
- The strong implications concerning potential policy irrelevance, have, however, lead researchers to search for alternative models of incomplete nominal adjustment; namely those with *sticky* prices and wages.

3 Sticky prices

- In the Lucas misperceptions model, money shocks have real effects due to imperfect information; but prices are modelled as flexible. Many economists, however, believe that short-run price flexibility is a strong assumption. I.e., some *nominal rigidity* is believed to prevail in the short run
  - If this is the case, monetary phenomena will have much stronger impact
  - E.g., with rigid prices, expansive monetary policy will likely translate directly into higher production

- E.g., with rigid wages, monetary policy affects prices, thus directly affecting the real wage and employment

- Lot of research therefore examines the consequences of rigidities in wages and/or prices for the transmission of monetary policy shocks, and for the possibility of using monetary policy to stabilize the economy against shocks.

3.1 Sticky wages in MIU model

- A log-linearized version of the stochastic MIU model of Walsh (2003, Chapter 2) with endogenous labor supply is set up. The main differences and simplifications are:

  - The central utility function parameters: $b = \Phi$ is set to 1. This means that focus is on the parameter configuration which with flexible prices caused money to be superneutral. By this assumption, one isolates the real effects of money due to sticky wages.
  - There is no capital

- Hence, any real effects of money will have to arise from nominal rigidities (otherwise the model exhibits the classical dichotomy)

- Nominal rigidity is introduced — in the simplest possible form — by assuming that the *nominal* wage for period t is set in period $t-1$. It cannot change in period t (e.g., for contractual reasons). Wage determination is materialized under a simple assumption: the nominal wage is set such that the *expected* real wage equals the expected marginal product of labor (with flexible wages, the actual real wage would equal the marginal product). With a Cobb-Douglas production function $Y_t = \mathrm{e}^{e_t} N^{1-\alpha}$, $0 < \alpha < 1$, the marginal product is given by

$$(1-\alpha)\,\mathrm{e}^{e_t} N^{-\alpha} = (1-\alpha)\, Y_t / N_t.$$

  In log-deviations from steady state, the real-wage-equal-to-the-marginal-product condition is

$$w_t - p_t = y_t - n_t \tag{5.9}$$

- To attain this condition in expectations, the nominal wage is in period $t-1$ set according to:

$$\begin{aligned} w_t^c &= \mathrm{E}_{t-1} p_t + \mathrm{E}_{t-1} y_t - \mathrm{E}_{t-1} n_t \qquad (5.15') \\ &= \mathrm{E}_{t-1} p_t + \mathrm{E}_{t-1} \omega_t^* \end{aligned}$$

  - Higher expected prices, leads to higher contract wage so as to secure the real wage "target" (= expected marginal product of labor)

– Higher $\mathrm{E}_{t-1}y_t-$ $\mathrm{E}_{t-1}n_t$, higher contract wage to "match" higher expected marginal product

- **Actual** employment for a given contract wage is therefore given by

$$\begin{aligned} n_t &= y_t - (w_t^c - p_t) \\ &= (y_t - \mathrm{E}_{t-1}y_t) + (p_t - \mathrm{E}_{t-1}p_t) + \mathrm{E}_{t-1}n_t \end{aligned}$$

where the second line follows from insertion of (5.15'). Hence, actual employment in period t is higher than period-$t-1$ expected employment if

  – $y_t > \mathrm{E}_{t-1}y_t$ as the marginal product then is higher than expected by wage-setters, which the contract wage fails to "incorporate"

  – $p_t > \mathrm{E}_{t-1}p_t$ as the actual real wage then is lower than expected, making firms hire more labor

- To further examine the implications of wage rigidity on actual output, consider the production function in logs:

$$y_t = (1-\alpha)\,n_t + e_t \tag{5.7}$$

Insert the expression for employment from above to get:

$$y_t = (1-\alpha)\left[(y_t - \mathrm{E}_{t-1}y_t) + (p_t - \mathrm{E}_{t-1}p_t) + \mathrm{E}_{t-1}n_t\right] + e_t$$

This can then be solved for output:

$$\alpha y_t = -(1-\alpha)\left[\mathrm{E}_{t-1}y_t - (p_t - \mathrm{E}_{t-1}p_t) - \mathrm{E}_{t-1}n_t\right] + e_t,$$

or, by making the "trick" of adding and subtracting $\mathrm{E}_{t-1}e_t$ and using that $\mathrm{E}_{t-1}y_t = (1-\alpha)\mathrm{E}_{t-1}n_t+$ $\mathrm{E}_{t-1}e_t$,:

$$\begin{aligned} \alpha y_t &= \mathrm{E}_{t-1}e_t - (1-\alpha)\left[\mathrm{E}_{t-1}y_t - (p_t - \mathrm{E}_{t-1}p_t) - \mathrm{E}_{t-1}n_t\right] \\ &\quad + e_t - \mathrm{E}_{t-1}e_t \\ &= \mathrm{E}_{t-1}y_t - (1-\alpha)\left[\mathrm{E}_{t-1}y_t - (p_t - \mathrm{E}_{t-1}p_t)\right] + e_t - \mathrm{E}_{t-1}e_t \end{aligned}$$

This finally yields:

$$\begin{aligned} y_t &= a\,(p_t - \mathrm{E}_{t-1}p_t) + \mathrm{E}_{t-1}y_t + (1+a)\,\varepsilon_t \\ \varepsilon_t &\equiv e_t - \mathrm{E}_{t-1}e_t, \qquad a \equiv (1-\alpha)/\alpha > 0 \end{aligned} \tag{5.17'}$$

Hence, output exceeds its expected value if $p_t > \mathrm{E}_{t-1}p_t$

  – I.e., if prices are *surprisingly* high, output is higher than expected (and/or if productivity shock is higher than expected)

- Note that this is analogous to the employment and output equations of the Lucas misperceptions model! But for a very different underlying reason:
  - In the Lucas model, nominal "surprises" cause confusion about relative prices
  - In sticky wage model, nominal "surprises" cause the actual real wage to differ from the expected, since the nominal wage are "locked in"
- Numerical example shows that the impact of a monetary shock is *much* larger that in the flexible price model. If aggregate demand is

$$y_t = m_t - p_t \tag{5.19}$$

(i.e., a simple quantity equation in logs) output is given by

$$y_t = (1 - \alpha)(m_t - \mathrm{E}_{t-1}m_t) + \varepsilon_t \tag{5.20'}$$

- Consider a one percent positive money shock

=> $(1 - \alpha)$ percent increase in output

=> With $1 - \alpha = 0.64$ (labor's share of income), this is a *significant* increase in output (much larger that what has been seen in previous models)

- Then, what about the persistent effect of money shocks found in data? It is *not* present here. After a shock has emerged, wages have adjusted fully after one period (the assumed contract length). This is also the case also of persistent shocks. So there are no real effects of money shocks above the length of the wage contract. One could imagine that the assumption of no capital accumulation could be responsible for this lack of persistence. This, however, is not the case, cf. the weak persistence even in flex-price model. So adding capital accumulation will not give rise to medium-run effects of money shocks (as long as contracts only last for one period).
- Hence, while the introduction of wage rigidity in the MIU model gives much stronger effects of monetary shocks (compared to the flex-price versions), the effects are not persistent. Also, the transmission mechanism implies a countercyclical real wage which is not in accordance with data (where real wages are acyclical or mildly procyclical). Moreover, the introduction of a nominal rigidity raises the issue: *Who* is actually setting the nominal variable? To address this final issue properly, one must explicitly model price- or wage setting agents. This, on the other hand, must involve introducing monopoly power into the model.
- This is exemplified in a model extension where intermediate goods producers are price-setters and are acting under *monopolistic competition* (they have *monopoly*

power over their own, unique, intermediate good, but *competes* with other intermediate goods producers). Note that monopoly power does not in itself create monetary non-neutrality. Therefore, prices are assumed to be fixed for some time. Also, they are assumed to be set (and reset), in an asynchronized — "staggered" — fashion; i.e., all monopolists do not set/reset prices at exact same dates. The latter feature will be seen to be a potential way of generating persistent effects of money shocks.

3.2 "Staggered" price setting and persistence of money shocks

3.2.1 Model of price setting under imperfect competition

- The economy has two sectors. A final goods sector operating under perfect competition, using intermediate goods in production process. An intermediate goods sector, where a continuum of monopolists set the price on their unique intermediate good

- The production function for final goods producers is

$$Y_t = \left[\int_0^1 Y_t(i)^q \, \mathrm{d}i\right]^{\frac{1}{q}}, \qquad 0 < q \leq 1 \tag{5.21}$$

  where Y_t is the final good and $Y_t(i)$ is intermediate good i, $i \in [0,1]$. Profits of the final goods producers:

$$P_t Y_t - \int_0^1 P_t(i) \, Y_t(i) \, \mathrm{d}i$$

  where P_t is the price on the final good, and $P_t(i)$ is the price on intermediate good i. Using the production function, profits are

$$P_t \left[\int_0^1 Y_t(i)^q \, \mathrm{d}i\right]^{\frac{1}{q}} - \int_0^1 P_t(i) \, Y_t(i) \, \mathrm{d}i$$

- Final goods producers take all prices as given (perfect competition), and choose profit-maximizing demands for intermediate goods. From the first-order condition one gets:

$$P_t \left[\int_0^1 Y_t(i)^q \, \mathrm{d}i\right]^{\frac{1-q}{q}} Y_t(i)^{q-1} = P_t(i), \qquad i \in [0,1],$$

  which essentially states that the marginal revenue of each intermediate good equals its marginal cost. This is rewritten by use of the production function:

$$P_t Y_t^{1-q} Y_t(i)^{q-1} = P_t(i) \qquad i \in [0,1],$$

  and the resulting demand functions for intermediate goods are

$$Y_t(i) = \left[\frac{P_t}{P_t(i)}\right]^{\frac{1}{1-q}} Y_t \qquad i \in [0,1]. \tag{5.22}$$

Note that the demand of intermediate good i is falling in its the relative price ($1/(1-q)$ is elasticity of substitution between intermediate factor inputs). Tedious algebra shows that zero profits in the final goods sector requires (see Appendix B)

$$P_t = \left[\int_0^1 P_t(i)^{\frac{q}{q-1}} \mathrm{d}i\right]^{\frac{q-1}{q}}.$$

- Each intermediate goods producer has the profit function

$$\begin{aligned}\pi_t(i) &= P_t(i) Y_t(i) - r_t K_t(i) - W_t L_t(i) \\ &= [P_t(i) - P_t V_t] Y_t(i)\end{aligned}$$

where V_t is the minimized real costs of producing one unit of the good. $P_t V_t$ thus denotes the nominal marginal cost of $Y_t(i)$. As mentioned, the intermediate producer has monopoly power, and it sets its price so as to maximize profits, knowing the demand function for its product. I.e., it maximizes

$$\pi_t(i) = [P_t(i) - P_t V_t]\left[\frac{P_t}{P_t(i)}\right]^{\frac{1}{1-q}} Y_t$$

with respect to $P_t(i)$ (taking as given P_t; thereby the term monopolistic competition). From the first-order condition one readily recover a familiar monopoly-theory result:

$$P_t(i) = \frac{1}{q} P_t V_t, \qquad \frac{1}{q} > 1 \tag{5.25}$$

The price is set as a *mark up*, $1/q > 0$, over marginal costs.[2]

- Note that imperfect competition does not in itself provide monetary non-neutrality (but $q < 1$ implies inefficiently low production and too high prices): In a symmetric equilibrium, $P_t(i) = P_t$, $L_t(i) = L_t$ and proportional changes in prices and nominal wages have no real effects.

- Assume therefore that price setters cannot adjust prices freely (this may even be optimal under so-called "menu cost" arguments; i.e., there are some small costs of changing prices, which make it optimal to leave prices unchanged).

3.2.2 The model with sticky prices and staggered price setting

- It is now assumed that intermediate goods producers set a price, which has a duration of two periods. Moreover, prices are assumed to be set in a *staggered* fashion: Half of the producers set prices in a given period, the other half in the next period:

  - Half sets prices in t, $t+2$, $t+4$, $t+6$, ...

[2] Note that $q \to 1$ implies that intermediate goods become perfect substitutes. Hence, there is no monopoly power, and the mark up becomes 1. This corresponds to the case of perfect competition in the intermediate goods sectors.

  - Other half sets prices in $t+1$, $t+3$, $t+5$, $t+7$, ...
  - Notationally, $\overline{P}_{t+j}$ denotes a price fixed for periods $t+j$ and $t+j+1$

- Note the following important result:

  - Any monopolist i setting prices in a period, cares about the *relative* price of its product, $P_t(i)/P_t$ in the period it sets the price *and* the expected next-period relative price
  - Two-period profit-maximization will therefore lead to a price-setting rule depending on *current and expected future* aggregate prices

- From the first-order condition for two-period profit maximization, one gets the optimal price set in t (in log deviations from steady state) (see Appendix C) :

$$\overline{p}_t = \frac{1}{2}\left(p_t + \mathrm{E}_t p_{t+1}\right) + \frac{1}{2}\left(v_t + \mathrm{E}_t v_{t+1}\right) \tag{5.30}$$

  Indeed, $\overline{p}_t$ depends on current aggregate prices and real marginal costs, as well as *expected future* values of these variables.

- Now note that the aggregate price is by definition a function of the prices set in the period, as well as the prices set in the previous period:

$$p_t = \frac{1}{2}\left(\overline{p}_t + \overline{p}_{t-1}\right)$$

  Therefore, the optimal price in period t will depend on past period's optimal prices and expected future optimal prices:

$$\overline{p}_t = \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\mathrm{E}_t\overline{p}_{t+1} + \left(v_t + \mathrm{E}_t v_{t+1}\right)$$

  This will, as will hopefully be clear in a moment, account for potential *gradual adjustment* of aggregate prices, and thus potential persistence of monetary shocks

- Implications of this form of price rigidity is best analyzed under very simple assumptions (special cases of the MIU equations):

  - First, real marginal costs are assumed to be linearly related to output:

$$v_t = \gamma y_t, \qquad \gamma > 0$$

  - Secondly, aggregate demand is characterized by a quantity equation:

$$m_t - p_t = y_t$$

  - Finally, nominal money supply follows a random walk:

$$\mathrm{E}_t m_{t+1} = m_t$$

- With these assumptions, one can solve for $\overline{p}_t$ by the method of undetermined coefficients, in order to find (see Appendix D) :

$$\overline{p}_t = a\overline{p}_{t-1} + (1-a)\, m_t, \qquad a = \frac{1-\sqrt{\gamma}}{1+\sqrt{\gamma}}, \qquad |a| < 1 \tag{5.32}$$

- The associated solution for aggregate prices, remembering $p_t = (1/2)\left(\overline{p}_t + \overline{p}_{t-1}\right)$, is

$$p_t = ap_{t-1} + \frac{1}{2}(1-a)(m_t + m_{t-1})$$

  Hence, for $|a| \neq 0$ aggregate prices exhibit *inertia*, implying *prolonged* real effects of a monetary shock

- An important parameter for the degree of inertia is γ, the sensitivity of real marginal costs to output (could depend inversely on labor supply wage elasticity)

  – Low sensitivity is often labelled a situation with high degree of *real rigidity*

- For $\gamma = 1$ highly sensitive real marginal costs, and $a = 0$, so aggregate price adjustment is immediate; real effect of money shock dies out after one period

  – With $\gamma = 1$, a one percent increase in nominal money will for given prices increase output by one percent. This increases real marginal costs by one percent, and all firms who can adjust prices adjust by raising prices by one percent. Aggregate prices will raise by 0.5 percent, and output raises in equilibrium by 0.5 percent (see Figure 5.1 in Walsh, 2003)

  – In next period, when all firms have had the opportunity to adjust prices, the aggregate price adjustment is complete, and $y = 0$ again

- With $\gamma < 1$, on the other hand, there are less sensitive real marginal costs, $a > 0$, and aggregate price adjustment is inertial; the real effect of a money shock is persistent

  – Hence, even after all firms have had the opportunity to adjust prices, aggregate price adjustment is incomplete, and y has not returned to steady-state

  – Source of inertia is the interplay between prices' dependence on past prices and expected future prices

  – Those adjusting in period t do not adjust fully to a monetary shock as their real marginal cost does not rise sufficiently. Hence, aggregate prices will adjust by less than half of the change in the money supply

  – This will feed into next period's price adjustment, which will be dampened as it depends on the past period's dampened adjustment

- This is known by current price setters, and given that expected next-period prices are important, equilibrium adjustment of current prices are further reduced
- This process of less than full adjustment in each period continues into the future, making adjustment only gradual
- See Figure 5.1 for "$a = 0.61$" (different "a" than in wage contract model!)

• This case of staggered price-setting therefore provides a framework for generating persistent real effects of monetary shocks. The parameter γ, however, has to be rather low; i.e., a high degree of real rigidity is necessary

- This could be a situation where real labor market imperfections (efficiency-wage considerations, unionized wage setting), implies a rather rigid real wage. Then, real marginal costs will be rather insensitive to output
- So, *high degree of real rigidity* (low γ) provides a case for *high degree of persistent nominal rigidity*

3.3 Summary

• Nominal rigidities give rise to substantial effects of monetary shocks as compared with full-information flexible price models and, to some extent, misperceptions models

• One-period nominal wage (or price) contracts give rise to no persistence of monetary shocks (in a dynamic general equilibrium model *with* capital accumulation; extremely little persistence)

• Modelling sticky price (or wage) setting calls for models of monopoly power

- Monopoly power in itself not a source of non-neutrality of money
- Particular price-setting structure important for the transmission of money shocks
- With staggered price setting, persistent effect of money shocks; in particular with high degrees of real rigidity

Appendix

A Deriving the first-order conditions in the Island model

The agent on island i has the per-period utility function

$$u\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right),$$

and wishes to maximize the expected discounted sum of this. As noted, the relevant budget constraint, when transfers are viewed as being proportional to real money holdings, is (where $Y_t^i = \left(N_t^i\right)^{1-\alpha}$ has been substituted in):

$$C_t^i + \frac{M_t^i}{P_t^i} = \left(N_t^i\right)^{1-\alpha} + \frac{T_t}{1+\pi_t^i}\frac{M_{t-1}^i}{P_{t-1}^i}.$$

Let λ^i denote the Lagrange multiplier on the budget constraint. We then get the following necessary first-order conditions for optimal choices of C_t^i, M_t^i/P_t^i and N_t^i:

$$u_C\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) - \lambda_t^i = 0, \tag{1}$$

$$u_{M/P}\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) - \lambda_t^i + \beta \mathrm{E}_t^i \lambda_{t+1} \frac{T_{t+1}}{1+\pi_{t+1}} = 0, \tag{2}$$

$$-u_{1-N}\left(C_t^i, M_t^i/P_t^i, N_t^i\right) + \lambda_t^i (1-\alpha)\left(N_t^i\right)^{-\alpha} = 0, \tag{3}$$

Now note that (1) and (2) can immediately be combined into

$$u_{1-N}\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) = \left[(1-\alpha)\frac{Y_t^i}{N_t^i}\right] u_C\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right),$$

which is the intratemporal consumption-leisure optimality condition.

Then note that equations (1) and (2) can be combined into

$$\begin{aligned} u_C\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) &= u_{M/P}\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) \\ &\quad + \beta \mathrm{E}_t^i \frac{T_{t+1}}{1+\pi_{t+1}} u_C\left(C_{t+1}, M_{t+1}/P_{t+1}, 1-N_{t+1}\right), \end{aligned}$$

which is the condition for optimal money demand.

B Derivation of P_t consistent with zero profits

Profits are zero if revenues equals total costs, i.e., if

$$P_t Y_t = \int_0^1 P_t(i)\, Y_t(i)\, \mathrm{d}i.$$

Then replace $Y_t(i)$ with the demand function, (5.22) to get

$$P_t Y_t = \int_0^1 P_t(i) \left[\frac{P_t}{P_t(i)}\right]^{\frac{1}{1-q}} Y_t \mathrm{d}i.$$

We immediately see that Y_t drops out such that

$$P_t = \int_0^1 P_t(i) \left[\frac{P_t}{P_t(i)}\right]^{\frac{1}{1-q}} \mathrm{d}i.$$

The condition is then rewritten as

$$P_t^{1-\frac{1}{1-q}} = \int_0^1 P_t(i)^{1-\frac{1}{1-q}} \mathrm{d}i,$$

$$P_t^{\frac{q}{q-1}} = \int_0^1 P_t(i)^{\frac{q}{q-1}} \mathrm{d}i,$$

and finally:

$$P_t = \left[\int_0^1 P_t(i)^{\frac{q}{q-1}} \mathrm{d}i\right]^{\frac{q-1}{q}}.$$

C Derivation of expression (5.30)

The expression is an approximation of (5.29) where discounting is ignored (i.e., $R_t = 1$). One therefore has to take a first-order Taylor approximation of the expression

$$\overline{P}_t = \frac{\mathrm{E}_t\left[P_t^{\theta} V_t Y_t + P_{t+1}^{\theta} V_{t+1} Y_{t+1}\right]}{q\mathrm{E}_t\left[P_t^{\frac{1}{1-q}} Y_t + P_{t+1}^{\frac{1}{1-q}} Y_{t+1}\right]},$$

around the steady state (where variables are constants; i.e., the price level is constant and thus there is assumed no inflation). The left-hand-side needs no approximation. Now, let an undated variable indicate its steady-state value. A first-order Taylor expansion of the right-hand-side then yields:

$$\frac{\mathrm{E}_t\left[P_t^{\theta}V_tY_t+P_{t+1}^{\theta}V_{t+1}Y_{t+1}\right]}{q\mathrm{E}_t\left[P_t^{\frac{1}{1-q}}Y_t+P_{t+1}^{\frac{1}{1-q}}Y_{t+1}\right]}$$

$$\approx \frac{2P^{\theta}VY}{q2P^{\frac{1}{1-q}}Y}+\frac{\theta P^{\theta-1}VY\left(2qP^{\frac{1}{1-q}}Y\right)-q\frac{1}{1-q}P^{\frac{1}{1-q}-1}Y\left(2P^{\theta}VY\right)}{\left(q2P^{\frac{1}{1-q}}Y\right)^2}\left(\overline{P}_t-P\right)$$

$$+\frac{\theta P^{\theta-1}VY\left(2qP^{\frac{1}{1-q}}Y\right)-q\frac{1}{1-q}P^{\frac{1}{1-q}-1}Y\left(2P^{\theta}VY\right)}{\left(q2P^{\frac{1}{1-q}}Y\right)^2}\mathrm{E}_t\left(\overline{P}_{t+1}-P\right)$$

$$+\frac{P^{\theta}Y}{q\left(2P^{\frac{1}{1-q}}Y\right)}\left(V_t-V\right)+\frac{P^{\theta}Y}{q\left(2P^{\frac{1}{1-q}}Y\right)}\mathrm{E}_t\left(V_{t+1}-V\right)$$

$$+\frac{P^{\theta}V\left(2qP^{\frac{1}{1-q}}Y\right)-qP^{\frac{1}{1-q}}\left(2P^{\theta}VY\right)}{q\left(2P^{\frac{1}{1-q}}Y\right)^2}\left(Y_t-Y\right)$$

$$+\frac{P^{\theta}V\left(2qP^{\frac{1}{1-q}}Y\right)-qP^{\frac{1}{1-q}}\left(2P^{\theta}VY\right)}{q\left(2P^{\frac{1}{1-q}}Y\right)^2}\mathrm{E}_t\left(Y_{t+1}-Y\right).$$

This expression is reduced to

$$\frac{\mathrm{E}_t\left[P_t^{\theta}V_tY_t+P_{t+1}^{\theta}V_{t+1}Y_{t+1}\right]}{\mathrm{E}_t\left[P_t^{\frac{1}{1-q}}Y_t+P_{t+1}^{\frac{1}{1-q}}Y_{t+1}\right]}$$

$$\approx \frac{1}{q}P^{\theta-\frac{1}{1-q}}V+2qV\frac{\theta P^{\theta-1}\left(P^{\frac{1}{1-q}}\right)-\frac{1}{1-q}P^{\frac{1}{1-q}-1}\left(P^{\theta}\right)}{\left(q2P^{\frac{1}{1-q}}\right)^2}\left(\overline{P}_t-P\right)$$

$$+2qV\frac{\theta P^{\theta-1}\left(P^{\frac{1}{1-q}}\right)-\frac{1}{1-q}P^{\frac{1}{1-q}-1}\left(P^{\theta}\right)}{\left(q2P^{\frac{1}{1-q}}\right)^2}\mathrm{E}_t\left(\overline{P}_{t+1}-P\right)$$

$$+\frac{1}{2q}P^{\theta-\frac{1}{1-q}}\left(V_t-V\right)+\frac{1}{2q}P^{\theta-\frac{1}{1-q}}\mathrm{E}_t\left(V_{t+1}-V\right)$$

Now use that $\theta = (2-q)/(1-q)$ in order to get

$$
\begin{aligned}
&\frac{\mathrm{E}_t\left[P_t^{\theta}V_tY_t + P_{t+1}^{\theta}V_{t+1}Y_{t+1}\right]}{\mathrm{E}_t\left[P_t^{\frac{1}{1-q}}Y_t + P_{t+1}^{\frac{1}{1-q}}Y_{t+1}\right]} \\
\approx\ & \frac{1}{q}P^{\theta-\frac{1}{1-q}}V + 2qV\frac{\frac{2-q}{1-q}P^{\frac{1}{1-q}}\left(P^{\frac{1}{1-q}}\right) - \frac{1}{1-q}P^{\frac{1}{1-q}-1}P^{\frac{2-q}{1-q}}}{\left(q2P^{\frac{1}{1-q}}\right)^2}\left(\overline{P}_t - P\right) \\
&+2qV\frac{\frac{2-q}{1-q}P^{\frac{1}{1-q}}\left(P^{\frac{1}{1-q}}\right) - \frac{1}{1-q}P^{\frac{1}{1-q}-1}P^{\frac{2-q}{1-q}}}{\left(q2P^{\frac{1}{1-q}}\right)^2}\mathrm{E}_t\left(\overline{P}_{t+1} - P\right) \\
&+\frac{1}{2q}P^{\theta-\frac{1}{1-q}}\left(V_t - V\right) + \frac{1}{2q}P^{\theta-\frac{1}{1-q}}\mathrm{E}_t\left(V_{t+1} - V\right),
\end{aligned}
$$

and thus

$$
\begin{aligned}
\frac{\mathrm{E}_t\left[P_t^{\theta}V_tY_t + P_{t+1}^{\theta}V_{t+1}Y_{t+1}\right]}{\mathrm{E}_t\left[P_t^{\frac{1}{1-q}}Y_t + P_{t+1}^{\frac{1}{1-q}}Y_{t+1}\right]} \approx\ & \frac{1}{q}P^{\theta-\frac{1}{1-q}}V + \frac{1}{2q}V\left(\overline{P}_t - P\right) \\
&+\frac{1}{2q}V\mathrm{E}_t\left(\overline{P}_{t+1} - P\right) \\
&+\frac{1}{2q}P^{\theta-\frac{1}{1-q}}\left(V_t - V\right) + \frac{1}{2q}P^{\theta-\frac{1}{1-q}}\mathrm{E}_t\left(V_{t+1} - V\right),
\end{aligned}
$$

We thus have the approximated relationship

$$
\begin{aligned}
\overline{P}_t =\ & \frac{1}{q}P^{\theta-\frac{1}{1-q}}V + \frac{1}{2q}V\left(\overline{P}_t - P\right) + \frac{1}{2q}V\mathrm{E}_t\left(\overline{P}_{t+1} - P\right) \\
&+\frac{1}{2q}P^{\theta-\frac{1}{1-q}}\left(V_t - V\right) + \frac{1}{2q}P^{\theta-\frac{1}{1-q}}\mathrm{E}_t\left(V_{t+1} - V\right),
\end{aligned}
$$

or as $\overline{P} = \frac{1}{q}P^{\theta-\frac{1}{1-q}}V = \frac{1}{q}PV$:

$$
\begin{aligned}
\overline{P}_t - \overline{P} =\ & \frac{1}{2q}V\left(\overline{P}_t - P\right) + \frac{1}{2q}V\mathrm{E}_t\left(\overline{P}_{t+1} - P\right) \\
&+\frac{1}{2q}P\left(V_t - V\right) + \frac{1}{2q}P\mathrm{E}_t\left(V_{t+1} - V\right),
\end{aligned}
$$

Divide the left-hand side by $\overline{P}$ and the right-hand side by $\frac{1}{q}PV$, and the expression becomes

$$
\begin{aligned}
\frac{\overline{P}_t - \overline{P}}{\overline{P}} =\ & \frac{1}{2}\frac{\overline{P}_t - P}{P} + \frac{1}{2}\mathrm{E}_t\left(\frac{\overline{P}_{t+1} - P}{P}\right) \\
&+\frac{1}{2}\frac{V_t - V}{V} + \frac{1}{2}\mathrm{E}_t\left(\frac{V_{t+1} - V}{V}\right).
\end{aligned}
$$

Letting lower-case letters denote percentage deviations from steady state one recovers

$$\overline{p}_t = \frac{1}{2}\left(p_t + \mathrm{E}_t p_{t+1}\right) + \frac{1}{2}\left(v_t + \mathrm{E}_t v_{t+1}\right),$$

which is equation (5.30).

D Derivation of (5.32)

With the assumptions that $v_t = \gamma y_t$ and $m_t - p_t = y_t$, the expression

$$\overline{p}_t = \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\mathrm{E}_t\overline{p}_{t+1} + (v_t + \mathrm{E}_t v_{t+1})$$

becomes

$$\overline{p}_t = \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\mathrm{E}_t\overline{p}_{t+1} + \gamma\left[m_t - p_t + \mathrm{E}_t m_{t+1} - \mathrm{E}_t p_{t+1}\right].$$

One then uses that the aggregate price level is an average of prices set in the current and last period; i.e.,

$$p_t = \frac{1}{2}\left(\overline{p}_{t-1} + \overline{p}_t\right).$$

This is inserted into the equation above:

$$\begin{aligned}\overline{p}_t &= \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\mathrm{E}_t\overline{p}_{t+1} \\ &\quad + \gamma\left[m_t - \frac{1}{2}\left(\overline{p}_{t-1} + \overline{p}_t\right) + \mathrm{E}_t m_{t+1} - \mathrm{E}_t\frac{1}{2}\left(\overline{p}_t + \overline{p}_{t+1}\right)\right]\end{aligned}$$

Solving for $\overline{p}_t$ yields:

$$\begin{aligned}\overline{p}_t\left(1 + \frac{\gamma}{2} + \frac{\gamma}{2}\right) &= \frac{1}{2}\overline{p}_{t-1} + \frac{1}{2}\mathrm{E}_t\overline{p}_{t+1} \\ &\quad + \gamma\left[m_t - \frac{1}{2}\overline{p}_{t-1} + \mathrm{E}_t m_{t+1} - \mathrm{E}_t\frac{1}{2}\overline{p}_{t+1}\right]\end{aligned}$$

This immediately yields

$$\overline{p}_t = \frac{1}{2}\frac{1-\gamma}{1+\gamma}\left[\overline{p}_{t-1} + \mathrm{E}_t\overline{p}_{t+1}\right] + \frac{\gamma}{1+\gamma}\left[m_t + \mathrm{E}_t m_{t+1}\right], \tag{5.31}$$

which is a second-order difference equation in $\overline{p}_t$. This can be solved by the method of undetermined coefficients. One makes the conjecture that the solution of (5.31) takes the form

$$\overline{p}_t = a\overline{p}_{t-1} + bm_t.$$

Forward this one period and take period t expectations:

$$\mathrm{E}_t\overline{p}_{t+1} = a\overline{p}_t + bm_t,$$

where it has been used that money supply follows a random walk ($\mathrm{E}_t m_{t+1} = m_t$). Insert this expression in (5.31) (and use again that $\mathrm{E}_t m_{t+1} = m_t$):

$$\overline{p}_t = \frac{1}{2}\frac{1-\gamma}{1+\gamma}\left[\overline{p}_{t-1} + a\overline{p}_t + bm_t\right] + \frac{\gamma}{1+\gamma}2m_t,$$

and solve for $\overline{p}_t$:

$$\overline{p}_t\left[1 - \frac{a}{2}\frac{1-\gamma}{1+\gamma}\right] = \frac{1}{2}\frac{1-\gamma}{1+\gamma}\left[\overline{p}_{t-1} + bm_t\right] + \frac{\gamma}{1+\gamma}2m_t,$$

$$\overline{p}_t = \frac{1-\gamma}{2(1+\gamma) - a(1-\gamma)}\overline{p}_{t-1} + \frac{(1-\gamma)b + 4\gamma}{2(1+\gamma) - a(1-\gamma)}m_t.$$

This verifies the form of the conjecture, and shows that the undetermined coefficients must satisfy

$$\begin{aligned} a &= \frac{1-\gamma}{2(1+\gamma) - a(1-\gamma)}, \\ b &= \frac{(1-\gamma)b + 4\gamma}{2(1+\gamma) - a(1-\gamma)}. \end{aligned}$$

The first determines a from

$$-a^2(1-\gamma) + 2(1+\gamma)a - (1-\gamma) = 0,$$

or from

$$a^2 - \frac{2(1+\gamma)}{(1-\gamma)}a + 1 = 0,$$

as stated by Walsh (2003). The coefficient b can then be determined subsequently. Indeed, it is easy to verify that $b = 1 - a$.

E Key concepts you should know

Imperfect information as a source of non-neutrality of money

- Friedman's informal hypothesis
- The essentials of the Island model: Imperfect information
- Agents cannot distinguish whether money shocks are aggregate or local
  - If shocks are known to be aggregate they have no real effect
  - If shocks are known to be local, they have real effects
- The signal extraction problem
- The linear least squares projection
- The role of the relative variances of local and aggregate shocks in agents' estimation of shock
- The real effects of aggregate shocks under imperfect information
- The role of unanticipated money
- The policy irrelevance hypothesis
  - Weak and strong form

One-period sticky wages and prices

- Simple one-period nominal wage contracts in MIU model
- The employment effects of higher-than-expected prices
- Real effects of unanticipated money
- Lack of persistence of money shocks with one-period wage rigidity

Staggered price setting

- Model of imperfect competition in intermediate goods market
- Prices as mark-up over marginal costs
- Imperfect competition in itself does not create non-neutrality of money
- Sticky, and staggered price setting
- The importance of current prices and expected future prices for each firms' pricing behavior
- Aggregate implications:
  - Current prices depend on past prices and expected future prices
  - A role for gradual adjustment of prices and thus persistent effects of money shocks
- Real rigidity versus nominal rigidity
  - High degree of real rigidity increases persistence