5. Operating Procedures, Interest Rates and Monetary Policy*

Henrik Jensen
Department of Economics
University of Copenhagen

April 19, 2006

Abstract

Notes for the course "Monetary Economics: Macro Aspects," Spring 2006. The relevant literature behind these notes is:

Walsh (2003, Chapter 9, pp. 429-448);
Walsh (2003, Chapter 10, pp. 499-512).
Recommended reading (not required): Taylor (1993); Clarida et al. (1998); Walsh (2003, Chapter 10, pp. 473-480; 488-499).

1 Introductory remarks

In models so far, the choice variables, or instruments, of the central bank has interchangeably been the nominal money supply, the nominal interest rate, or even the inflation rate. This is permissible when the models are used to getting various points through in the most transparent manner. In the real world, however, things are more complicated and not so innocent. I.e., it is of relevance to ask what is the actual policy instrument of a central bank? Moreover, what is the *best* instrument to use given the uncertainties that inevitable are a part of policymaking? The latter question amounts to asking which *operating procedures* a central bank should use.

In reality, the central bank can only exercise close control over the money base and the (very) short term nominal interest rate. Even though the money base is the actual instrument, the central bank can very well behave as if the market nominal interest rate is its instrument: It will adjust the monetary base to attain some desired value of the nominal interest rate. This would be an example of an interest rate target procedure. Alternatively, it could adjust the money base to keep stable some broader money aggregate (e.g., M1). This would be an example of a money supply target procedure. But what is best to do?

General lesson from the model examples presented here:

- It depends on the ultimate goals of monetary policy (i.e., what are the relevant policy objectives?)
- It depends on the relative variability of the shocks hitting the economy when shocks cannot be observed directly
- It depends on which variables provide good information about the ultimate goal variables

Moreover, the exercises reveal that changes in monetary aggregates (broad money supply, nominal interest rates) tell little about deliberate monetary policy shifts if one does not take into account under which operating procedures the central bank acts (cf. the identification problems in the empirical VAR literature)

In real life today, most central banks use the interest rate as the monetary policy instrument (i.e., acts under an interest rate target procedure). This can be theoretically justified by use of the model examples, but it introduces a set of important issues.

- Theoretically, interest targeting procedures can give problems with price level determination, and in principle pave the roads for self-fulfilling economics fluctuations.
- Moreover, with the nominal money stock "out of the picture," long-term interest rates become more important, and thus the link between the short interest rate (the policy instrument) and the long interest rates (the rate typically affecting demand) becomes relevant to examine.

- Finally, an important issue is how does various interest rate policy rules perform in terms of stabilizing the economy (this is examined both from a positive and normative point of view, and some international empirical evidence on interest rate rules are reviewed.

2 Operating procedures and choice of monetary policy instrument

2.1 The Poole (1970) model of instrument choice

- The model due to William Poole highlights in a very simple way how the nature of economic disturbances is crucial when one decides on the operating procedure in monetary policymaking. More specifically, the model considers the case where the choice is between using the money supply or the nominal interest rate as the policy instrument. The condition determining which operating procedure to prefer, is the seen to depend in an intuitive way on the relative variances of shocks affecting money and goods markets.

- The model is a simple one-period IS/LM model in logarithms (the price level is $p_t = 0$ by normalization). The IS and LM curves are, respectively,

$$y_t = -\alpha i_t + u_t, \qquad \alpha > 0 \tag{9.1}$$

$$m_t = -c i_t + y_t + v_t, \qquad c > 0 \tag{9.2}$$

  Note that in a usual (y, i)-space, the slope of the LM curve is $1/c$, and the slope of IS curve is $-1/\alpha$.

- The shocks u_t and v_t are mean-zero, independent shocks with variances σ_u^2 and σ_v^2, respectively. It is assumed that the objective of policymaking is very simple. It is the aim to minimize output variance

$$\mathrm{E}\left[y_t\right]^2. \tag{9.3}$$

  (Note that it is subsumed that all variables are log-deviations from a zero equilibrium, i.e., $\mathrm{E}[y_t] = 0$.) The monetary policy under either operating target is conducted before shocks u_t and v_t hits

- When money supply is instrument, y_t is solved in terms of m_t (graphically, this is output at the cross of the IS and LM curve)

$$y_t = \frac{\alpha m_t + c u_t - \alpha v_t}{\alpha + c}$$

The optimal policy is then simply $m_t = 0$. The associated output variance, and, thus, value of the objective function:

$$\mathrm{E}_m [y_t]^2 = \frac{c^2\sigma_u^2 + \alpha^2\sigma_v^2}{(\alpha + c)^2}, \tag{9.4}$$

where E_m denotes expectations under a money supply operating target.

- When the interest rate is the instrument, then IS curve gives output independent of the money market:

$$y_t = -\alpha i_t + u_t.$$

The optimal policy is then simply $i_t = 0$. Actual output is then,

$$y_t = u_t$$

and the associated output variance is

$$\mathrm{E}_i [y_t]^2 = \sigma_u^2, \tag{9.5}$$

where E_i denotes expectations under an interest rate operating target.

- The two operating procedures can now be ranked, by comparing their implied output variance. An interest rate operating procedure is preferred iff

$$\mathrm{E}_i [y_t]^2 < \mathrm{E}_m [y_t]^2,$$

which from the solutions is

$$\sigma_u^2 < \frac{c^2\sigma_u^2 + \alpha^2\sigma_v^2}{(\alpha + c)^2}$$

or

$$\left(1 + \frac{2c}{\alpha}\right)\sigma_u^2 < \sigma_v^2$$

- Hence, in favor of interest rate targeting procedure is
  - High money demand volatility. In that case, it is desirable to keep the nominal interest rate stable, as the money market shock will then not affect output at all. Instead, the money supply will adjust to secure money market equilibrium.
  - Low aggregate demand volatility. In that case, output will not fluctuate so much under an interest rate operating procedure. Under a money supply target, a goods demand shock will be partly stabilized by the associate interest rate increase. So, for an interest rate target to be preferable, goods demand shock must be relative unimportant, i.e., the variance of u_t must be small.
  - A flat IS curve (high α). In that case, money market shocks moving the LM curve, would have relatively large output effects (e.g., an increase in money demand puts upward pressure on the interest rate, causing a relatively large drop in output when α is large). Thus, it is favorable to insulate the economy from money market shocks by adopting an interest rate operating procedure.

– A steep LM curve (low c). In that case, money market shocks requires large interest changes to restore money market equilibrium, and the result is high output volatility. Again, an interest rate target would be desirable.

- The model thus provides a simple example of how relative variances of macroeconomic shocks matter for optimal choice of instrument. Taking the model at face value, it suggests that when money market volatility is high relative to goods market volatility, then one should use the nominal interest rate as an instrument. This could be a simple way of explaining why most modern central banks adopts an interest rate targeting procedure. It is because financial markets are relatively more volatile than goods markets.

2.2 Extension of the basic Poole model: Monetary base as potential instrument

- Note that central banks control the money base, but not, say, broader aggregates like M1. Does this overturn the basic message(s) of the Poole model? Fortunately not, as will we demonstrated here. The model is extended with an equation for the determination of money supply (M1) as a function of the instrument; the monetary base:

$$m_t = b_t + hi_t + \omega_t, \qquad h > 0 \tag{9.7}$$

Here, b_t is the money base. Note that $m_t - b_t$ is the (log) money multiplier (in levels the ration of the money supply to base money). ω_t is mean-zero money multiplier shock. Equation (9.7) posits that the money multiplier is increasing in the interest rate. This can be rationalized by assuming that banks for a higher interest rate want to lend more and/or that consumers want to hold less cash. Hence, expanding deposits are possible leading to an increase in M1 for given M0 (the monetary base).

- Note that with this extension, we now have an "LM curve" in written in b_t:

$$b_t = -(c+h)\, i_t + y_t + v_t - \omega_t$$

Accordingly, the distinction in instrument choice will be between a money base operating target and an interest rate operating target.

- The interest rate as an instrument gives solution as before; $\mathrm{E}_i [y_t]^2 = \sigma_u^2$

- The money base as instrument (again optimal to set $b_t = 0$), yields output as:

$$y_t = \frac{(c+h)\, u_t - \alpha v_t + \alpha \omega_t}{\alpha + c + h},$$

and the associated output variance is

$$\mathrm{E}_b [y_t]^2 = \frac{(c+h)^2 \sigma_u^2 + \alpha^2 \sigma_v^2 + \alpha^2 \sigma_\omega^2}{(\alpha + c + h)^2}.$$

The operating procedures can then be ranked, and the interest operating target is preferred iff:

$$\left[1 + \frac{2(c+h)}{\alpha}\right]\sigma_u^2 < \sigma_v^2 + \sigma_\omega^2$$

- This condition is a reinforcement of the simple Poole result: More volatility on money market/financial markets (here represented by shocks v_t and ω_t) makes a base operating target *less* attractive. Again, this may explain why real-life central banks are using interest rate operating procedures as money demand is unstable and/or financial markets are volatile. Note, however, that any normative result depends strongly on the assumed objectives of the monetary policymaker. Here, only output stability is favoured. Obviously, other objective would give other results.

2.3 Policy rules and information

- As mentioned, the monetary base is the *de facto* policy instrument (or a very short-term nominal interest rate), but one may still under an interest operating procedure think of the nominal interest rate as an instrument. In the evaluation of various procedures, the Poole analysis took an "either or" perspective. Something "in between" may, however, be optimal. This is illustrated by a money base policy rule, where the money base responds to the observed nominal interest rate:

$$b_t = \mu i_t \tag{9.8}$$

  We get various operating procedures as special cases:

  - $\mu = 0$: A base money operating procedure
  - $\mu = -h$: A money supply operating procedure (plug (9.8) into (9.7), and see the result)
  - $\mu \to \infty$: An interest rate operating procedure

- The solution for output with this base rule:

$$y_t = \frac{(c+\mu+h)\,u_t - \alpha(v_t - \omega_t)}{c+h+\mu+h},$$

  and the associated variance is

$$\mathrm{E}_\mu[y_t]^2 = \frac{(c+\mu+h)^2\sigma_u^2 + \alpha^2(\sigma_v^2+\sigma_\omega^2)}{(c+h+\mu+h)^2}$$

- What is the optimal rule, in terms of minimizing output variance? This is found by finding the optimal coefficient μ in the rule (9.8). I.e., one solves

$$\min_\mu \frac{(c+\mu+h)^2\sigma_u^2 + \alpha^2(\sigma_v^2+\sigma_\omega^2)}{(c+h+\mu+h)^2}$$

From the first-order condition one recovers the optimal solution for μ as

$$\mu^* = -(c+h) + \frac{\alpha\left(\sigma_v^2 + \sigma_\omega^2\right)}{\sigma_u^2}.$$

- Again, optimality depends upon relative variances and slopes of IS and LM curves. Indeed, approaching optimality of an interest rate operating procedure requires
  - High money market volatility
  - Low aggregate demand volatility
  - Flat IS curve (high α)
  - Steep "LM curve" (low $c+h$)
- Note, however, that even when $\sigma_v^2 = \sigma_\omega^2 = 0$, i.e., there is *no* financial market volatility, it is *not* optimal to have a "pure" base rule operating procedure. E.g., with $u_t > 0$ one can do better by contracting b_t so as to further increase the nominal interest rate ($\mu^* < 0$)
- With σ_v^2, $\sigma_\omega^2 > 0$ "leaning against the wind" may become optimal, as an interest rate increase may reflect either $v_t > 0$ or $\omega_t < 0$ ($\mu^* > 0$)
- There is a close analogy between finding the optimal value of μ in the policy rule and a "signal extraction" problem (like in Lucas' Island model). Central bank observes the nominal interest rate, but not the underlying shocks. Hence, it attempts to forecast the shocks based on the "signal," the nominal interest rate. Formally, *assume* that the policy rule could respond to shocks:

$$b_t = \mu_u u_t + \mu_v v_t + \mu_\omega \omega_t$$

Output then satisfies

$$y_t\left(1 + \frac{c}{a} + \frac{h}{\alpha}\right) = (\mu_v - 1)\, v_t + \left(\mu_u + \frac{h}{\alpha} + \frac{c}{\alpha}\right) u_t + (1 + \mu_\omega)\,\omega_t.$$

Hence, the rule

$$b_t = -\frac{c+h}{\alpha} u_t + v_t - \omega_t$$

completely stabilizes output. However, shocks *cannot* be observed, so *estimates* of the shocks are made:

$$b_t = -\frac{c+h}{\alpha} \widehat{u}_t + \widehat{v}_t - \widehat{\omega}_t$$

Estimates are made conditional on observed i_t :

$$\widehat{u}_t = \mathrm{E}\left[u_t | i_t\right] = \delta_u i_t, \qquad \widehat{v}_t = \mathrm{E}\left[v_t | i_t\right] = \delta_v i_t, \qquad \widehat{\omega}_t = \mathrm{E}\left[\omega_t | i_t\right] = \delta_\omega i_t,$$

We then get

$$b_t = \left(-\frac{c+h}{\alpha}\delta_u + \delta_v - \delta_\omega \right) i_t \tag{9.11}$$

In Appendix A, it is shown that when the δs are chosen to minimize squared forecast errors, they indeed take on values such that the coefficient on i_t in (9.11) becomes μ^*.

3 Intermediate targets in policymaking

- A central lesson from previous analysis, is that forecasts of shocks may determine optimal monetary policy (and that forecasts depend on relative variances of shocks). In real life, central banks must rely on best possible information. This will then provide scope for adjusting policy in light of movements in variables that provides good information about movements in goal variables. This is known as *intermediate targeting.*

- It is illustrated in simple AS/IS/LM model with imperfect information about current shocks. The policy goal is simple (just for illustrative purposes), and involves "strict inflation targeting." Here, this means that the aim is to minimize inflation variance:

$$V = \mathrm{E}\left[\pi_t\right]^2 \tag{9.15'}$$

(as in previous analyses, normalize the value of the inflation target to zero; $\pi^* = 0$). The AS/IS/LM model used is:

$$\begin{aligned}
y_t &= a\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + z_t, \qquad a > 0 && \text{(AS 9.12)}\\
y_t &= -\alpha\left(i_t - \mathrm{E}_t\pi_{t+1}\right) + u_t, \qquad \alpha > 0 && \text{(IS 9.13)}\\
m_t - p_t &= m_t - \pi_t - p_{t-1} = y_t - ci_t + v_t, \qquad c > 0 && \text{(LM 9.14)}
\end{aligned}$$

All shocks follow AR(1) processes:

$$\begin{aligned}
z_t &= \rho_z z_{t-1} + e_t, \qquad 0 < \rho_z < 1,\\
u_t &= \rho_u u_{t-1} + \varphi_t, \qquad 0 < \rho_u < 1,\\
v_t &= \rho_v v_{t-1} + \psi_t, \qquad 0 < \rho_v < 1,
\end{aligned}$$

and all innovations to these processes, e_t, φ_t and ψ_t, are mean-zero, independent shocks (with variances σ_e^2, σ_φ^2, σ_ψ^2). The policy instrument is the nominal interest rate (so the issue now is not to find the best instrument, but how to let a given instrument react to various observable variables). With the simple policy goal, the solution of model is simplified as $\mathrm{E}_{t-1}\pi_t = \mathrm{E}_t\pi_{t+1} = 0$. The AS and IS curves provides the solution for inflation as function of interest rate

$$\pi_t = \frac{-\alpha i_t + u_t - z_t}{a} \tag{9.16'}$$

Clearly, the optimal shock-contingent interest rate is

$$i_t = (1/\alpha)(u_t - z_t). \tag{9.17'}$$

But, if the interest rate must be set before period t innovations are realized, the optimal interest rate is

$$\widehat{\imath}_t = (1/\alpha)(\rho_u u_{t-1} - \rho_z z_{t-1}) \tag{9.18'}$$

Given this policy, actual inflation becomes

$$\pi_t(\widehat{\imath}_t) = \frac{\varphi_t - e_t}{a}$$

Inflation fluctuates with the supply and demand shock innovations (not money demand shocks, as these play no role for outcomes under an interest rate targeting procedure; cf. the simple Poole analysis). The associated inflation variance is

$$V(\widehat{\imath}_t) = \frac{\sigma_\varphi^2 + \sigma_e^2}{a^2}.$$

One can equivalently find the money supply that gives the same inflation and interest rate (all relevant derivations are provided in Appendix B):

$$\widehat{m}_t = p_{t-1} - \frac{c}{\alpha}\rho_u u_{t-1} + \left(1 + \frac{c}{\alpha}\right)\rho_z z_{t-1} + \rho_v v_{t-1}$$

- Now assume that the central bank, when setting the interest rate, observes the actual value of m_t. The individual shocks are still unobservable, but m_t will provide information about the shocks — a signal. Adjusting the interest rate such that actual m_t becomes equal to $\widehat{m}_t$ *may* improve policymaking (it takes into account information about the unobservable shocks). In that case, m_t becomes an intermediate target for policymaking

- Example: $\sigma_\psi^2 = \sigma_e^2 = 0$; i.e., only aggregate demand shocks matter

  - $\varphi_t > 0$ will for given interest rate be reflected in higher output and inflation and higher m_t
  - Higher m_t thus provides information about $u_t > 0$, and the interest rate should be increased to dampen output and inflation, i.e., to bring back m_t to $\widehat{m}_t$
  - Intermediate targeting is therefore appropriate policy

- Example: $\sigma_\psi^2 = \sigma_\varphi^2 = 0$; only aggregate supply shocks matter

  - $e_t > 0$ will for given interest rate be reflected in lower inflation and lower m_t
  - Lower m_t thus provides information about $e_t > 0$, and the interest rate should be decreased to bring inflation back towards target, i.e., to bring back m_t to $\widehat{m}_t$

– Intermediate targeting is therefore appropriate policy

- Example: $\sigma_e^2 = \sigma_\varphi^2 = 0$; only money demand shocks matter
  - $\psi_t > 0$ will for given interest rate be reflected in higher m_t and no change in inflation
  - Higher m_t thus provides information about $\psi_t > 0$, and the interest rate should be left unchanged and leave m_t different from $\widehat{m}_t$
  - Intermediate targeting would therefore be **in**appropriate policy

- Overall, the desirability of a certain policy regime, here monetary intermediate targeting, will depend upon the relative variances of the shocks. An automatic interest rate procedure to keep $m_t = \widehat{m}_t$ improves by keeping $i_t = \widehat{\imath}_t$ if σ_e^2 and σ_φ^2 are relatively high, and σ_ψ^2 is relatively low (the intermediate target, m_t, must thus be "controllable")

- Note, the simple Poole model featured and "either or" choice which could be improved by an optimal policy rule, that optimally "processed" the new information about shocks. The same logic applies here, where one can formulate interest rate rule like

$$i_t = \widehat{\imath}_t + \mu x_t \tag{9.22}$$

where x_t is the new information obtained by observing m_t (a linear combination of unobserved shocks). The optimal value of μ will, e.g., not surprisingly be decreasing in σ_ψ^2. In this sense, the new information used by observing the intermediate target *is used optimally*, and it will improve over a simple intermediate targeting policy aimed at attaining $m_t = \widehat{m}_t$.

4 Price level (in)determinacy

- Using the interest rate as monetary policy instrument may render the price level *indeterminate*. This means that any value of the price level is consistent with a rational expectations equilibrium. To see this, consider a simple AS/IS model under an interest rate operating procedure:

$$y_t = a\left(p_t - \mathrm{E}_{t-1}p_t\right) + e_t, \quad a > 0 \tag{10.6'}$$

$$y_t = -\alpha r_t + u_t, \quad \alpha > 0 \tag{10.7'}$$

$$i_t^T = r_t + \left(\mathrm{E}_t p_{t+1} - p_t\right) \tag{10.8}$$

Note that the price level appears as

  - An expectational error in AS curve
  - An expected change in Fisher relationship

- Hence, the price level as such is not determined by the model. If, say, $\left(p_t^*, p_{t+1}^*\right)$ is a solution, then $\left(p_t^* + \kappa, p_{t+1}^* + \kappa\right)$ is a solution for *any* κ. The price expectation error and expected price change are *independent* of κ.

- The endogenous money supply will be indeterminate as well

  - If private agents suddenly expects higher prices, the expectation is self-fulfilling
  - The central bank must increase the nominal money supply to maintain i_t^T
  - . . . m_t is not determined by the model when p_t is not

- This is problematic, e.g., when writing nominal contracts, as the price level can suddenly shift for no fundamental reasons. One may then ask whether an interest rate operating procedure is disastrous? Not necessarily, as long as the central bank to some extent care about the price level

  - Either by adjusting the interest rate in response to price movements, e.g., $i_t = i_t^T + f\left(p_t\right)$
  - or, by letting policy to some extent depend on the nominal money supply

- In either case, the central bank's policy will then contain a *nominal anchor*, which will determine the price level. An example could be a policy depending on the money supply:

$$m_t = \mu_0 + m_{t-1} + \mu\left(i_t - i_t^T\right) \tag{10.9}$$

  This is equivalent to the money base rule considered above (note that for $\mu \to \infty$ this corresponds to a pure interest rate operating procedure). With this rule, the LM curve "re-enters" the model (to determine actual nominal interest rate):

$$m_t - p_t = -ci_t + y_t + v_t, \qquad c > 0, \tag{10.3}$$

  and the price level then enters in itself (and not just as an expectations error or expected change), and it will then be determined by the model

- For a high value of μ the central bank can still be interpreted as acting under an interest rate operating target

  - The "aggressive" response of the money supply towards $\left(i_t - i_t^T\right)$ minimizes deviations in actual interest rate from target value
  - . . . and the price level becomes determinate

- General lesson: Under an interest rate operating procedure, policy design should secure determinacy by letting the interest rate respond to the price level directly or indirectly.

5 The term structure of interest rates

- Under an interest rate operating procedure, money demand plays a minor role in transmission of monetary policy. Money demand depends on short-term interest rates, which are under closest control of the central bank. Aggregate goods demand, on the other hand, typically depends on long-term interest rates, which are market determined. Hence, under an interest rate operating procedure, the link between short- and long-term interest rates becomes of high importance. The link is denoted the *term structure.*

- Common theory for analyzing the link between long and short interest rates is the **expectations theory** of the term structure. It takes as starting point a simple no-arbitrage condition:

  - The nominal return of holding an n-period bond to maturity must equal the return of holding an n-period sequence of one-period bonds

- Define $i_{n,t}$ as the nominal yield to maturity on a n-period bond in period t. Define i_t as the nominal yield on a one-period bond. Then, the condition (which ignores realistic risk premia on long bonds) can be stated formally:

$$(1 + i_{n,t})^n = \prod_{i=o}^{n-1} (1 + i_{t+i}) .$$

Approximately it is (take logs and assume small yields)

$$i_{n,t} = \frac{1}{n} \sum_{i=0}^{n-1} i_{t+i}$$

Hence, the long interest rate is an unweighted average of current and future short interest rates

- Note that the condition can be written as a difference equation in the long interest rate. An n-period bond becomes an $n-1$-period bond in next period; therefore

$$(1 + i_{n,t})^n = (1 + i_t) (1 + i_{n-1,t+1})^{n-1} \qquad (*)$$

which approximately is

$$i_{n,t} = \frac{1}{n} i_t + \frac{n-1}{n} i_{n-1,t+1}$$

Notice how (*) can be interpreted as a no-arbitrage condition:

$$(1 + i_t) = \frac{(1 + i_{n,t})^n}{(1 + i_{n-1,t+1})^{n-1}}$$

- The right-hand side is the one-period gross return on the long bond. Why:

– Price of bond in period t is $P_{n,t} = 1/\left(1+i_{n,t}\right)^n$

– Price of bond in period $t+1$ is $P_{n-1,t+1} = 1/\left(1+i_{n-1,t+1}\right)^{n-1}$

– Gross return is $P_{n-1,t+1}/P_{n,t} = \left(1+i_{n,t}\right)^n / \left(1+i_{n-1,t+1}\right)^{n-1}$

- Notice that under uncertainty the relationships are not correct. The expected one-period return on the long bond is

$$\mathrm{E}_t \frac{\left(1+i_{n,t}\right)^n}{\left(1+i_{n-1,t+1}\right)^{n-1}}$$

The no-arbitrage condition is therefore

$$\left(1+i_t\right) = \mathrm{E}_t \frac{\left(1+i_{n,t}\right)^n}{\left(1+i_{n-1,t+1}\right)^{n-1}},$$

which does **not** imply (*) under uncertainty:

$$\left(1+i_{n,t}\right)^n = \left(1+i_t\right) \mathrm{E}_t \left(1+i_{n-1,t+1}\right)^{n-1}$$

This mistake is ignored here. Important matters are that:

– Long interest rate will be function of current and expected future interest rates

– If aggregate demand depends on long rates, expectations about future monetary policy, future short rates, becomes of importance

- Simple example with the "long" bond being a two period bond:

$$\left(1+I_t\right)^2 = \left(1+i_t\right)\left(1+\mathrm{E}_t i_{t+1}\right)$$

I_t is "long" interest rate. Approximately:

$$I_t = \frac{1}{2}\left(i_t + \mathrm{E}_t i_{t+1}\right)$$

The associated simple term structure:

$$I_t - i_t = \frac{1}{2}\left(\mathrm{E}_t i_{t+1} - i_t\right)$$

– Hence, long rate exceeds short rate if short rates are expected to increase

– If theory is correct, the interest rate spread — the yield curve — is **an indicator** for expectations about future monetary policy; and thus future economic activity!

– Empirically, the difference between long and short rates do have predictive power concerning, e.g., output

- Empirically, the expectations theory of the term structure has, however, had mixed success. The theory implies

$$\frac{1}{2}(i_{t+1} - i_t) = I_t - i_i + \theta_t$$

with $\theta_t = \frac{1}{2}(i_{t+1} - \mathrm{E}_t i_{t+1})$. A regression like

$$\frac{1}{2}(i_{t+1} - i_t) = a + b(I_t - i_i) + \theta_t$$

should therefore give estimates $a = 0$ and $b = 1$

  - This is rarely the case! Usually $b < 1$ (and even $b < 0$) in estimations!
  - However, the theory does not take monetary policymaking into consideration! (Ignores operating procedures)
  - *If* central bank tends to "smooth" interest rates b **should** not be one
    * Extreme (illuminating) case:
    * Policy is $i_t = i_{t-1}$. Then, an increase in I_t, e.g., due to an increase in the risk premium, will not lead to a change in the short rate
  - Also, if the central bank tries to dampen the economic effects of an increased risk premium (by lowering short rates now and in the future), then $b < 0$ is what theory says

- For lack of better alternative, we usually stick with the expectations theory of the term structure

- Important implication of the theory concerns **inflation expectations.** We have

$$\begin{aligned} I_t &= \frac{1}{2}(i_t + \mathrm{E}_t i_{t+1}) \\ &= \frac{1}{2}(r_t + \mathrm{E}_t r_{t+1} + \mathrm{E}_t \pi_{t+1} + \mathrm{E}_t \pi_{t+2}) \end{aligned}$$

where r_t is the short real interest rate. Assuming the real rate fluctuates little, $r_t = \mathrm{E}_t r_{t+1} \approx \overline{r}$ we get

$$\begin{aligned} I_t &= \frac{1}{2}(\overline{r} + \mathrm{E}_t \pi_{t+1} + \mathrm{E}_t \pi_{t+2}) \\ &= \frac{1}{2}(\overline{r} + \mathrm{E}_t \overline{\pi}_{t+2}) \end{aligned}$$

with $\overline{\pi}_{t+2} = p_{t+2} - p_t$. Hence, increases in the long rate may reflect increased inflation expectations. Empirically, changes in long rates indeed often reflect changes in inflation expectations

- Note if the **short** nominal interest rate is **increased**, the **long** rate may **decrease**, as it can reflect that the market believes that the central bank is trying to, and will be successful in, bringing down future inflation.

6 A model for monetary policy analysis and the impact of interest rate rule parameters

- The simple one-period sticky-wage variant of the MIU model, did not provide adequate output and inflation dynamics. Hence, it will not be used to evaluate various interest rate policy rules. Focus is on extended version of the model, which

  - allows for richer dynamics
  - allows for a distinction between long and short interest rate

- Model is due to Jeffrey Fuhrer and George Moore (1995, American Economic Review). It is a simple closed-economy IS/AS model. The IS curve is given as:

$$\begin{aligned} y_t &= a_1 y_{t-1} + a_2 y_{t-2} - a_3 r_{t-1} + u_t \qquad (10.35) \\ u_t &= \rho u_{t-1} + \varepsilon_t \end{aligned}$$

  - More lags (empirically, not theoretically explained)
  - Delayed response of demand to a change in the long real interest rate r_t

Term structure equation formulated as a difference equation in the long rate:

$$r_t - D\left(\mathrm{E}_t r_{t+1} - r_t\right) = i_t^f - \mathrm{E}_t \pi_{t+1} \qquad (10.36)$$

  - i_t^f is the one period nominal interest rate — the policy instrument
  - D is a measure of the duration of the long bond (equals the term for conventional discount bonds)
  - Note if $r_t > i_t^f - \mathrm{E}_t \pi_{t+1}$, agents must be expecting a capital loss on the long bond, i.e., $\mathrm{E}_t r_{t+1} > r_t$
  - Note that a given expected capital loss drives up the long rate by more, the longer term the bond has (an expected increase in the interest rate is more costly when the bond has a long term)

AS curve: A Fuhrer-Moore **inflation inertia equation**:

$$\pi_t = \frac{1}{2}\left(\pi_{t-1} + \mathrm{E}_t \pi_{t+1}\right) + \gamma q_t + \eta_t \qquad (10.40)$$

where q_t summarizes the business cycle stance (lagged, current and expected future output)

  - Price adjustment structure like staggered price setting, but here in terms of inflation rather than prices

– Empirically plausible; theoretically an ongoing issue to model it satisfactorily (Fuhrer and Moore provides just one attempt)

Model is closed by a specification of the monetary policy rule (a nominal interest rate rule):

$$i_t^f = b_1 i_{t-1}^f + b_2 \left(\pi_t - \pi^T\right) + b_3 y_t + b_4 \left(y_t - y_{t-1}\right) + \varphi_t, \quad (10.41)$$

φ_t is "policy shock"

- Model is calibrated, and simulated (with estimated parameters; cf. Table 10.1 in Walsh, 2003). The baseline show that a contractive interest rate shock give rise to output and inflation dynamics which reasonably mimic what is found in VAR analyses. The long rate becomes expansive after a while, in anticipation of future short interest rate cuts to bring back the economy to initial equilibrium

- Changing the policy rule parameters:

  – Increasing b_3 substantially lowers output variance and causes a faster return to initial equilibrium

  – Increasing b_2 also lowers output variance (a reflection of the fact that the demand shock pushes output and inflation in same directions; so does the interest rate)

  – The long rate will now respond less, in anticipation of the faster adjustment — hence a more aggressive policy rule stabilizes the long interest rate

- Very simple model captures empirical regularities, and is nice laboratory for analyzing **performance of alternative policy rules**

  – Models of this basic type play large role for real-life central banks (US Fed uses — larger — version of such model for policy analyses)

- Care must be taken: We have gone further and further away from micro-founded models, and the Lucas critique may start to "bite"!

- Looking positively at this: The good performance of the model invites a search for micro-founded models which, e.g., generate the lags in the IS equation. Some recent progress on this: Habit formation in utility function. Then, utility of consumption today depends on consumption yesterday. Implication: Current consumption will depend on past consumption, which translates into output persistence

- Immediate issues: Given that model is "good," what is then **the best** (=optimal) interest rate policy? Has industrialized countries followed policies that comes close to such a policy?

- First issue is now covered in a simplified Fuhrer-Moore model. The second issue is covered by subsequently examining some empirical evidence on interest rate rules.

7 Optimal interest rate rule in simple model for policy analysis

7.1 The modified Fuhrer-Moore model

- A simplified Fuhrer-Moore model is used to illustrate the computation of an optimal policy rule for the nominal interest rate. Simplifications are

  - Drop distinction between short- and long term interest rate (just have a short interest rate)
  - Drop forward-looking elements in the AS curve
  - Drop a lag in the IS curve (not done by Walsh, 2003)
  - Let shocks be mean-zero white-noise disturbances

  The model is now a simple *two-equation* IS/AS model (written explicitly in terms of the nominal interest rate and inflation, so Fisher relationship is not needed). The IS curve is:

$$y_t = a_1 y_{t-1} - a_3 \left(i_{t-1} - \mathrm{E}_{t-1}\pi_t\right) + u_t, \qquad a_1, a_3 > 0, \tag{10.42'}$$

  and the AS curve is

$$\pi_t = \pi_{t-1} + \gamma y_t + \eta_t, \qquad \gamma > 0 \tag{10.43}$$

  (i.e., an "Accelerationist Phillips-curve"). The model features a simple monetary transmission mechanism:

  - Nominal interest rate affects aggregate demand with a one period-lag
  - Through the Phillips curve, inflation is also affected with a one-period lag
  - Effects are persistent due to the lags in both the IS and AS curve

- What are the stability properties of system in absence of further policy response? It requires stability of the system

$$y_t = a_1 y_{t-1} + a_3 \pi_t \tag{*}$$
$$\pi_t = \pi_{t-1} + \gamma y_t \tag{**}$$

  To analyze stability, one usually formulates the system in matrix form:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \mathbf{A} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \end{bmatrix}$$

where **A** is a 2×2 matrix. Since the system has no forward-looking variables, and two predetermined variables, y_{t-1}, and π_{t-1}, stability requires that the real parts of the eigenvalues of **A** are both numerically smaller than one. Cf. Blanchard and Kahn, 1980, *Econometrica,* which is the standard technical reference for stability conditions in general, linear dynamic rational expectations.[1] Hence, find **A**. From (**) and (*) we get

$$y_t = a_1 y_{t-1} + a_3 \left(\pi_{t-1} + \gamma y_t\right)$$
$$y_t = \frac{a_1}{1-\gamma a_3} y_{t-1} + \frac{a_3}{1-\gamma a_3} \pi_{t-1}$$

Also we get

$$\pi_t = \pi_{t-1} + \gamma \left(a_1 y_{t-1} + a_3 \pi_t\right)$$
$$\pi_t = \frac{1}{1-\gamma a_3} \pi_{t-1} + \frac{\gamma a_1}{1-\gamma a_3} y_{t-1}$$

The system is thus

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \frac{a_1}{1-\gamma a_3} & \frac{a_3}{1-\gamma a_3} \\ \frac{\gamma a_1}{1-\gamma a_3} & \frac{1}{1-\gamma a_3} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \end{bmatrix},$$

such that

$$\mathbf{A} = \begin{bmatrix} \frac{a_1}{1-\gamma a_3} & \frac{a_3}{1-\gamma a_3} \\ \frac{\gamma a_1}{1-\gamma a_3} & \frac{1}{1-\gamma a_3} \end{bmatrix}$$

The eigenvalues λ_1 and λ_2 are computed from:

$$\begin{vmatrix} \frac{a_1}{1-\gamma a_3} - \lambda & \frac{a_3}{1-\gamma a_3} \\ \frac{\gamma a_1}{1-\gamma a_3} & \frac{1}{1-\gamma a_3} - \lambda \end{vmatrix} = 0$$

This gives a second-order polynomial in λ:

$$\lambda^2 - \frac{1+a_1}{1-\gamma a_3}\lambda + \frac{a_1^2}{(1-\gamma a_3)^2} - \frac{\gamma a_1 a_3}{(1-\gamma a_3)^2} = 0,$$

$$\lambda^2 - \frac{1+a_1}{1-\gamma a_3}\lambda + \frac{a_1}{1-\gamma a_3} = 0,$$

and the solutions are

$$\lambda = \frac{\frac{1+a_1}{1-\gamma a_3} \pm \sqrt{\left(\frac{1+a_1}{1-\gamma a_3}\right)^2 - 4\frac{a_1}{1-\gamma a_3}}}{2}$$

[1] This generalizes the simple and intuitive condition for stability of a one equation "system" with one predetermined variable: there, stability requires that there is one eigenvalue numerically smaller than one. In plain language, the coefficient on the predetermined variable is numerically smaller than one.

- Parameter values based on estimations on US data by Rudebusch and Svensson (1999, in John Taylor (ed.): "Monetary Policy Rules", Chicago University Press)
  - Persistence in aggregate demand; $a_1 = 0.91$
  - Aggregate demand's real interest rate sensitivity; $a_3 = 0.1$
  - Inflation's sensitivity to aggregate demand; $\gamma = 0.14$
- With these values we get

$$\begin{aligned} \lambda_1 &= 1.0918 \\ \lambda_2 &= 0.8453 \end{aligned}$$

- System is **unstable**! For a fixed nominal interest rate, shocks to the economy will lead to **explosive** output and inflation paths
  - Intuition: A positive u_t shock increases output, which leads to inflation. This **decreases** the real interest rate, and stimulates output and inflation further, and so on, and so on.....
- The purpose of the derivation of an optimal policy rule is therefore **two-fold:**
  - To secure that the economy **returns to its long-run equilibrium** following a shock
  - Secure the **optimal manner** by which it returns to the long-run equilibrium

7.2 Optimal monetary policymaking

- The criterion of monetary policy is to minimize the expected discounted sum of deviations in output and inflation from their long-run equilibrium values. In each period t, the loss function is assumed to be

$$\frac{\lambda}{2} y_t^2 + \frac{1}{2} \pi_t^2, \qquad \lambda > 0.$$

- To solve the model note first that
  - in period t policy cannot affect output and inflation
  - period t policy only affects period $t + 1$ values (and onwards)

  The objective function is therefore written as

$$L = \frac{1}{2} \mathrm{E}_t \sum_{i=1}^{\infty} \beta^i \left[\lambda y_{t+i}^2 + \pi_{t+i}^2 \right], \qquad 0 < \beta < 1, \tag{10.47}$$

  and the policy problem is to minimize L w.r.t. i_t subject to the IS/AS curves

- A trick is introduced in order to make the solution easier. Since it is period $t+1$ output that can be controlled, forward the IS curve one period:

$$y_{t+1} = a_1 y_t - a_3 \left(i_t - \mathrm{E}_t \pi_{t+1}\right) + u_{t+1}$$

Inflation expectations follow by forwarding the AS curve and take expectations:

$$\mathrm{E}_t \pi_{t+1} = \pi_t + \gamma \mathrm{E}_t y_{t+1}$$

Combine the two to get

$$y_{t+1} = a_1 y_t - a_3 \left(i_t - \pi_t - \gamma \mathrm{E}_t y_{t+1}\right) + u_{t+1}$$

Take period t expectations on both sides (remember $\mathrm{E}_t u_{t+1} = 0$)

$$\mathrm{E}_t y_{t+1} = a_1 y_t - a_3 \left(i_t - \pi_t - \gamma \mathrm{E}_t y_{t+1}\right)$$

This gives solution for $\mathrm{E}_t y_{t+1}$

$$\mathrm{E}_t y_{t+1} = \frac{a_1 y_t - a_3 \left(i_t - \pi_t\right)}{1 - \gamma a_3}$$

which is inserted into the IS curve:

$$y_{t+1} = \frac{a_1 y_t - a_3 \left(i_t - \pi_t\right)}{1 - \gamma a_3} + u_{t+1}$$

But in period t, y_t and π_t are predetermined from the perspective of policy. Hence, one can treat

$$\theta_t \equiv \frac{a_1 y_t - a_3 \left(i_t - \pi_t\right)}{1 - \gamma a_3} \qquad (10.51')$$

as the policy instrument! The IS now curve becomes:

$$y_{t+1} = \theta_t + u_{t+1}, \qquad (10.52)$$

and the AS curve becomes:

$$\begin{aligned} \pi_{t+1} &= \pi_t + \gamma \theta_t + \gamma u_{t+1} + \eta_{t+1} \\ &= \pi_t + \gamma \theta_t + v_{t+1} \qquad v_{t+1} \equiv \gamma u_{t+1} + \eta_{t+1} \end{aligned} \qquad (10.53)$$

One then finds the optimal value of θ_t, which by (10.51') residually provides i_t as a function of y_t and π_t

- Treating θ_t as the policy instrument and π_t as the state variable, one sets up a dynamic programing problem as

$$V\left(\pi_t\right) = \min_{\theta_t} \mathrm{E}_t \left\{ \frac{1}{2} \left(\lambda y_{t+1}^2 + \pi_{t+1}^2 + \beta V\left(\pi_{t+1}\right) \right) \right\},$$

where the minimization is subject to $y_{t+1} = \theta_t + u_{t+1}$ and $\pi_{t+1} = \pi_t + \gamma\theta_t + v_{t+1}$. These constraints, however, are inserted into the value function so as to give an unconstrained problem:

$$\begin{aligned} V(\pi_t) &= \min_{\theta_t} \mathrm{E}_t \left\{ \frac{1}{2} \left(\lambda (\theta_t + u_{t+1})^2 + (\pi_t + \gamma\theta_t + v_{t+1})^2 \right) \right. \\ & \left. + \beta V(\pi_t + \gamma\theta_t + v_{t+1}) \right\} \end{aligned}$$

The relevant first-order condition is then readily found as

$$\begin{aligned} & \mathrm{E}_t \{ \lambda (\theta_t + u_{t+1}) + \gamma (\pi_t + \gamma\theta_t + v_{t+1}) \\ & + \beta\gamma V_\pi (\pi_t + \gamma\theta_t + v_{t+1}) \} \\ = & \; 0, \end{aligned}$$

or,

$$\left(\lambda + \gamma^2\right) \theta_t + \gamma\pi_t + \beta\gamma \mathrm{E}_t V_\pi (\pi_{t+1}) = 0.$$

We have that the value function in optimum is given by

$$\begin{aligned} V(\pi_t) &= \mathrm{E}_t \left\{ \frac{1}{2} \left(\lambda (\theta_t + u_{t+1})^2 + (\pi_t + \gamma\theta_t + v_{t+1})^2 \right) \right. \\ & \left. + \beta V(\pi_t + \gamma\theta_t + v_{t+1}) \right\}, \end{aligned}$$

where θ_t is an optimal function of the state. Differentiating the value function then gives

$$\begin{aligned} V_\pi(\pi_t) &= \mathrm{E}_t \left\{ \lambda (\theta_t + u_{t+1}) \frac{\partial\theta_t}{\partial\pi_t} + (\pi_t + \gamma\theta_t + v_{t+1}) \left(1 + \frac{\partial\theta_t}{\partial\pi_t}\right) \right. \\ & \left. + \beta V_\pi (\pi_t + \gamma\theta_t + v_{t+1}) \left(1 + \gamma\frac{\partial\theta_t}{\partial\pi_t}\right) \right\}. \end{aligned}$$

By the envelope theorem, all coefficients to $\partial\theta_t/\partial\pi_t$ can be ignored (as they are zero by the first-order condition for optimal choice of θ_t; cf. above); hence:

$$V_\pi(\pi_t) = \pi_t + \gamma\theta_t + \beta \mathrm{E}_t V_\pi(\pi_{t+1}).$$

Multiply this equation by γ on both sides:

$$\gamma V_\pi(\pi_t) = \gamma\pi_t + \gamma^2\theta_t + \beta\gamma \mathrm{E}_t V_\pi(\pi_{t+1}),$$

and eliminate $\beta\gamma \mathrm{E}_t V_\pi(\pi_{t+1})$ from the first-order condition,

$$\left(\lambda + \gamma^2\right)\theta_t + \gamma\pi_t + \gamma V_\pi(\pi_t) - \gamma\pi_t - \gamma^2\theta_t = 0,$$

such that

$$\lambda\theta_t = -\gamma V_\pi(\pi_t).$$

Forward this one period, multiply by β, and take period t expectations:

$$\lambda\beta \mathrm{E}_t\theta_{t+1} = -\beta\gamma \mathrm{E}_t V_\pi\left(\pi_{t+1}\right),$$

so as to finally get from the first-order condition

$$\left(\lambda+\gamma^2\right)\theta_t + \gamma\pi_t - \lambda\beta \mathrm{E}_t\theta_{t+1} = 0.$$

This expectational difference equation is solved for θ_t by conjecturing $\theta_t = B\pi_t$, which implies $\mathrm{E}_t\theta_{t+1} = B\mathrm{E}_t\pi_{t+1} = B\left(\pi_t+\gamma\theta_t\right)$, where the last equality follows from the Phillips curve. Hence, we have:

$$\left(\lambda+\gamma^2\right)\theta_t + \gamma\pi_t - \lambda\beta B\left(\pi_t+\gamma\theta_t\right) = 0,$$

which by use of the conjecture becomes

$$\left(\lambda+\gamma^2\right)B\pi_t + \gamma\pi_t - \lambda\beta B\left(\pi_t+\gamma B\pi_t\right) = 0.$$

This validates the conjecture, and identifies B from

$$-\left(\lambda\left(1-\beta\right)+\gamma^2\right)B - \gamma + \lambda\beta\gamma B^2 = 0.$$

The solutions are

$$B = \frac{\lambda\left(1-\beta\right)+\gamma^2 \pm \sqrt{\left(\lambda\left(1-\beta\right)+\gamma^2\right)^2 + 4\lambda\beta\gamma^2}}{\lambda\beta\gamma}.$$

I.e., one is negative and one is positive. As the inflation equation becomes $\pi_{t+1} = \left(1+\gamma B\right)\pi_t + \eta_{t+1}$, only the negative solution to B is relevant (the negative value cannot be optimal as it implies explosive inflation and output). One can then, from the definition of θ_t compute the associated optimal interest rule:

$$B\pi_t = \frac{a_1}{1-a_3\gamma}y_t - \frac{a_3}{1-a_3\gamma}\left(i_t-\pi_t\right),$$

and thus

$$i_t = \left[1-\frac{\left(1-a_3\gamma\right)B}{a_3}\right]\pi_t + \frac{a_1}{a_3}y_t.$$

with $B<0$ and a function of the model parameters.

Qualitative Implications

- The optimal coefficient on inflation is larger than one. This *secures stability* of model:
  - If $u_{t+1}>0$, output and inflation increase

  - This increases the real interest rate, when the nominal rate responds by more than one to inflation
  - This reduces output and inflation, and brings back the economy towards long-run equilibrium

- Higher weight on output stabilization, that is higher λ in the loss function, leads to a lower coefficient on inflation in the optimal interest rate rule ($B < 0$ increases)

- Note, somewhat remarkably, that if there is **no** weight on output stabilization, i.e., if $\lambda = 0$, there is still a response to output in the optimal rule

  - Hence, arguments in optimal policy rules say *nothing* about policy goals!
  - Here, it is optimal to respond to output even for $\lambda = 0$, as higher output today gives information about higher output tomorrow (when $a_1 > 0$) and higher inflation;
  - Output is **an intermediate target** in this model as it provides relevant information about future inflation

Quantitative Implications:

- Use the estimated values of the parameters to assess the optimal coefficients. Walsh's parameterization of model equations, along with $\lambda = 1$ and $\beta = 0.989$, yield

$$i_t = 1.50\pi_t + 4.37y_t$$

  Other parameterization (Ball), yields:

$$i_t = 1.48\pi_t + 0.8y_t$$

  The Svensson/Rudebusch estimates yield (with $\lambda = 1$ and $\beta = 0.96$):

$$i_t = 9.11\pi_t + 9.1y_t$$

  (*much* higher coefficients as the aggregate demand is less sensitive to real interest rate changes than in the other calibrations)

- These differences highlight that one should be careful when interpreting observed interest rate coefficients:

  - Observing a relatively high coefficient on inflation does not necessarily signal a "strong preference" for inflation stability

  - It could just as well be that the structural parameters in the economy *requires* a strong response to inflation in order to satisfy a "mild preference" for inflation stability.

- In any case, by adopting an interest rate rule the central bank:
  - Secures stability of the economy when it is hit by shocks
  - Minimize the weighted sum of output and inflation variability

8 International evidence on interest rate rules

- The derivation of optimal interest rate policy provides testable implications:
  - A central bank aiming at stabilizing the economy's inflation rate and output should follow an interest rule which has a higher than one coefficient on inflation
  - Also, a positive coefficient on output should be observed

- Most famous empirical account of the relationship between nominal interest rates and macroeconomic aggregates is the article:

John B. Taylor (1993):
"Discretion versus Policy Rules in Practice"

- The article summarized nicely the research frontier at that time:
  - Acknowledgement of Lucas critique
  - Monetary policy is not ineffective
  - Importance of credibility
  - Time inconsistency problems lead to favor of rules over discretion
  - "Flexible" rules responding to economic developments preferable to rigid rules

Taylor stresses that one should not follow rules mechanically, and also that they should be relatively **simple**. Even though not to be followed mechanically, the "rules" approach is to be meant as not to fall into discretionary policymaking ("start from scratch every period"); it is merely viewed as providing systematic behavior. He cites studies that consider performance of nominal interest rate rules in large-scale econometric models, and find that consensus is that output and inflation stability are best secured by responding towards these variables directly. It is confirmed by Taylor's own research (on a G7-multicountry econometric model)

- The representative policy rule:

$$r = p + 0.5y + 0.5\,(p - 2) + 2 \tag{1}$$

  where r is the nominal interest rate, p is inflation, y is output deviation from trend (inflation target and steady-state real interest rate is 2). Note that an inflation increased raises the nominal rate by 1.5, i.e., raises the real interest rate by 0.5. Equation (1) is now know as **<u>the</u> Taylor rule**. Taylor shows that it describes actual US monetary policy remarkably well

- One example of **not** being a mechanical rule: Oil price increases following the first Gulf War
  - Mechanical adherence to rule would have called for increased r (inflation rose more than output fell)
  - But the increase in inflation was *judged* to be temporary, so increasing r could be harmful

- Another example: Rising long interest rates in early 1990
  - As we know, higher long rates can be signal of expectations about higher future inflation
  - Could call for a monetary tightening
  - But, it was *judged* that real interest rate had increased due to expected German unification (causing German budget deficit, increased demand for capital), spilling over to the US through integrated world capital markets

Evidence for interest rate rules in other countries

- Richard Clarida, Jordi Galí and Mark Gertler (1998, *European Economic Review*) estimate "Taylor type rules" for Germany, Japan, US, UK. France and Italy (1979 and onwards)

- Main differences from conventional Taylor rule:
  - Responses are to expected future inflation ("Forward-looking Taylor rules")
  - Partial adjustment of interest rates to economic developments ≈ interest rate "smoothing"

- Main results for Germany, US and Japan:

- The forward-looking Taylor rule describes actual policymaking well (better than "backward-looking specifications")
- When expected inflation increases, the nominal rate is raised sufficiently to raise the real interest rate
- Some response to output also ("modest stabilization component")
- Interesting that Germany with its history as a "monetary targeter," does not respond significantly to money growth (**typo** in last column of Table 1! Coefficient in money supply row should be 0.07 (not 0.7))

• Main results for UK, France and Italy:

- Less convincing results — **not surprising** as for the most part of the period, these countries had no monetary autonomy (they pegged their currencies vis-à-vis the German mark; German interest rate is a highly significant explanatory variable)
- What **if** they had responded to internal economic conditions?

  => How far off were actual interest rates from those under such counterfactuals? May give indication about the causes of the breakdown of the EMS
- Indeed, around the breakup, these countries had much higher interest rates than had they followed a Taylor rule and responded to own economic developments
- A "blow" against trying to obtain monetary credibility by pegging to a credible bank (the Bundesbank)?
- A "blow" against EMU?

Appendix

A Equivalence of (9.11) and (9.10) in Walsh (2003) when (9.11) involves linear projections of the shocks on the nominal interest rate

This appendix shows how equation (9.11) implies the optimal value of the coefficient in the extended Poole model with the money base rule as a linear function of the nominal interest rate, when in (9.11) the forecasts of the shocks are linear projections on the interest rate.

We have the resulting interest rate in the model with $b_t = \mu i_t$, and equations

$$y_t = -\alpha i_t + u_t \tag{9,1}$$
$$m_t = -c i_t + y_t + v_t \tag{9.2}$$
$$m_t = b_t + h i_t + \omega_t \tag{9.7}$$

given as

$$i_t = \frac{v_t - \omega_t + u_t}{\alpha + c + \mu + h}. \tag{9.9}$$

The resulting solution for output is given by

$$\begin{aligned} y_t &= -\alpha \frac{v_t - \omega_t + u_t}{\alpha + c + \mu + h} + u_t \\ &= \frac{(c + \mu + h) u_t - \alpha (v_t - \omega_t)}{\alpha + c + \mu + h}, \end{aligned}$$

with the associated variance

$$\sigma_y^2 = \frac{(c + \mu + h)^2 \sigma_u^2 - \alpha^2 (\sigma_v^2 + \sigma_\omega^2)}{(\alpha + c + \mu + h)^2}.$$

Minimizing this with respect to μ gives the first-order condition

$$\frac{2(c + \mu + h)\sigma_u^2 (\alpha + c + \mu + h)^2 - 2(\alpha + c + \mu + h)\left[(c + \mu + h)^2 \sigma_u^2 - \alpha^2 (\sigma_v^2 + \sigma_\omega^2)\right]}{(\alpha + c + \mu + h)^4} = 0,$$

and thus

$$2(c + \mu + h)\sigma_u^2 (\alpha + c + \mu + h)^2 - 2(\alpha + c + \mu + h)\left[(c + \mu + h)^2 \sigma_u^2 - \alpha^2 \left(\sigma_v^2 + \sigma_\omega^2\right)\right] = 0,$$

$$(c + \mu + h)\sigma_u^2 (\alpha + c + \mu + h) - (c + \mu + h)^2 \sigma_u^2 + \alpha^2 \left(\sigma_v^2 + \sigma_\omega^2\right) = 0,$$

$$(c + \mu + h)(\alpha + c + \mu + h) - (c + \mu + h)^2 + \alpha^2 \frac{(\sigma_v^2 + \sigma_\omega^2)}{\sigma_u^2} = 0,$$

$$(c + \mu + h)\left[(\alpha + c + \mu + h) - (c + \mu + h)\right] + \alpha^2 \frac{(\sigma_v^2 + \sigma_\omega^2)}{\sigma_u^2} = 0,$$

and thus

$$\mu^* = -(c + h) + \alpha^2 \frac{(\sigma_v^2 + \sigma_\omega^2)}{\sigma_u^2}. \tag{9.10}$$

as the **optimal coefficient** in the policy rule.

The alternative is that shocks are observable, so that the base rule can be stated as

$$b_t = \mu_u u_t + \mu_v v_t + \mu_\omega \omega_t.$$

Inserting this into (9.1), (9.2) and (9.7) gives:

$$\begin{aligned} m_t &= \mu_u u_t + \mu_v v_t + \mu_\omega \omega_t + h i_t + \omega_t \\ &= -c i_t + y_t + v_t \end{aligned}$$

Eliminating i by (9.1) yields

$$\mu_u u_t + \mu_v v_t + \mu_\omega \omega_t + h \frac{u_t - y_t}{\alpha} + \omega_t = -c \frac{u_t - y_t}{\alpha} + y_t + v_t$$

from which we get the solution for y_t:

$$y_t \left(1 + \frac{c}{a} + \frac{h}{\alpha}\right) = (\mu_v - 1) v_t + \left(\mu_u + \frac{h}{\alpha} + \frac{c}{\alpha}\right) u_t + (1 + \mu_\omega) \omega_t$$

We immediately see that

$$b_t = -\frac{c+h}{\alpha} u_t + v_t - \omega_t$$

completely stabilizes output. However, shocks cannot be observed, so *estimates* of the shocks are made based on the observed interest rate, so that the policy rule becomes

$$b_t = -\frac{c+h}{\alpha} \widehat{u}_t + \widehat{v}_t - \widehat{\omega}_t$$

where

$$\widehat{u}_t = \mathrm{E}\left[u_t | i_t\right], \quad \widehat{v}_t = \mathrm{E}\left[v_t | i_t\right], \quad \widehat{\omega}_t = \mathrm{E}\left[\omega_t | i_t\right].$$

are the estimates of the shocks based on the observed interest rate. We assume that these estimates are made by linear projections on i_t with the aim of minimizing the squared forecast errors.

But what is the nominal interest rate under this rule? From (9.1) and (9.2) one immediately get

$$m_t = -(c + \alpha) i_t + u_t + v_t$$

Using (9.7) on gets

$$b_t + h i_t + \omega_t = -(c + \alpha) i_t + u_t + v_t$$

and thus

$$i_t = -\frac{b_t}{\alpha + c + h} + \frac{u_t + v_t - \omega_t}{\alpha + c + h}$$

As the shock forecasts are linear projections on i: $\widehat{u}_t = \mathrm{E}[u_t|i_t] = \widehat{\delta}_u i_t$, $\widehat{v}_t = \mathrm{E}[v_t|i_t] = \widehat{\delta}_v i_t$ and $\widehat{\omega}_t = \mathrm{E}[\omega_t|i_t] = \widehat{\delta}_\omega i_t$, where $\widehat{\delta}_u$, $\widehat{\delta}_v$ and $\widehat{\delta}_\omega$ are estimation coefficients to be determined, we have that

$$b_t = \left(-\frac{c+h}{\alpha} \widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega\right) i_t \tag{9.11}$$

Therefore we get an expression for the interest rate from

$$i_t(\alpha + c + h) = -\left(-\frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega\right) i_t + u_t + v_t - \omega_t,$$

so that the solution for the interest rate becomes

$$i_t = \frac{u_t + v_t - \omega_t}{\alpha + c + h - \frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega}.$$

Hence, the estimates are found from

$$\begin{aligned} \widehat{u}_t &= \delta_u i_t \\ &= \delta_u \frac{u_t + v_t - \omega_t}{\alpha + c + h - \frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega}, \end{aligned}$$

where δ_u minimizes the squared forecast error. I.e., it solves

$$\begin{aligned} &\min_{\delta_u} \quad \mathrm{E}\left[\widehat{u}_t - u_t\right]^2 \\ = &\min_{\delta_u} \quad \mathrm{E}\left[\delta_u \frac{u_t + v_t - \omega_t}{\alpha + c + h - \frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega} - u_t\right]^2 \\ = &\min_{\delta_u} \quad \left[\delta_u^2 \frac{\sigma_v^2 + \sigma_\omega^2 + \sigma_u^2}{\left(\alpha + c + h - \frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega\right)^2} + \sigma_u^2 - 2\delta_u \frac{\sigma_u^2}{\alpha + c + h - \frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega}\right] \end{aligned}$$

The first-order condition is:

$$2\delta_u \frac{\sigma_v^2 + \sigma_\omega^2 + \sigma_u^2}{\left(\alpha + c + h - \frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega\right)^2} - 2\frac{\sigma_u^2}{\alpha + c + h - \frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega} = 0,$$

or,

$$\delta_u \frac{\sigma_v^2 + \sigma_\omega^2 + \sigma_u^2}{\alpha + c + h - \frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega} - \sigma_u^2 = 0 \qquad (*)$$

We then find the estimation coefficient $\widehat{\delta}_v$ from

$$\begin{aligned} &\min_{\delta_v} \quad \mathrm{E}\left[\widehat{v}_t - v_t\right]^2 \\ = &\min_{\delta_v} \quad \mathrm{E}\left[\delta_v \frac{u_t + v_t - \omega_t}{\alpha + c + h - \frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega} - v_t\right]^2 \end{aligned}$$

The first-order condition becomes

$$\delta_v \frac{\sigma_v^2 + \sigma_\omega^2 + \sigma_u^2}{\alpha + c + h - \frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega} - \sigma_v^2 = 0 \qquad (**)$$

Likewise, the first order condition determining δ_ω becomes

$$\delta_\omega \frac{\sigma_v^2 + \sigma_\omega^2 + \sigma_u^2}{\alpha + c + h - \frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega} + \sigma_\omega^2 = 0 \qquad (***)$$

Combining (*), (**) and (***) reveals that the estimates are

$$\begin{aligned}\widehat{\delta}_u &= \alpha, \\ \widehat{\delta}_v &= \frac{\alpha\sigma_v^2}{\sigma_u^2}, \\ \widehat{\delta}_\omega &= -\frac{\alpha\sigma_\omega^2}{\sigma_u^2}.\end{aligned}$$

Hence, the forecast-based rule for b is given by

$$\begin{aligned}b_t &= \left(-\frac{c+h}{\alpha}\widehat{\delta}_u + \widehat{\delta}_v - \widehat{\delta}_\omega\right) i_t \\ &= \left[-(c+h) + \frac{\alpha\left(\sigma_v^2 + \sigma_\omega^2\right)}{\sigma_u^2}\right] i_t \\ &= \mu^* i_t.\end{aligned}$$

I.e., the exact same optimal rule as derived before by (9.11).

B Deriving (9.21) in Walsh (2003)

This appendix shows how to derive the nominal interest rate securing that the actual money supply always is equal to the value securing the average inflation target. I.e., the nominal interest rate that uses the actual money supply as an intermediate target.

The model is given by

$$\begin{aligned}y_t &= a\left(\pi_t - E_{t-1}\pi_t\right) + z_t \\ y_t &= -\alpha\left(i_t - E_t\pi_{t+1}\right) + u_t \\ m_t - p_t &= m_t - \pi_t - p_{t-1} = y_t - ci_t + v_t\end{aligned}$$

What money supply would give an inflation target of π^*? The trick is to acknowledge that with the strict inflation targeting preferences, in expected value we have inflation on target. I.e., $\mathrm{E}_{t-1}\pi_t = \mathrm{E}_t\pi_{t+1} = \pi^*$. Hence the model is rewritten as

$$\begin{aligned}y_t &= a\left(\pi_t - \pi^*\right) + z_t \\ y_t &= -\alpha\left(i_t - \pi^*\right) + u_t \\ m_t - p_t &= m_t - \pi_t - p_{t-1} = y_t - ci_t + v_t\end{aligned}$$

Now, the LM curve is inserted into the IS curve to eliminate i_t:

$$y_t = -\alpha\frac{y_t + v_t - m_t + \pi_t + p_{t-1}}{c} + \alpha\pi^* + u_t,$$

and

$$\begin{aligned}y_t\left(1 + \frac{\alpha}{c}\right) &= \alpha\frac{-v_t + m_t - \pi_t - p_{t-1}}{c} + \alpha\pi^* + u_t, \\ y_t\frac{\alpha + c}{c} &= \alpha\frac{-v_t + m_t - \pi_t - p_{t-1}}{c} + \alpha\pi^* + u_t,\end{aligned}$$

$$y_t = \frac{\alpha}{\alpha + c}(m_t - \pi_t - p_{t-1} - v_t) + \frac{c}{\alpha + c}(\alpha\pi^* + u_t).$$

We then find the *actual* inflation rate by combining this expression with the "modified" Lucas supply schedule:

$$\frac{\alpha}{\alpha + c}(m_t - \pi_t - p_{t-1} - v_t) + \frac{c}{\alpha + c}(\alpha\pi^* + u_t) = a(\pi_t - \pi^*) + z_t,$$

from which we get the solution for the inflation rate for a given money supply:

$$\frac{\alpha}{\alpha + c}(m_t - \pi_t - p_{t-1} - v_t) + \frac{c}{\alpha + c}(\alpha\pi^* + u_t) = a(\pi_t - \pi^*) + z_t,$$

$$\pi_t\left(a + \frac{\alpha}{\alpha + c}\right) = \frac{\alpha}{\alpha + c}(m_t - p_{t-1} - v_t) + \frac{c}{\alpha + c}(\alpha\pi^* + u_t) + a\pi^* - z_t$$

$$\pi_t\frac{a(\alpha + c) + \alpha}{\alpha + c} = \frac{\alpha}{\alpha + c}(m_t - p_{t-1} - v_t) + \frac{c}{\alpha + c}(\alpha\pi^* + u_t) + a\pi^* - z_t$$

and therefore

$$\pi_t = \frac{\alpha}{a(\alpha + c) + \alpha}(m_t - p_{t-1} - v_t) + \frac{c}{a(\alpha + c) + \alpha}(\alpha\pi^* + u_t) + \frac{a(\alpha + c)\pi^* - (\alpha + c)z_t}{a(\alpha + c) + \alpha}$$

We then solve for the value of m_t that secures $\pi_t = \pi^*$. I.e., this value must satisfy

$$\pi^* = \frac{\alpha}{a(\alpha + c) + \alpha}(m_t - p_{t-1} - v_t) + \frac{c}{a(\alpha + c) + \alpha}(\alpha\pi^* + u_t) + \frac{a(\alpha + c)\pi^* - (\alpha + c)z_t}{a(\alpha + c) + \alpha},$$

from which we get

$$\pi^*\left[1 - \frac{c\alpha + a(\alpha + c)}{a(\alpha + c) + \alpha}\right] = \frac{\alpha}{a(\alpha + c) + \alpha}(m_t - p_{t-1} - v_t) + \frac{c}{a(\alpha + c) + \alpha}u_t - \frac{(\alpha + c)}{a(\alpha + c) + \alpha}z_t,$$

and

$$\pi^*\frac{a(\alpha + c) + \alpha - c\alpha - a(\alpha + c)}{a(\alpha + c) + \alpha} = \frac{\alpha}{a(\alpha + c) + \alpha}(m_t - p_{t-1} - v_t) + \frac{c}{a(\alpha + c) + \alpha}u_t - \frac{(\alpha + c)}{a(\alpha + c) + \alpha}z_t,$$

$$\pi^*\frac{\alpha(1 - c)}{a(\alpha + c) + \alpha} = \frac{\alpha}{a(\alpha + c) + \alpha}(m_t - p_{t-1} - v_t) + \frac{c}{a(\alpha + c) + \alpha}u_t - \frac{(\alpha + c)}{a(\alpha + c) + \alpha}z_t,$$

and finally

$$m_t = p_{t-1} + v_t + (1 - c)\pi^* - \frac{c}{\alpha}u_t + \frac{\alpha + c}{\alpha}z_t.$$

As shocks are unobservable, the optimal target of m_t is given by

$$\widehat{m}_t = p_{t-1} + \rho_v v_{t-1} + (1 - c)\pi^* - \frac{c}{\alpha}\rho_u u_{t-1} + \frac{\alpha + c}{\alpha}\rho_z z_{t-1} \tag{9.19}$$

The actual money supply, for a given interest rate $\widehat{i_t}$, follows from the LM curve as

$$m_t\left(\widehat{i_t}\right) = \pi_t\left(\widehat{i_t}\right) + p_{t-1} + y_t\left(\widehat{i_t}\right) - c\widehat{i_t} + v_t. \tag{*}$$

Note that we have that

$$\widehat{i_t} = \pi^* + \frac{1}{\alpha}\left(\rho_u u_{t-1} - \rho_z z_{t-1}\right) \tag{9.17}$$

and

$$\pi_t\left(\widehat{i_t}\right) = \pi^* + \frac{\varphi_t - e_t}{a} \tag{9.18}$$

We can then find $y_t\left(\widehat{i_t}\right)$ by inserting $\pi_t\left(\widehat{i_t}\right)$ into the Lucas supply schedule:

$$\begin{aligned} y_t\left(\widehat{i_t}\right) &= a\left(\pi^* + \frac{\varphi_t - e_t}{a} - \pi^*\right) + z_t \\ &= \varphi_t - e_t + z_t \\ &= \varphi_t + \rho_z z_{t-1} \end{aligned}$$

Then insert the found expressions for $\pi_t\left(\widehat{i_t}\right)$, $y_t\left(\widehat{i_t}\right)$ and $\widehat{i_t}$ into (*):

$$\begin{aligned} m_t\left(\widehat{i_t}\right) &= \pi^* + \frac{\varphi_t - e_t}{a} + p_{t-1} + \varphi_t + \rho_z z_{t-1} \\ &\quad -c\left[\pi^* + \frac{1}{\alpha}\left(\rho_u u_{t-1} - \rho_z z_{t-1}\right)\right] + v_t \\ &= (1-c)\,\pi^* + p_{t-1} + v_t + \frac{1+a}{a}\varphi_t - \frac{c}{\alpha}\rho_u u_{t-1} - \frac{1}{a}e_t + \frac{\alpha + c}{\alpha}\rho_z z_{t-1} \quad (**) \end{aligned}$$

From (9.19) note that

$$\widehat{m}_t - \rho_v v_{t-1} = p_{t-1} + (1-c)\,\pi^* - \frac{c}{\alpha}\rho_u u_{t-1} + \frac{\alpha + c}{\alpha}\rho_z z_{t-1},$$

which applied on (**) yields

$$\begin{aligned} m_t\left(\widehat{i_t}\right) &= \widehat{m}_t - \rho_v v_{t-1} + v_t + \frac{1+a}{a}\varphi_t - \frac{1}{a}e_t \\ &= \widehat{m}_t + \psi_t + \frac{1+a}{a}\varphi_t - \frac{1}{a}e_t \end{aligned} \tag{9.20}$$

Now, when actual m_t conditional on $\widehat{i_t}$ changes relative to $\widehat{m}_t$, it is time to change i_t such that $m_t = \widehat{m}_t$ again. What value of the interest rate will accomplish that? I.e., how do we derive equation (9.21) on page 443 in Walsh (2003)?

The trick is to solve the model for m_t as a function of *any* value of the interest rate, and then find the interest rate that delivers $m_t = \widehat{m}_t$. This can be accomplished by the central bank, as it observes m_t even though it doesn't observe the various period-t disturbances.

As the model is

$$\begin{aligned} y_t &= a\left(\pi_t - \pi^*\right) + z_t \\ y_t &= -\alpha\left(i_t - \pi^*\right) + u_t \\ m_t - p_t &= m_t - \pi_t - p_{t-1} = y_t - c i_t + v_t \end{aligned}$$

we first combine the AS and IS curve to find inflation as a function of the interest rate:

$$a\left(\pi_t - \pi^*\right) + z_t = -\alpha\left(i_t - \pi^*\right) + u_t$$

and thus

$$\pi_t = \frac{a+\alpha}{a}\pi^* - \frac{\alpha}{a}i_t + \frac{1}{a}(u_t - z_t)$$

We have output a function of the interest rate directly from the IS curve:

$$y_t = -\alpha(i_t - \pi^*) + u_t$$

We can the use this in the LM relationship to find

$$\begin{aligned} m_t &= \frac{a+\alpha}{a}\pi^* - \frac{\alpha}{a}i_t + \frac{1}{a}(u_t - z_t) + p_{t-1} - \alpha(i_t - \pi^*) + u_t - ci_t + v_t \\ &= -\frac{\alpha(1+a)+ca}{a}i_t + \frac{a+\alpha+\alpha a}{a}\pi^* + p_{t-1} + \frac{1+a}{a}u_t + v_t - \frac{1}{a}z_t \end{aligned}$$

Securing that $m_t = \widehat{m}_t$ requires that we use (9.19) and find the value of i_t that secures this:

$$\begin{aligned} & -\frac{\alpha(1+a)+ca}{a}i_t + \frac{a+\alpha+\alpha a}{a}\pi^* + p_{t-1} + \frac{1+a}{a}u_t + v_t - \frac{1}{a}z_t \\ = \; & p_{t-1} + \rho_v v_{t-1} + (1-c)\pi^* - \frac{c}{\alpha}\rho_u u_{t-1} + \frac{\alpha+c}{\alpha}\rho_z z_{t-1}, \end{aligned}$$

or,

$$\begin{aligned} & -\frac{\alpha(1+a)+ca}{a}i_t + \frac{a+\alpha+\alpha a}{a}\pi^* + \frac{1+a}{a}u_t + v_t - \frac{1}{a}z_t \\ = \; & \rho_v v_{t-1} + (1-c)\pi^* - \frac{c}{\alpha}\rho_u u_{t-1} + \frac{\alpha+c}{\alpha}\rho_z z_{t-1}, \end{aligned}$$

$$\begin{aligned} & -\frac{\alpha(1+a)+ca}{a}i_t + \frac{a+\alpha+\alpha a}{a}\pi^* + \frac{1+a}{a}\rho_u u_{t-1} \\ & +\frac{1+a}{a}\varphi_t + \psi_t - \frac{1}{a}\rho_z z_{t-1} - \frac{1}{a}e_t \\ = \; & (1-c)\pi^* - \frac{c}{\alpha}\rho_u u_{t-1} + \frac{\alpha+c}{\alpha}\rho_z z_{t-1}, \end{aligned}$$

$$\begin{aligned} & -\frac{\alpha(1+a)+ca}{a}i_t + \frac{a+\alpha+\alpha a}{a}\pi^* + \left[\frac{1+a}{a} + \frac{c}{\alpha}\right]\rho_u u_{t-1} \\ & +\frac{1+a}{a}\varphi_t + \psi_t - \left(\frac{1}{a} + \frac{\alpha+c}{\alpha}\right)\rho_z z_{t-1} - \frac{1}{a}e_t \\ = \; & (1-c)\pi^* \end{aligned}$$

$$\begin{aligned} & \left(\frac{a+\alpha+\alpha a}{a} + c - 1\right)\pi^* + \left[\frac{1+a}{a} + \frac{c}{\alpha}\right]\rho_u u_{t-1} \\ & +\frac{1+a}{a}\varphi_t + \psi_t - \left(\frac{1}{a} + \frac{\alpha+c}{\alpha}\right)\rho_z z_{t-1} - \frac{1}{a}e_t \\ = \; & \frac{\alpha(1+a)+ca}{a}i_t \end{aligned}$$

$$\begin{aligned}
&\left(\frac{a+\alpha+\alpha a}{a}+c-1\right)\pi^* + \frac{\alpha(1+a)+ca}{a\alpha}\rho_u u_{t-1} \\
&+\frac{1+a}{a}\varphi_t + \psi_t - \frac{\alpha + a(\alpha+c)}{a\alpha}\rho_z z_{t-1} - \frac{1}{a}e_t \\
= &\ \frac{\alpha(1+a)+ca}{a}i_t
\end{aligned}$$

An thus

$$\begin{aligned}
i_t\frac{\alpha(1+a)+ca}{a} &= \frac{\alpha(1+a)+ca}{a}\pi^* \\
&+\frac{\alpha(1+a)+ca}{a\alpha}\left(\rho_u u_{t-1} - \rho_z z_{t-1}\right) \\
&+\frac{1+a}{a}\varphi_t + \psi_t - \frac{1}{a}e_t,
\end{aligned}$$

which finally gives

$$\begin{aligned}
i_t &= \pi^* + \frac{1}{\alpha}\left(\rho_u u_{t-1} - \rho_z z_{t-1}\right) \\
&+\frac{(1+a)\varphi_t - e_t + a\psi_t}{\alpha(1+a)+ca}.
\end{aligned}$$

Using the result for $\widehat{i_t}$, equation (9.17), this readily reduces to

$$i_t = \widehat{i_t} + \frac{(1+a)\varphi_t - e_t + a\psi_t}{\alpha(1+a)+ca} \equiv i_t^T$$

which is equation (9.21) in Walsh (2003).

C Key concepts you should know

Choice of monetary policy instrument/operating procedures

- The Poole (1970) model
- The importance of relative variances of shocks
- The importance of the goals of policy for the determination of optimal instrument
- Money base as potential instrument: more "financial" instability causing it more likely that interest rate is optimal instrument
- A money base rule covering three "extreme" operating procedures
  - Optimal rule as an intermediate case
- The importance of forecasts of shocks
- The importance of operating procedure for identification of monetary policy

Intermediate targets

- Adjusting policy instrument towards variable providing good information about ultimate goal variables
- Example of unobservable shocks, but observable money supply, under strict inflation targeting
- Adjusting interest rate to attain intermediate target for money supply
  - Good if demand and supply shocks are important
  - Bad if money demand shocks are important
- Desirability of an intermediate target variable depends on relative variances of shocks

Price level (in)determinacy

- Using the nominal interest rate as instrument may render the price level indeterminate
- Circumvention of problem by having the price level or money supply re-enter the model
  - Feedback interest rate rule towards price level
  - Feedback interest rate rule towards money supply

The term structure of interest rates

- The link between short and long interest rates
- The expectations theory of the term structure
  - Long rates as average of expected current and future short rates
  - The role of credibility of future short interest rate setting
- The yield curve as an indicator for expectations about future monetary policy
- Empirical problems with the expectations hypothesis
  - The importance of actual policymaking for the empirical failure of the expectations hypothesis

- The relationship between long rates and inflation expectations

Impact of interest rate rule parameters in simple model MIU style model

- Changes in policy rule parameters change impact of shocks
  - No "policy irrelevance"

Optimal interest rate rule parameters in simpler model

- Instability of economy for fixed nominal interest rate
- Optimal policy rule must:
  - Secure stability
  - Minimize output and inflation fluctuations
- Trick of treating expected demand as policy instrument
- Solution of optimal expected demand by dynamic programming
- Finding explicit solution for expected demand by method of undetermined coefficients
- Properties of optimal interest rate rule
  - Higher inflation increases nominal interest rate by more => higher real interest rate => stability
  - More weight on output stabilization; less weight on inflation in policy rule
- Optimal to respond to output, even if inflation is all that matters
  - Arguments in policy rules tells nothing about the ultimate goals of policy
  - Size of response to variable says nothing about policy preferences (may as well reflect the economic structure)

International evidence for interest rate rules

- The Taylor rule looks very much like the optimal rule derived above
  - Coefficient on inflation higher than one
  - Positive coefficient on output gap

- Not to be seen as a mechanical rule
- Other countries' policy rules also look like Taylor-type rules (but may be "forward-looking")