7. Open-economy Aspects and Monetary Policy Coordination*

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Abstract

Notes for the course "Monetary Economics: Macro Aspects," Spring 2006. The relevant literature behind these notes is:

Walsh (2003, Chapter 6, pp. 269-297).

Recommended reading (not required): Clarida et al. (2002); Benigno (2002); Walsh (2003, Chapter 6, pp. 297-304)

1 Introductory remarks

- All analyses so far have been confined to closed-economy models. This obviously misses out important monetary transmission mechanisms in most real-life economies, which are engaged in trade with other economies. E.g., the exchange rate channel is ignored, as is the impact of external shocks (policy- and non-policy shocks). Therefore, we now look into open-economy aspects of monetary economics.

- First, a tractable, monetary general equilibrium model for open economies is formulated. It builds closely on the, now famous, paper by Maurice Obstfeld and Kenneth Rogoff (1995, *Journal of Political Economy*): "Exchange Rate Dynamics Redux." The paper and model initiated a new generation of micro-founded, open-economy models with incomplete nominal adjustment. A large (and still growing) literature is now known as the "New Open Economy Macroeconomics." While it is inherently a sticky-price model, the foundations of the model are easiest seen in a flexible price version. Incomplete nominal adjustment, in the form of price rigidity, is then introduced subsequently.

- Secondly, we look into the issue of policy coordination. Such an issue becomes of relevance when one country's policy actions affect another country's economy, and vice versa. Monetary policy is no exception, and a simple model of international monetary policy interactions are examined.

2 The Obstfeld-Rogoff Two-Country Model

2.1 The flexible-price version

Basics

- The world economy is made up two countries: Domestic/Home and Foreign
  - A continuum $z \in [0, n]$ of agents live in Home country
  - A continuum $z \in (n, 1]$ of agents live in Foreign country

  Each agent is monopolist supplier of a distinct good, which is an imperfect substitute with all other goods. I.e., a monopolistic competition market structure is assumed

- The utility of a representative domestic agent j is given by:

$$U^j = \sum_{t=0}^{\infty} \beta^t \left[\log C_t^j + b \log \frac{M_t^j}{P_t} - \frac{k}{2} y_t(j)^2 \right], \quad b, \ k > 0 \tag{6.1}$$

The consumer index, C^j, is defined as:

$$C^j \equiv \left[\int_0^1 c^j(z)^q \, \mathrm{d}z \right]^{\frac{1}{q}}, \qquad 0 < q < 1. \tag{6.2}$$

Hence, it is an aggregate of all goods $c(z)$ produced in the world. Note that since $q < 1$, goods are imperfect substitutes. Correspondingly, the domestic consumer price index is given by (see Appendix B for a derivation, but note that one has to have read Appendix A on demand for each good $c(z)$ in order to follow these computations; so, wait a little):

$$P = \left[\int_0^1 p(z)^{\frac{q}{q-1}} \, \mathrm{d}z \right]^{\frac{q-1}{q}}. \tag{6.3}$$

I.e., an aggregate of all goods prices $p(z)$. The foreign agents' utility functions are similar

- The nominal budget constraint of representative domestic agent j is given by:

$$P_t C_t^j + M_t^j + P_t T_t + P_t B_t^j \leq p_t(j) \, y_t(j) + R_{t-1} P_t B_{t-1}^j + M_{t-1}^j$$

  - B_t^j is an internationally freely tradable bond with real (gross) return R_t
  - T_t are real government taxes
  - Balanced government budget and no government spending:

$$0 = P_t T_t + (M_t - M_{t-1})$$

    (note mistake in Walsh, 2003, p. 271: "$P_t T_t = (M_t - M_{t-1})$)"
  - Money supplies (Home and Foreign) are the policy instruments in the model

Budget constraint in real terms:

$$C_t^j + \frac{M_t^j}{P_t} + T_t + B_t^j \leq \frac{p_t(j)}{P_t} y_t(j) + R_{t-1} B_{t-1}^j + \frac{1}{1+\pi_t} \frac{M_{t-1}^j}{P_{t-1}}$$

where π_t is domestic consumer-price inflation

- There are no barriers to trade in goods. Hence, the price of a good z is the same Home and abroad measured in common currency:

$$p(z) = S p^*(z)$$

  - S is nominal exchange rate defined as the price of foreign currency in terms of domestic currency. Hence, when S goes up we have a domestic depreciation
  - $p^*(z)$ is foreign currency price of good z (stars indicate foreign variables)

- This law-of-one-price leads to PPP — purchasing power parity:

$$P_t = S_t P_t^* \tag{6.8}$$

  * Note! Empirically problematic assumption in short run....

Optimal behavior

- Each agent must determine:
  - Aggregate consumption dynamics (and thus asset holdings)
  - Relative demand for different goods (i.e., the "composition" of aggregate cpm-sumption)
  - Production effort of "own" good

Optimal production decision

- Optimal demand for good z by individual j is found as (see Appendix **??**

$$c^j(z) = \left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C^j.$$

  - Demand for good z is increasing in total consumption
  - Demand for good z is decreasing in the real price of the good, $p(z)/P$
  - Note that $1/(1-q)$ is the elasticity of substitution between goods
  - Hence, the limiting case of perfect competition applies when $q \to 1$ (horizontal demand curve and thus no monopoly power)

Total demand in the two countries of a good z is n times domestic demand plus $1-n$ times foreign demand. A representative foreign consumer's demand function is

$$c^{*j}(z) = \left[\frac{p^*(z)}{P^*}\right]^{-\frac{1}{1-q}} C^{*j}.$$

Since the nominal exchange rate is S, this is the same as

$$c^{*j}(z) = \left[\frac{Sp^*(z)}{SP^*}\right]^{-\frac{1}{1-q}} C^{*j},$$

and thus, by the law of one price,

$$c^{*j}(z) = \left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C^{*j}$$

Total demand, $y^d(z)$, is therefore given by

$$\begin{aligned} y^d(z) &= n\left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C + (1-n)\left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C^* \\ &= \left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} [nC + (1-n)C^*] \\ &= \left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C^w, \end{aligned}$$

where $C^w \equiv nC + (1-n)C^*$ is world consumption. Now, optimal production given by standard condition determining the consumption-leisure trade-off:

$$\frac{MUTILIY_{leisure}}{MUTILITY_{consumption}} = MRETURN\ OF\ WORK$$

or,

$$-\frac{MLOSS_{work}}{MUTILITY_{consumption}} = MRETURN\ OF\ WORK$$

We have by the specific form of the utility function:

$$-\frac{MLOSS_{work}}{MUTILITY_{consumption}} = \frac{ky_t^j}{(1/C_t^j)}.$$

What is the marginal return from work? The total real return is

$$\frac{p_t(z)\,y_t(z)}{P_t};$$

i.e., the total earned income, $p_t(z)\,y_t(z)$, deflated by the price index. Hence, the marginal return is found as

$$\frac{p_t(z)}{P_t} + \left[\partial\left(\frac{p_t(z)}{P_t}\right)/\partial y_t(z)\right] y_t(z).$$

To determine $\partial\left(\frac{p_t(z)}{P_t}\right)/\partial y_t(z)$, the real price is found by "inverting" the demand curve:

$$\frac{p_t(z)}{P_t} = \left[\frac{y_t^d(z)}{C_t^w}\right]^{q-1}$$

Note that with $q < 1$ the marginal return for work effort is lower than the real price, as more work reduces the relative price by reducing $p_t(z)$ — the standard monopoly effect. Only with $q \to 1$ is the price taken as given, and perfect competition mimicked. The marginal product of work effort is therefore

$$\frac{p_t(z)}{P_t} + (q-1)\left[\frac{y_t^d(z)}{C_t^w}\right]^{q-2} \frac{1}{C_t^w} y_t(z) < \frac{p_t(z)}{P_t},$$

or,

$$\left[\frac{y_t^d(z)}{C_t^w}\right]^{q-1} + (q-1)\left[\frac{y_t^d(z)}{C_t^w}\right]^{q-1},$$

and thus

$$q\left[\frac{y_t^d(z)}{C_t^w}\right]^{q-1}$$

The optimal consumption-leisure choice is therefore characterized by

$$\frac{ky_t^j}{\left(1/C_t^j\right)} = q\left[\frac{y_t^j}{C_t^w}\right]^{q-1} \tag{6.6'}$$

Optimal money demand decision

- This is derived from the by now standard, general condition:

$$\frac{MUTILITY_{money}}{MUTILITY_{consumption}} = \frac{i_t}{1+i_t}$$

with $i_t = R_t(1+\pi_{t+1}) - 1$ being the nominal interest rate. With the assumed functional form of the utility function this condition becomes

$$\frac{b\left[1/\left(M_t^j/P_t\right)\right]}{1/C_t^j} = \frac{i_t}{1+i_t},$$

which results in a simple money demand function:

$$\frac{M_t^j}{P_t} = bC_t^j\frac{1+i_t}{i_t}. \tag{6.7}$$

Optimal intertemporal consumption allocation

- This allocation is obtained by a version of the Keynes-Ramsey rule:

$$MUTILITY_{consumption}(t) = \beta R_t MUTILITY_{consumption}(t+1),$$

which with the functional form of the utility function becomes

$$\frac{1}{C_t^j} = \beta R_t\frac{1}{C_{t+1}^j},$$

or,

$$C_{t+1}^j = \beta R_t C_t^j \tag{6.5}$$

Equilibrium conditions and flex-price implications

- The equilibrium satisfies the optimality conditions (Home and Foreign); the budget constraints, the PPP condition, and two additional market clearing conditions:

- Goods market clearing:

$$C_t^w = n\frac{p_t(h)}{P_t}y_t(h) + (1-n)\frac{p_t^*(f)}{P_t^*}y_t^*(f) \equiv Y_t^w$$

with $p_t(i)$ and $y_t(i)$ being representative price and production in country i

- Bond-market clearing:

$$nB_t + (1-n)B_t^* = 0$$

- As is well known, monopolistic competition gives no real effects of money in itself. Nevertheless, it is a natural modelling framework when introducing sticky prices further on. The lack of real effects under flexible prices is also confirmed in this model. E.g., an increase in M_t leads to a proportional increase in $p_t(h)$ and P_t and a proportional increase in S_t (a nominal depreciation). *All* real variables and foreign prices are unchanged

- The model also exhibits superneutrality. An increase in the growth rate of M_t will increase Home CPI inflation and Home nominal interest rate, and create a higher rate of growth in S_t. All real variables and foreign prices are unchanged (this follows since the utility function is separable in money and real variables

- These properties are emphasized by the steady-state solution for domestic and foreign consumption (from the budget constraints):

$$\begin{aligned} C &= \frac{p(h)}{P}y(h) + (R-1)B & (6.9)\\ C^* &= \frac{p^*(f)}{P^*}y^*(f) - (R-1)\frac{n}{1-n}B & (6.10) \end{aligned}$$

Real steady-state consumption equals real steady-state income (real value of output plus/minus bond income)

A linear approximation

- As indicated, the nominal exchange rate plays a role under flexible prices to neutralize real effects of monetary shocks. A log-linearized version of model serves to highlight this (lower-case letters are log deviations from steady state). The Home optimality conditions are

$$\begin{aligned} y_t &= \frac{1}{1-q}[p_t - p_t(h)] + c_t^w, & (6.13)\\ (2-q)y_t &= (1-q)c_t^w - c_t, & (6.18)\\ m_t - p_t &= c_t - \delta(r_t + \pi_{t+1}), \qquad \delta = \beta/\left(\overline{\Pi} - \beta\right), & (6.20)\\ c_{t+1} &= c_t + r_t. & (6.16) \end{aligned}$$

Domestic price index written in terms of foreign producer prices and the nominal exchange rate:

$$p_t = np_t(h) + (1-n)\left[s_t + p_t^*(f)\right]. \tag{6.11}$$

These five equations plus the five Foreign analogues equations, and the definition of c_t^w, will determine the eleven endogenous variables:

- Home and foreign production (y_t, y_t^*)
- Home, foreign and world consumption (c_t, c_t^*, c_t^w)
- Prices and the nominal exchange rate $(p_t(h), p_t, p_t^*(f), p_t^*, s_t)$
- The real interest rate (r_t)

- Money neutrality is seen immediately from linearized model. The two price indices, imply the PPP relationship:

$$s_t = p_t - p_t^*$$

Used in domestic price index:

$$p_t = np_t(h) + (1-n)\left(p_t - p_t^* + p_t^*(f)\right)$$
$$0 = n\left[p_t(h) - p_t\right] + (1-n)\left[p_t^*(f) - p_t^*\right]$$

or written compactly,

$$0 = n\chi_t + (1-n)\chi_t^*.$$

The demand functions are correspondingly re-written as

$$y_t = \frac{1}{1-q}\chi_t + c_t^w,$$
$$y_t^* = \frac{1}{1-q}\chi_t^* + c_t^w$$

- The last three equations will along with the two consumption Euler equations, the two production decision equations, and the definition of c_t^w determine the **real allocation**: χ_t, χ_t^*, y_t, y_t^*, c_t, c_t^*, c_t^w, r_t. Hence, changes in m_t or m_t^* have no real effects. There are only effects on prices and the nominal exchange rate (as mentioned previously, an increase in m_t increase p_t, $p_t(h)$ and s_t proportionally)

- To see how the nominal exchange rate is determined in the model, subtract the money market equilibrium conditions:

$$m_t - m_t^* - (p_t - p_t^*) = (c_t - c_t^*) - \delta\left(\pi_{t+1} - \pi_{t+1}^*\right)$$

(note that the inflation differential measures the nominal interest differential, as Home and Foreign face same real interest rate). Then use the PPP relationship in level and change form:

$$m_t - m_t^* - s_t = (c_t - c_t^*) - \delta\left(s_{t+1} - s_t\right). \tag{6.23}$$

This is a first-order (expectational) difference equation in s_t:

$$s_t = \frac{\delta}{1+\delta} s_{t+1} + \frac{1}{1+\delta} \left[(m_t - m_t^*) - (c_t - c_t^*) \right]$$

Solving (6.23) successively forward and imposing $\lim_{i \to \infty} \left(\delta / \left(1+\delta \right) \right)^i s_{t+i} = 0$ yields

$$s_t = \frac{1}{1+\delta} \sum_{i=0}^{\infty} \left(\frac{\delta}{1+\delta} \right)^i \left[\left(m_{t+i} - m_{t+i}^* \right) - \left(c_{t+i} - c_{t+i}^* \right) \right]$$

Hence, the nominal exchange rate equals the present value of current and future relative money supplies and demands (the latter quantified by the consumption differential). This expression can be simplified as any consumption differential is permanent since

$$c_{t+1} = r_t + c_t$$

and

$$c_{t+1}^* = r_t + c_t^*$$

imply

$$c_{t+1} - c_t = c_{t+1}^* - c_t^*$$

and

$$c_{t+1} - c_{t+1}^* = c_t - c_t^*.$$

Hence,

$$s_t = -\left(c_t - c_t^* \right) + \frac{1}{1+\delta} \sum_{i=0}^{\infty} \left(\frac{\delta}{1+\delta} \right)^i \left(m_{t+i} - m_{t+i}^* \right)$$

(since $\frac{1}{1+\delta} \sum_{i=0}^{\infty} \left(\frac{\delta}{1+\delta} \right)^i = 1$)

- Note therefore that if
  - $m_t > m_t^*$: relative supply of home currency increases, and the relative value of the currency falls (s_t increases; home currency depreciates)
  - $m_{t+i} > m_{t+i}^*$: future relative supply of home currency increases; future inflation differential increases, and reduces relative demand for home currency; relative value of the currency falls (s_t increases; home currency depreciates)
  - $c_t > c_t^*$: current or future money demand shifts towards home currency; relative value of the currency increases (s_t decreases; home currency appreciates)

- An alternative exposition features the relationship between nominal interest rates and the nominal exchange rates; *uncovered interest parity.* The real interest rate is identical across countries:

$$i_t - \pi_{t+1} = i_t^* - \pi_{t+1}^*$$

and thus

$$i_t - i_t^* = \pi_{t+1} - \pi_{t+1}^*$$

From the PPP relationship in first differences:

$$i_t - i_t^* = s_{t+1} - s_t$$

Hence, in a world with perfect capital mobility and floating exchange rates, the nominal interest rate differential equals the (expected) depreciation rate of home currency. Only then is holding Home and Foreign bonds equally attractive

- Empirically, tests of the "UIP" relationship have been unsuccessful
  - Not necessarily reason for abandoning model
  - Note, as with estimations of term-structure relationships covered in Chapter 10, the manner which monetary policy is conducted will matter for regression results:
    - If the nominal interest rate was instrument, and was responding to the exchange rate, a regression of the type

      $$s_{t+1} - s_t = a + b\left(i_t - i_t^*\right) + \varphi_{t+1}$$

      should not necessarily give $b = 1$ (as $i_t - i_t^*$ would be a positive function of s_t)

2.2 The sticky-price version and money shocks

- Now, a sticky-price version of the Obstfeld-Rogoff model is examined. Questions addressed:
  - What are the impact of monetary shocks?
  - What are the impacts of an asymmetric monetary shock on the Home and Foreign economies?
  - What are the "spill-overs" of unilateral policy?
  - What are the welfare effects of money shocks?
- The flex-price Obstfeld-Rogoff model featured money neutrality. As mentioned, to introduce real effects of money shocks in the Home and Foreign countries, price stickiness is introduced. The manner this is done is the following. Producers set their prices for one period, one period in advance. I.e., in any period $p_t(z)$ and $p_t^*(z)$ are now exogenous variables. It is assumed that these prices are set equal to the steady-state value of the previous period. For simplicity we thus have $p_t(z) = p_t^*(z) = 0$.

- Note that stickiness in producer-prices does *not* imply stickiness in consumer prices:

$$\begin{aligned} p_t &\equiv np_t(h) + (1-n)\left[s_t + p_t^*(f)\right] \\ &= (1-n)\,s_t \end{aligned} \tag{6.11}$$

$$\begin{aligned} p_t^* &\equiv (1-n)\,p_t^*(f) + n\left[p_t(h) - s_t\right] \\ &= -ns_t \end{aligned} \tag{6.12}$$

  Changes in the nominal exchange rate will change consumer prices. E.g., a nominal depreciation, $s_t \uparrow$, increases p_t and decreases p_t^*. This will change the real prices of goods and thereby real demand and production!

- Policy experiment. Consider an unanticipated, permanent increase in the Home money supply. Premise for "evaluation" of experiment: After the period of the shock, all prices re-adjust and economy returns to (potentially new!) steady state.

  - Hence, we distinguih between **short-run effects** (one period) and **long-run effects** (steady state)

- Central for transmission of shock in short run is nominal exchange rate effect

- Central for (potential) transmission of shock in long run is wealth redistribution between countries. I.e., if short-run effects imply a current account imbalance, a country will accumulate claims on the other countyr. This induces permanent wealth effects (in absence of further shocks).

- To reiterate: The nominal exchange rate is determined from money market equilibrium conditions:

$$m_t - m_t^* - (p_t - p_t^*) = (c_t - c_t^*) - \delta\left(\pi_{t+1} - \pi_{t+1}^*\right)$$

$$m_t - m_t^* - s_t = (c_t - c_t^*) - \delta\left(s_{t+1} - s_t\right) \tag{6.23}$$

  Solving (6.23) successively forward led to

$$s_t = \frac{1}{1+\delta}\sum_{i=0}^{\infty}\left(\frac{\delta}{1+\delta}\right)^i \left[\left(m_{t+i} - m_{t+i}^*\right) - \left(c_{t+i} - c_{t+i}^*\right)\right].$$

- Now, since a change in relative money supplies from the policy shock is permanent (by assumption), any consumption differential is permanent (by the Euler equations and perfect capital mobility implying equal real interest rates). We thus get

$$s_t = \Omega - \mathcal{C} \tag{6.28}$$

where $\Omega > 0$ is money differential and where $\mathcal{C}$ is the consumption differential. Thus, if the consumption differential increases by less than the money differential, the money shock depreciates the nominal exchange rate. This will indeed be the case as tedious algebra shows that $\mathcal{C} = \Omega / (1 + \psi)$, with $\psi > 0$. Hence, the policy shock induces a less than proportional nominal exchange rate depreciation (in contrast with the flexible price case).

- Short-run implications:
  - Depreciation of exchange rate increases the Home CPI, p_t
  - This reduces real price of domestic goods, and demand increases relative to demand for foreign goods
  - Home and foreign consumption **increases** and Home production increases (thus causing Home consumption to increase the most).
  - World consumption increases and (by global market clearing) world production increases
    - Foreign production may or may not increase:
    - Demand for foreign goods falls due to decrease in p_t^*, but increases due to increase in c_t^w
  - Home output increases by more than Home consumption
    - => Home current account surplus
    - => Accumulation of claims on Foreign country; i.e., Home net foreign assets go up
    - (b goes up, and b^* goes down)

- Long-run implications:
  - Higher Home wealth implies permanently higher home consumption (and lower production — country is continuously financing its trade deficit by the interest income on higher asset holdings)
  - So, long-run effects of the monetary shock due to wealth redistribution

- If policy experiment involved symmetric increase in money supplies, no nominal exchange rate effect, and only a one-period symmetric expansion, and no wealth redistribution.

- What are the welfare implications of the (asymmetric) shock? This is an intricate but interesting issue (where utility of real money per se is downplayed). Note first that the experiment represents a *marginal* change in home money supply (otherwise the linearization is not valid). Then note that in "pre-shock equilibrium" **all agents are behaving optimally**

  - This implies that changes in Home and Foreign work effort due to relative price changes are of **second order** in welfare terms (e.g., the higher income for Home work cancels out the higher disutility of work)
  - This implies that changes in consumption dynamics following current account imbalances — i.e., the long-run effects — are of **second order** in welfare terms (when starting from an equilibrium with optimal consumption smoothing)

  What is left to provide first-order welfare effects? The part in the initial equilibrium that is **suboptimal:** The inefficient output due to monopolistic competition. The short-run increase in world consumption affects this distortion in a **first-order** welfare-improving way

  - Hence, *both* countries' welfare increases *by the same amount* following **the asymmetric money shock** (remember, both production schedules are scaled by *world* consumption)[1]

- Unilateral money expansion has positive spill-overs

  - Countries have then incentives to perform expansive monetary policies; either unilaterally or coordinated

- Important caveat: It is **unanticipated** money increases that have real effects

  - If anticipated, they would have been build into price setting behavior
  - As a result attempts to boost output above the (inefficient) flex-price level, could lead to an inflation bias (the model thus provides micro foundations for the Barro-Gordon model)

[1] This is the case even when the net effect of the Home policy expansion is that foreign output decreases. In that case, the relative price effect dominates the aggregate demand effect. But as the *relative price effect has no effects on welfare* (when we consider marginal changes in policy; as we do), but the *aggregate demand effect has positive welfare implications*, the net effect on welfare is always positive. (And *the same* in both countries; Home agents "only" get a welfare gain from the aggregate demand increase; the increased production due to the relative price decrease, has no first-order welfare effects.)

- The Obstfeld-Rogoff model highlights the role of the nominal exchange rate for transmission of money shocks in open economies. It emphasizes importance of micro foundations (the welfare implications could in no way have been assessed properly in an ad hoc model). It highlights the spill-overs of unilateral policies, which immediately invites to an analysis of policy coordination
- This is, however, somewhat involved to analyze in the O-R model; therefore the issue, and the general messages are conducted in an *ad hoc* model.

3 Policy coordination

- With international spill-overs of unilateral policies, coordination may be desirable:
  - "Inward" oriented policies ignore by nature external effects => international inefficiencies
  - Through coordination, the external effects are internalized to benefit of all
  - ....an example is provided as illustration (not within a micro-founded model.....)
- The world portrayed is a symmetric log-linear two-country model of the AS/AD style. Home and foreign AS curves are given by

$$y_t = -b_1\rho_t + b_2\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + \varepsilon_t \tag{6.35'}$$

$$y_t^* = b_1\rho_t + b_2\left(\pi_t^* - \mathrm{E}_{t-1}\pi_t^*\right) + \varepsilon_t \tag{6.36'}$$

  - Inflation surprises increase output (e.g., due to one-period nominal wage rigidity)
  - A real exchange rate depreciation, $\rho_t \equiv s_t + p^* - p$ goes up, reduces Home supply
    - as imported inputs becomes more expensive,
    - and/or as the real product wage rises relative to the real consumer wage

  Home and foreign AD curves:

$$y_t = a_1\rho_t - a_2 r_t \tag{6.37'}$$

$$y_t^* = -a_1\rho_t - a_2 r_t^* \tag{6.38'}$$

  - Home demand increases by a real exchange rate depreciation (competition effect)
  - Home demand decreases with the real interest rate r_t

– (Compared with Walsh, 2003, spill-overs from other country's output ignored for simplicity; $a_3 = 0$; also, demand shocks are ignored $u_t = u_t^* = 0$)

UIP in real terms:

$$r_t - r_t^* = \mathrm{E}_t \rho_{t+1} - \rho_t \tag{6.39'}$$

- Monetary policy instruments are for simplicity taken to be the inflation rates
- "Welfare functions" of Home and Foreign country (conventional *ad hoc* quadratic loss functions)

$$V_t = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\lambda y_{t+i}^2 + \pi_{t+i}^2\right], \quad \lambda > 0, \tag{6.40}$$

$$V_t^* = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\lambda \left(y_{t+i}^*\right)^2 + \left(\pi_{t+i}^*\right)^2\right]. \tag{6.41}$$

Notice, "no-above-steady-state target" for output. Hence, there will be no inflation bias considerations

- In absence of shocks, $\varepsilon_t = 0$, everything is in steady state, and there is no reason for any policy. Assuming that the shock is mean-zero and serially uncorrelated, the policy problem(s) when a shock hits become a static one — the economy is in expectation back in steady state next period. Implication: $\mathrm{E}_t \rho_{t+1} = 0$

Solving the model under coordinated monetary policy

- Under coordination, central banks jointly choose π_t, π_t^* so as to minimize

$$V_t + V_t^*$$

subject to the model's equations.

- First, solve the model in terms of the real exchange rate, and thus outputs (to create unconstrained problem). The real exchange rate:
  – Demand differential:

$$y_t - y_t^* = 2a_1 \rho_t - a_2 \left(r_t - r_t^*\right)$$

$$\begin{aligned} y_t - y_t^* &= 2a_1 \rho_t + a_2 \rho_t \\ &= \left(2a_1 + a_2\right) \rho_t \end{aligned}$$

A real depreciation increases relative Home demand through two channels:

    * The direct relative demand shift channel
    * The decrease in the real interest rate differential (as a real appreciation is expected)

  – Supply differential:

$$y_t - y_t^* = -2b_1\rho_t + b_2\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) - b_2\left(\pi_t^* - \mathrm{E}_{t-1}\pi_t^*\right)$$

  A real depreciation decreases relative Home supply from one source:

    * The direct cost channel(s)

Equilibrium real exchange rate found from "intersection" of relative demand and supply schedules:

$$\begin{aligned}\rho_t &= \frac{b_2}{B'}\left[\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) - \left(\pi_t^* - \mathrm{E}_{t-1}\pi_t^*\right)\right] \\ B' &\equiv 2a_1 + a_2 + 2b_1\end{aligned} \tag{6.42'}$$

A Home inflation surprise causes a real depreciation as it increases the supply differential

  – The real exchange rate must depreciate to secure goods market equilibrium: i.e., increase the demand differential

With solution for real exchange rates, outputs are found as functions of policy instruments from AS-curves:

$$\begin{aligned}y_t &= -b_1\rho_t + b_2\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + \varepsilon_t \\ &= -b_1\frac{b_2}{B'}\left[\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) - \left(\pi_t^* - \mathrm{E}_{t-1}\pi_t^*\right)\right] \\ &\quad +b_2\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + \varepsilon_t\end{aligned}$$

and thus

$$y_t = b_2A_1'\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + b_2A_2'\left(\pi_t^* - \mathrm{E}_{t-1}\pi_t^*\right) + \varepsilon_t \tag{6.43'}$$

$$\begin{aligned}A_1' &\equiv \frac{2a_1 + a_2 + b_1}{B'} \\ A_2' &\equiv b_1/B'\end{aligned}$$

The international policy spill-over: Foreign inflation surprise is expansionary on Home output as it appreciates the real exchange rate. It reduces supply differential, and to equilibrate goods market equilibrium, relative demand must shift towards foreign goods necessitating a real appreciation.

- Equivalent output equation for the foreign country:

$$y_t^* = b_2A_1'\left(\pi_t^* - \mathrm{E}_{t-1}\pi_t^*\right) + b_2A_2'\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + \varepsilon_t \tag{6.44'}$$

- Under cooperation, the policy problem is to choose π_t, π_t^* to minimize $V_t+V_t^*$ subject to output schedules. The relevant first-order conditions are:

$$\begin{aligned}\lambda b_2 A_1' y_t + \pi_t + \lambda b_2 A_2' y_t^* &= 0, \\ \lambda b_2 A_1' y_t^* + \pi_t^* + \lambda b_2 A_2' y_t &= 0.\end{aligned}$$

This implies (remember the shock is mean zero), $\mathrm{E}_{t-1}\pi_t = \mathrm{E}_{t-1}\pi_t^* = 0$, and thus the first condition becomes

$$\begin{aligned}&\lambda b_2 A_1' \left(b_2 A_1' \pi_t + b_2 A_2' \pi_t^* + \varepsilon_t\right) \\ &+\pi_t + \lambda b_2 A_2' \left(b_2 A_1' \pi_t^* + b_2 A_2' \pi_t + \varepsilon_t\right) \\ = \;& 0.\end{aligned}$$

The cooperative solution is symmetric, due to the symmetric structure, so $\pi_t = \pi_t^*$ in equilibrium. Hence,

$$\begin{aligned}&\lambda b_2 A_1' \left(b_2 A_1' \pi_t + b_2 A_2' \pi_t + \varepsilon_t\right) \\ &+\pi_t + \lambda b_2 A_2' \left(b_2 A_1' \pi_t + b_2 A_2' \pi_t + \varepsilon_t\right) \\ = \;& 0,\end{aligned}$$

and thus

$$\pi_{c,t} = \pi_{c,t}^* = -\frac{\lambda b_2}{1+\lambda b_2^2}\varepsilon_t \tag{6.45}$$

A "lean-against-the-wind" policy: If $\varepsilon_t < 0$, central banks jointly expand policies up to point where inflation becomes too high. These are standard stabilization policy implications. The supply shock is "spread out" on output and inflation rates to a relative extent determined by λ.

Solving the model under noncooperative monetary policy

- Now, what if central banks do not coordinate? It is then assumed that each central bank conducts policy with aim of minimizing "own" loss function, *ignoring* external effects, and taking foreign policy as given. This results in policy *reaction functions*. Their intersection provides the Nash (1950) equilibrium: Policy profiles from which unilateral deviation cannot pay

- Home policy problem: Choose π_t to minimize V_t subject to Home output, taking π_t^* as given. The relevant first-order condition

$$\lambda b_2 A_1' y_t + \pi_t = 0.$$

Compared to the condition for optimal π_t under coordination: Foreign output effects are ignored. If $\varepsilon_t < 0$ this implies too little Home expansion (the beneficial output effect of the Home expansion is ignored). Using output equation (and, again, $\mathrm{E}_{t-1}\pi_t = \mathrm{E}_{t-1}\pi_t^* = 0$):

$$\lambda b_2 A_1' \left(b_2 A_1' \pi_t + b_2 A_2' \pi_t^* + \varepsilon_t\right) + \pi_t = 0.$$

This defines the negatively sloped reaction function; higher π_t^* is met by a Home contraction. The Nash equilibrium is symmetric, so $\pi_t = \pi_t^*$ is imposed. Hence,

$$\lambda b_2 A_1' \left(b_2 A_1' \pi_t + b_2 A_2' \pi_t + \varepsilon_t\right) + \pi_t = 0,$$

and thus

$$\pi_{N,t} = \pi_{N,t}^* = -\frac{\lambda b_2 A_1'}{1 + \lambda b_2^2 A_1'} \varepsilon_t \tag{6.48}$$

A "leaning-against-the-wind" policy again.

- Comparison with coordinated policy:
  - When, e.g., $\varepsilon_t < 0$ uncoordinated policy expands too little
  - Why?
  - A unilateral expansion is for given foreign policy perceived to cause a real depreciation, which imply output costs that refrain sufficient expansion
  - Foreign central bank thinks the same!
  - *In* Nash equilibrium, the real exchange rate does not move and a more expansive policy would be preferable (as under coordination)
- Result: Noncooperative monetary policy leads to too unstable output and too stable inflation. Implication: Coordination is beneficial as it internalizes the externalities of unilateral policymaking! Here: Positive externalities, so non-cooperation leads to too little "policy activism."

- Is policy coordination generally preferable? "It depends" It may be undesirable if it changes "third party behavior" in adverse directions
  - E.g., with three countries, coordination by two, may induce adverse behavior by the third
  - E.g., in this model with inflation bias considerations, private sector expectations may change adversely under cooperation (Rogoff, 1985, *Journal of International Economics*):

        * Under non-cooperation the perceived real depreciation from monetary expansion is indeed a cost that will reduce equilibrium inflation; under coordination inflation will be higher => coordination may be counterproductive

- Is coordination, if it is beneficial, of quantitative importance? This is an unsettled issue in literature
    - So far, "old-style" analyses based on large-scale econometric models generally found modest gains from coordination
    - New models with microfoundations have yet in calibrated versions also found modest gains
    - However, these models often have very few distortions (to make them tractable) so different policy regimes generally shows small welfare differences

4 Concluding remarks

- The Obstfeld-Rogoff model highlights
    - The exchange rate channel for macroeconomic equilibrium
    - Wealth redistribution following asymmetric policy shocks
    - Importance of microfoundations for determination of welfare effects of policy interventions
    - The spill-overs of unilateral monetary policymaking in an integrated world
    - Begs the issue of policy coordination
- Policy coordination was seen to be beneficial in simple stylized two-country model
    - Coordinated policies internalize external effects of policy
    - In model, policy had positive externalities, and non-cooperative behavior ignored these and were "too passive"
    - Had policy negative externalities, non-cooperative behavior would ignore the harmful effects abroad and be "too aggressive"
    - Generally, difference between cooperative and non-cooperative policies depends on the nature of the externalities
    - Generally, coordination is beneficial as long as "third parties" do not change behavior

Appendix

A Derivation of relative demand

We have that the consumption index of agent j is given by

$$C^j = \left[\int_0^1 c^j (z)^q \, \mathrm{d}z \right]^{\frac{1}{q}}, \qquad 0 < q < 1. \tag{1}$$

To concentrate on the determination of relative demand for individual goods, assume that total nominal expenditures are given by Z. I.e.,

$$\int_0^1 p(z) \, c^j (z) \, \mathrm{d}z = Z \tag{2}$$

The optimal demand for a given good z, is therefore found by maximizing (1) w.r.t $c^j(z)$ subject to (2). The relevant first-order condition is

$$\left[\int_0^1 c^j (z)^q \, \mathrm{d}z \right]^{\frac{1}{q}-1} c^j (z)^{q-1} = \lambda p(z), \tag{3}$$

where λ is the Lagrange-multiplier associated with (2). From this, we get

$$\left[\int_0^1 c^j (z)^q \, \mathrm{d}z \right]^{\frac{1}{q}-1} c^j (z)^{q} = \lambda p(z) \, c^j (z),$$

and thus (by integrating over all zs)

$$\begin{aligned} \left[\int_0^1 c^j (z)^q \, \mathrm{d}z \right]^{\frac{1}{q}-1} \left[\int_0^1 c^j (z)^q \, \mathrm{d}z \right] &= \lambda \int_0^1 p(z) \, c^j (z) \, \mathrm{d}z, \\ \left[\int_0^1 c^j (z)^q \, \mathrm{d}z \right]^{\frac{1}{q}} &= \lambda \int_0^1 p(z) \, c^j (z) \, \mathrm{d}z \end{aligned}$$

We then find λ by using the definitions of C^j and Z from

$$C^j = \lambda Z,$$

i.e.,

$$\lambda = \frac{C^j}{Z},$$

which is the ratio of real total consumption to total nominal expenditures. Denoting P the price index associated with C^j, one can then write $Z = PC^j$, and it follows that

$$\lambda = \frac{1}{P}. \tag{4}$$

This result is used in (3) to yield

$$\left[\int_0^1 c^j (z)^q \, \mathrm{d}z \right]^{\frac{1}{q}-1} c^j (z)^{q-1} = \frac{p(z)}{P},$$

and thus

$$\left[\int_0^1 c^j(z)^q \, \mathrm{d}z\right]^{\frac{1}{q}} c^j(z)^{-1} = \left[\frac{p(z)}{P}\right]^{\frac{1}{1-q}},$$

which then provides the demand for the individual good z as

$$c^j(z) = \left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C^j. \tag{5}$$

We see that the demand for good z is decreasing in the real price of the good, $p(z)/P$. Note that $1/(1-q)$ is the elasticity of substitution between goods. (Hence, the limiting case of perfect competition applies when $q \to 1$.) Total demand in the two countries of a good z will thus be n times domestic demand, and $1-n$ times foreign demand. A representative foreign consumer's demand function is

$$c^{*j}(z) = \left[\frac{p^*(z)}{P^*}\right]^{-\frac{1}{1-q}} C^{*j}$$

As the nominal exchange rate is S, this is the same as

$$c^{*j}(z) = \left[\frac{Sp^*(z)}{SP^*}\right]^{-\frac{1}{1-q}} C^{*j},$$

and thus, by the law of one price,

$$c^{*j}(z) = \left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C^{*j}$$

Total demand is therefore

$$\begin{aligned} y^d(z) &= n\left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C + (1-n)\left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C^* \\ &= \left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} [nC + (1-n)C^*] \\ &= \left[\frac{p(z)}{P}\right]^{-\frac{1}{1-q}} C^w, \end{aligned}$$

where $C^w \equiv nC + (1-n)C^*$ is total world real consumption. This demand function is the relevant one for producer z, when determining production.

B Derivation of the price index

Now note that the precise form of the price index P follows from

$$PC^j = \int_0^1 p(z)\, c^j(z) \, \mathrm{d}z$$

and the optimality condition (5). I.e., we get

$$PC^j = \int_0^1 p(z)\left[\frac{p(z)}{P}\right]^{\frac{1}{q-1}} C^j \mathrm{d}z$$

$$\begin{aligned} P &= \int_0^1 p(z)\left[\frac{p(z)}{P}\right]^{\frac{1}{q-1}} \mathrm{d}z \\ P^{1+\frac{1}{q-1}} &= \int_0^1 p(z)^{1+\frac{1}{q-1}} \mathrm{d}z \\ P^{\frac{q}{q-1}} &= \int_0^1 p(z)^{\frac{q}{q-1}} \mathrm{d}z, \end{aligned}$$

and finally

$$P = \left[\int_0^1 p(z)^{\frac{q}{q-1}} \mathrm{d}z\right]^{\frac{q-1}{q}}$$

which is equation (6.3) in Walsh. Note that the index is therefore not "just a definition," but actually the only specification that is consistent with optimal behavior.

Alternatively, and equivalently, one could have derived P as the index which secures minimization of nominal expenditures subject to real consumption of one unit of the consumption index. I.e.,

$$\min_{c^j(z)} PC^j = \int_0^1 p(z)\, c^j(z)\, \mathrm{d}z$$

subject to $C^j = 1$. The first-order condition is

$$\begin{aligned} p(z) &= \mu\left[\int_0^1 c^j(z)^q \mathrm{d}z\right]^{\frac{1}{q}-1} c^j(z)^{q-1} \\ &= \mu\left[C^j\right]^{1-q} c^j(z)^{q-1} \\ &= \mu c^j(z)^{q-1} \end{aligned}$$

where μ is the Lagrange multiplier on $C^j = 1$, and where the last line uses that $C^j = 1$ indeed applies. One then gets

$$c^j(z) = \left[p(z)/\mu\right]^{\frac{1}{q-1}}, \tag{6}$$

which inserted into $C^j = 1$ gives

$$\begin{aligned} \left[\int_0^1 \left[p(z)/\mu\right]^{\frac{q}{q-1}} \mathrm{d}z\right]^{\frac{1}{q}} &= 1, \\ \left[\int_0^1 \left[p(z)\right]^{\frac{q}{q-1}} \mathrm{d}z\right]^{\frac{1}{q}} \mu^{\frac{1}{1-q}} &= 1, \end{aligned}$$

and thus

$$\mu^{\frac{1}{q-1}} = \left[\int_0^1 \left[p(z)\right]^{\frac{q}{q-1}} \mathrm{d}z\right]^{\frac{1}{q}}.$$

Insert this back into (6):

$$c^j(z) = p(z)^{\frac{1}{q-1}}\left[\int_0^1 \left[p(z)\right]^{\frac{q}{q-1}} \mathrm{d}z\right]^{-\frac{1}{q}}.$$

Then use

$$\begin{aligned} PC^j &= \int_0^1 p(z)\, c^j(z)\, \mathrm{d}z, \\ P &= \int_0^1 p(z)\, c^j(z)\, \mathrm{d}z, \end{aligned}$$

to find

$$\begin{aligned} P &= \int_0^1 p(z)\, p(z)^{\frac{1}{q-1}} \left[\int_0^1 [p(z)]^{\frac{q}{q-1}} \mathrm{d}z \right]^{-\frac{1}{q}} \mathrm{d}z, \\ P &= \int_0^1 p(z)^{\frac{q}{q-1}} \left[\int_0^1 [p(z)]^{\frac{q}{q-1}} \mathrm{d}z \right]^{-\frac{1}{q}} \mathrm{d}z, \\ &= \left[\int_0^1 [p(z)]^{\frac{q}{q-1}} \mathrm{d}z \right]^{\frac{q-1}{q}}; \end{aligned}$$

which again is equation (6.3) in Walsh.

C Key concepts you should know

The Obstfeld-Rogoff model under flexible prices

- The representative agents' utility functions and budget constraints
- The law of one price
- Optimal consumption decisions
  - Intertemporally and intratemporally
- Optimal production (=pricing) decisions
- Optimal money demand decisions
- Neutrality of money:
  - Increase in, e.g., domestic money supply raises domestic prices and nominal exchange rate (depreciation) proportionally
- Superneutrality of money
  - Follows from separability of utility function
- The log-linearized version
  - Money neutrality readily established

- Nominal exchange rate determination
  - Present values of money and consumption differentials
- Uncovered interest parity condition

The Obstfeld-Rogoff model with one-period fixed producer prices

- Sticky producer prices and flexible consumer prices
- Money shocks influence nominal exchange rate and consumer prices and demand and production
- Unanticipated, permanent increase in domestic money supply
  - Short-run effects (one period); the role of the nominal exchange rate depreciation; higher domestic consumption and production; current account surplus
  - Long-run effects (steady state); the role of wealth redistribution; higher consumption
- Welfare implications of shock:
  - The short run gains from higher production; the welfare irrelevance of long-run effects
- Spill-overs of unilateral monetary policy
- Scope for coordination?

Policy coordination

- Simple two-country AS/AD style model
- The spill over of foreign surprise inflation: Higher domestic output as the real exchange rate is appreciated (reducing production costs)
- Coordinated policies takes both countries' welfare into account
- "Leaning against the wind" policies in face of supply shocks
- Uncoordinated policies ignores other country's welfare => Nash equilibrium in policies
  - To little stabilization of the supply shock (each country ignores the positive international externality of their policies)

- The general possibility of counterproductive policy coordination

Small open economy under flexible and fixed nominal exchange rates (not compulsory, but check out Section 6.4 of Walsh, 2003)

- Simple *ad hoc* model for small open economy
- Flexible nominal exchange rates
  - Home prices are insulated from foreign price shocks
  - Home money shocks affect both real output and real exchange rate
  - Dornbusch "overshooting"
- Fixed nominal exchange rates
  - Home prices now proportional to foreign
  - No role for money market shocks
  - With real demand shocks more volatile output
- Optimal regime? Poole (1970) revisited.....