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[Notes 4]
On the "enforcement equation" in the repeated game version of the Barro and Gordon model

Q How is the enforcement equation (equation 8.13) on page 11 obtained?

A This equation is quantifying the net loss of cheating on the private sector's expectation of $\overline{\pi}$ in a given period, say, period t. By the nature of the private sector's "tit-for-tat" punishment strategy, the net loss occur in period $t+1$ only (after that, the "good" inflation equilibrium with $\pi = \overline{\pi}$ continues until a potential cheat happens again). By the punishment strategy, the private sector punishes by expecting $\pi^e_{t+1} = \lambda a$; the inflation rate under discretion. The best policy response in that punishment period, will simply be $\pi_{t+1} = \lambda a$. This will also induce $\pi^e_{t+2} = \overline{\pi}$ by the punishment strategy, i.e., a reversal to the "good" inflation rate from period $t+2$ onwards.

Now, the central bank's utility of enduring the punishment in period $t+1$ is given by

$$\lambda \left(a \left(\pi_{t+1} - \pi^e_{t+1} \right) \right) - \frac{1}{2} \left(\pi_{t+1} \right)^2,$$

or, with the strategies $\pi^e_{t+1} = \lambda a$, $\pi_{t+1} = \lambda a$, inserted:

$$-\frac{1}{2} \left(\lambda a \right)^2.$$

Had the central bank not cheated from the promised inflation rate $\overline{\pi}$ in period t, the good equilibrium would have prevailed in period $t+1$, and given the utility

$$\begin{aligned} & \lambda \left(a \left(\overline{\pi} - \overline{\pi} \right) \right) - \frac{1}{2} \left(\overline{\pi} \right)^2 \\ = & -\frac{1}{2} \left(\overline{\pi} \right)^2. \end{aligned}$$

The net *loss* of cheating in period t is therefore

$$\frac{1}{2} \left(\lambda a \right)^2 - \frac{1}{2} \left(\overline{\pi} \right)^2.$$

This measures in utility terms the loss of cheating in period t: The central bank gets a utility of $-\frac{1}{2} \left(\lambda a \right)^2$, while it by not cheating would have gotten $-\frac{1}{2} \left(\overline{\pi} \right)^2$. As long as $\left| \overline{\pi} \right| < \lambda a$, the net loss is positive.

Finally, this net loss is compared to the net gain of cheating. This gain accrues in period t. To make a meaningful comparison between the net gain and net loss, one must discount the period-$t+1$ net loss by β in order to make it comparable to period-t utility. Hence, the relevant net loss of cheating, or the *enforcement*, is given by

$$\beta \left[\frac{1}{2} (\lambda a)^2 - \frac{1}{2} (\overline{\pi})^2 \right]$$

which is equation (8.13).