Monetary Economics: Macro Aspects, Spring 2006
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F.A.Q., May 10

[Notes 5]
On the variable D, the duration of bonds

Q I couldn't understand the variable D in equation (10.36), and the associated Footnote 18 in Walsh (2005). Also, what does the "term on conventional discount bonds" mean?

A Equation (10.36),

$$r_t - D\left(\mathrm{E}_t r_{t+1} - r_t\right) = i_t^f - \mathrm{E}_t \pi_{t+1},$$

is a no-arbitrage condition, where the left-hand-side is the real return during a period on investment in the long-term bond, and the right-hand side is the real return during a period on investing at the short term interest rate. With free capital markets, and risk-free investors, the returns must equal. The left-hand side is clearly the most complicated one (the right-hand side is simply the real short interest rate). The reason for the complication is that a long term bond both pays interest (in terms of coupon payments, and ultimately payment of the principal), and also varies in price leading to capital gains and losses.

In particular, computing capital gain and losses on a bond investment is in nature associated with uncertainty. This uncertainty will on the other hand greatly depend on how the *payment structure* of the bond is. If most payments of a bond is placed early in its life, the risk of holding it to maturity (i.e., until its term ends), is, of course, smaller than in the case where all payments are made at maturity date. Indeed, conventional, or pure, discount bonds has this feature. They promise a single payment at maturity (and to be an attractive investment object, they sell below face value; thereby the name a "discount" bond). In finance, one therefore uses the concept of duration as a measure of the average time which all payments of principal and interest on a financial instrument are made. So, for the aforementioned discount bond, the duration is equal to the term, or, time to maturity. For bonds where payments also occur during the lifetime of the bond, the duration will be smaller. Frederick R. Macaulay (1938) made the specific expression for duration, which here takes on the letter D. You don't need to know the exact expression, but in words "*it is a weighted average of the time until the payments on the bond are received; the weighting function is* *the ratio of the present value of the payment stream to the value of the bond*" (Fuhrer and Moore, 1995, *American Economic Review*, p. 233).

The second term on the left-hand-side of equation (10.36), thus expresses the expected capital gain or loss on the long bond. The longer is the duration, the larger impact does a capital gain or loss have on a period's expected return. In finance, one says that the bond is then more exposable to interest rate risk (and, thus, bond pice risk).