Written exam for the M.Sc in Economics
Department of Economics, University of Copenhagen

Monetary Economics: Macro Aspects

Semester: Spring 2006

June 15, 2006

4-hour exam

This set contains three pages. Both questions must be answered.
In the evaluation, the two questions will be weighted equally

Any material is allowed

To be answered in Danish or English

QUESTION 1:

Consider an economy where output determination can be compactly described as

$$y_t = \alpha\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + \gamma\pi_t + \varepsilon_t, \qquad \alpha > 0, \qquad \gamma \gtreqless 0, \tag{1}$$

where y_t is log of output, π_t is the inflation rate (the monetary policy instrument), and ε_t is a mean-zero, serially uncorrelated shock with variance $\mathrm{E}_{t-1}\varepsilon_t^2 \equiv \sigma^2$. E_{t-1} is the rational expectations operator conditional on information up to and including period $t-1$.

(i) Discuss thoroughly the three components of output determination as described by (1). Explain in particular which theories can explain the various components.

(ii) Discuss the output-inflation trade off of monetary policy in the short and long run implied by (1).

The objective of the economy's central bank in period t is to conduct monetary policy so as to minimize the loss function

$$L = \frac{1}{2}\left[\lambda\left(y_t - k\right)^2 + \pi_t^2\right], \qquad \lambda > 0, \qquad k > 0. \tag{2}$$

(iii) Derive the solution for discretionary monetary policy and the associated solution for output. Describe the solutions intuitively.

(iv) Assume now that the central banker has the ability to commit to a policy before inflation expectations are formed. Derive the solution for monetary policy and output under this assumption, and discuss any differences with the discretionary solutions found under (iii). [Hint: Assume the solution for monetary policy takes the form $\pi_t = a - b\varepsilon_t$. Determine $\mathrm{E}_{t-1}\pi_t$ and find the values of a and b that minimize $\mathrm{E}_{t-1}L$.]

(v) Assume that the relevant welfare measure of the economy is indeed L. Discuss then whether it can be advantageous for the economy to appoint a "conservative" central banker with the loss function

$$L = \frac{1}{2}\left[\lambda^c (y_t - k)^2 + \pi_t^2\right], \qquad \lambda > \lambda^c > 0, \tag{3}$$

if commitment is not feasible?

QUESTION 2:

Assume a model of a closed economy formulated in discrete time, where representative individuals have utility functions

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1, \tag{1}$$

with

$$u(c_t) \equiv \frac{(c_t)^{1-\Phi} - 1}{1-\Phi}, \qquad \Phi > 0,$$

and budget constraints

$$f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + \frac{1}{1+\pi_t} m_{t-1} = c_t + k_t + m_t, \tag{2}$$

where c_t is consumption, m_t is real money balances at the end of period t, k_{t-1} is physical capital at the end of period $t-1$, τ_t are monetary transfers by the government, $0 < \delta < 1$ is the depreciation rate of capital, and π_t is the inflation rate. The function f is defined as

$$f(k_{t-1}) \equiv k_{t-1}^{\alpha}, \qquad 0 < \alpha < 1.$$

Purchases of consumption goods as well as investment in physical capital is subject to a cash-in-advance constraint. Formally,

$$c_t + k_t - (1-\delta) k_{t-1} \leq \tau_t + \frac{1}{1+\pi_t} m_{t-1}. \tag{3}$$

(i) Discuss briefly the model as portrayed by (1)-(3).

(ii) Derive the relevant first-order conditions for optimal individual behavior, For this purpose, use the value function

$$\begin{aligned} V\left(k_{t-1}, m_{t-1}\right) &= \max \left\{u\left(c_t\right) + \beta V\left(k_t, m_t\right),\right. \\ &\quad \left. -\mu_t\left[c_t + k_t - (1-\delta) k_{t-1} - \tau_t - \left(1/\left(1+\pi_t\right)\right) m_{t-1}\right]\right\} \end{aligned}$$

where μ_t is the multiplier on (3), and where the maximization is over c_t, m_t and k_t and subject to (2). [Hint: Simplify the problem by using (2) to substitute out k_t.]

(iii) Interpret the first-order conditions intuitively, and show that they can be combined (along with the expressions for the partial derivatives of the value function) into the following steady-state conditions:

$$\left(c^{ss}\right)^{-\Phi} = \beta V_k\left(k^{ss}, m^{ss}\right),$$

$$V_k\left(k^{ss}, m^{ss}\right) = \beta V_k\left(k^{ss}, m^{ss}\right)(1-\delta) + \beta V_m\left(k^{ss}, m^{ss}\right) \alpha \left(k^{ss}\right)^{\alpha-1},$$

$$V_m\left(k^{ss}, m^{ss}\right) = \beta V_k\left(k^{ss}, m^{ss}\right) \frac{1}{1+\pi^{ss}},$$

where superscipt "ss" denotes steady-state values.

(iv) Derive the steady-state value of k, and discuss whether or not the model exhibits superneutrality. Explain the result intuitively.

(v) In a similar economy where there is no cash-in-advance constraint, the steady-state value of k is given by

$$k^{ss} = \left[\frac{\alpha}{\frac{1}{\beta} - 1 + \delta}\right]^{\frac{1}{1-\alpha}}.$$

Derive the steady-state monetary policy (i.e., inflation rate) that supports this steady-state value of k in the cash-in-advance economy considered above. Explain the properties of this policy.