Monetary Economics: Macro Aspects, 14/2 2008
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Plan for today:

1. Money in the utility function (ended)
   Welfare costs of inflation
   Potential non-superneuutrality of money
   Dynamics and calibration

   Literature: Walsh (2003, Chapter 2, pp. 59-80, but check the Appendix as well)

Welfare Costs of Inflation

- Inflation affects real money holdings by affecting the nominal interest rate (the opportunity cost of holding money):
  \[
  \frac{m^{s*}}{m} = \frac{i^{ss}}{1 + i^{ss}} = I
  \]  
  (money demand)

  \[
  i^{ss} = r^{ss} + \pi^{ss}
  \]  
  (Fisher relationship)

  Within the MIU model framework with household utility as welfare measure, what are the welfare effects of inflation?

- Is there an optimal rate of inflation?

On the optimal long-run inflation rate

- Bailey/Friedman intuition:
  - Private marginal cost of holding money is increasing in the nominal interest rate
  - Social marginal cost of creating money is essentially zero
  - Equating private and social marginal cost requires a zero nominal interest rate
  - By the Fisher relationship it follows that \( \pi^{ss} = -r^{ss} < 0 \) is optimal
  - I.e., the optimal rate of change in prices involves deflation equal to the real interest rate (“The Friedman Rule”)
• This is formally confirmed in the model when finding the utility maximizing nominal money growth rate subject to resource constraint of economy

Solve:

$$\max_{m^*} u(c^s, m^*)$$
$$\max_{\theta^s} u(f(k^s) - \delta k^s, m^*)$$

First-order condition:

$$u_c(f(k^s) - \delta k^s, m^*) \frac{\partial (f(k^s) - \delta k^s)}{\partial m^s} \frac{\partial m^s}{\partial \theta^s} + u_m(c^s, m^s) \frac{\partial m^s}{\partial \theta^s} = 0.$$ 

So,

$$u_m(c^s, m^s) = 0$$

• With

$$\frac{u_m(c^s, m^s)}{u_c(c^s, m^s)} = \frac{i^s}{1 + i^s},$$

this implies \(i^s = 0\), and the condition determines what Friedman called the “optimal quantity of money”

• Note that with some finite \(\overline{m}\) defined as \(u_m(c^s, \overline{m}) = 0\), and \(u_m(c^s, m^*) < 0\) for \(m^* > \overline{m}\), this \(\overline{m}\) is the optimal quantity of money

What are the welfare costs of inflation?

• Could be computed as the area under the money demand curve at a given positive nominal interest rate (Figure 2.2, p. 61)

  – This is the “consumer surplus” lost by a positive nominal interest rate. Some estimates indicate that inflation at 10% corresponds to 1-3% of GDP per year

• Could, as Lucas suggests, be computed as the percentage increase in steady-state consumption needed to compensate for a suboptimal low real money stock caused by \(i/(1 + i) \equiv \Upsilon > 0\)

  – Normalizing \(c^s = 1\), this implies that the cost of inflation \(w(\Upsilon)\) is implicitly given as

$$u(1 + w(\Upsilon), m(\Upsilon)) = u(1, m^*)$$

where \(m^*\) is the optimal quantity of real money balances, and \(m(\Upsilon)\) is the money demand function, \(m' < 0\).

  – With specific form of the utility function, and using numbers from estimated money demand functions, Lucas finds a 10% nominal interest rate is equivalent of around 1.3% lost steady-state consumption

  – A large number or not? In present value terms it is much higher (as it is 1.3% each and every year.....)

• Note that MIU model ignores other cost of inflation (e.g., its variability, impact on relative prices, etc.)
Potential non-superneutrally of money

- Is superneutrally of money a robust feature of the MIU model?
  In the model of previous lecture, endogenous savings behavior uniquely defines steady-state capital
  - Capital is accumulated or decumulated until its net marginal product (real interest rate) equals households’ subjective real interest rate:
    \[ f_k (k^{ss}) + 1 - \delta = \frac{1}{\beta} \]
  - Hence, long-run superneutrality can only fail if the marginal product of capital is affected by inflation

- Possible if production function contains another endogenous input factor, which is affected by inflation
  “Candidates”? The obvious one: Money in the production function: \( y_t = f (k_{t-1}, m_t) \), \( f_m > 0 \)
  - E.g., if \( f_{km} > 0 \) (more money makes capital more productive)
    higher inflation leads to lower real money balances and lower steady-state capital

- More natural possibility is endogenous labor input in production
  Arises in MIU model if it is amended by a labor supply choice by households
  This is achieved by having leisure enter in utility function:
  \[ u_t = u (c_t, m_t, l_t) \]
  Assuming that \( l \) is the fraction of time spent on leisure, the production function is
  \[ y_t = f (k_{t-1}, 1 - l_t) \]
  or,
  \[ y_t = f (k_{t-1}, n_t) \]
  with \( n_t = 1 - l_t \) being fraction of time spent on work (NB: \( n_t \) IS NOT THE POPULATION GROWTH RATE!)

- Households now face an additional decision: How much time should be devoted to work; how much to leisure?
  The relevant optimality condition is
  \[ u_t (c_t, m_t, l_t) = u_c (c_t, m_t, l_t) f_m (k_{t-1}, 1 - l_t) \]  \( (2.34') \)
  Marginal gain of leisure is equated to the marginal cost, which is the utility loss from lower consumption times the marginal product of labor (the real wage)
In steady state three relationships apply:

\[ u_l(c^{ss}, m^{ss}, l^{ss}) = u_c(c^{ss}, m^{ss}, l^{ss}) f_k(k^{ss}, 1 - l^{ss}) \]

\[ f_k(k^{ss}, 1 - l^{ss}) + 1 - \delta = \frac{1}{\beta} \]

\[ c^{ss} = f(k^{ss}, 1 - l^{ss}) - \delta k^{ss} \]

Note: If \( u_l(c^{ss}, m^{ss}, l^{ss}) / u_c(c^{ss}, m^{ss}, l^{ss}) \) is independent of \( m^{ss} \), these equations determine \( k^{ss}, l^{ss} \) and \( c^{ss} \). Long-run superneu-
trality holds!

- This will be the case if utility is separable in money; e.g.
  \( u = v(c, l) g(m) \) (\( u_l \) and \( u_c \) are affected by \( m \) in the same
  way)

- Also, of course, it will be the case if \( u_l \) and \( u_c \) are not affected
  by \( m \) at all

But if \( u_l(c^{ss}, m^{ss}, l^{ss}) / u_c(c^{ss}, m^{ss}, l^{ss}) \) depends on \( m^{ss} \), long-run superneu-
trality will not hold

- Note that if \( u_l \) and \( u_c \) are independent of \( m \), then superneu-
trality holds in the short run as well — dynamics “collapse” into a real
Ramsey-style model (the “Keynes-Ramsey rule” depicting the
evolution of marginal utility of consumption will no longer be
affected by money)

Specific functional form of utility function:

\[ u(c_t, m_t, l_t) = \frac{(a e^{1-b} + (1 - a) m_t^{1-b})^{1-\psi}}{1 - \Phi} + \frac{\psi}{1 - \eta} l_t^{1-\eta}, \]

\( 0 < a < 1, b > 0, \eta > 0, \Phi > 0, \Psi > 0 (b, \eta, \Phi \neq 1) \)

- \( \Phi \) is coefficient of relative risk aversion
- \( b \) becomes inverse nominal interest rate elasticity of money
demand — see (2.25), p. 57

What is \( u_l(c^{ss}, m^{ss}, l^{ss}) / u_c(c^{ss}, m^{ss}, l^{ss}) \) with this specification?

\[ \frac{u_l}{u_c} = \frac{\psi l_t^{-\eta}}{a \left( a e_t^{1-b} + (1 - a) m_t^{1-b} \right) l_t^{1-\eta} c_t^{-b}} \]

- Hence, if \( \Phi = b \), \( u_{cm} = 0 \) and superneu-
trality holds in the
  short and the long run
- If \( \Phi < b \) (empirically plausible), then \( u_{cm} > 0 \).
  * Higher expected inflation will reduce real money balances
    and decrease marginal utility of consumption
  * Households substitute towards leisure, and labor supply decreases
  * Superneu-
    trality fails in the short and long run
- If \( \Phi > b \) (empirically less plausible), then \( u_{cm} < 0 \) and super-
neutrality fails “in the opposite direction”

Note that it is anticipated changes in inflation that cause real
effects. An unanticipated, temporary, change in \( \pi_t \) has no effects,
as it does not affect the nominal interest rate and money demand.
Only when \( \pi_{t+1} \) is affected, is the nominal interest rate affected
through the Fisher relationship, \( l_t = \pi_{t+1} + r_t \)
Dynamics and calibration

- Given that superneutrality may fail in the short run due to endogenous labor choice, a relevant issue is whether the MIU model has short-run properties which match the data.
- I.e., how is monetary shocks transmitted to the real economy, and how will monetary policy be able to play a stabilizing role?
- For this purpose a stochastic version of the model is formulated.
- Exogenous shocks bringing the economy away from steady state will be technology shocks and shocks to the growth rate of nominal money supply.

Model and private sector optimization. General case (a la Appendix in Walsh)

- Production function is amended to
\[ y_t = f(k_{t-1}, 1 - l_t, z_t) \]
where \( z_t \) is a technology shock.
- Assumption:
\[ z_t = \rho z_{t-1} + e_t, \quad |\rho| < 1, \]
with \( e_t \) being a mean-zero, white-noise shock.

- Nominal money growth is assumed to be
\[ \theta_t = \theta^{\ast} + u_t \]
where \( u_t \) is a shock to the growth rate.
- Assumption:
\[ u_t = \gamma u_{t-1} + \phi z_{t-1} + \varphi_t, \quad 0 \leq \gamma < 1, \quad \phi \leq 0 \]
with \( \varphi_t \) being a mean-zero, white-noise shock.

- Note that there may or may not be serial correlation in the shocks to nominal money growth.
- Note that money growth may or may not respond toward past technology shocks, and may be either procyclical (\( \phi > 0 \)) or countercyclical (\( \phi < 0 \)).
- Per-period utility function and budget constraint are
\[ u(c_t, m_t, l_t) \]
and
\[ y_t + \tau_t + (1 - \delta) k_{t-1} + \frac{1}{1 + \pi_t} m_{t-1} = c_t + k_t + m_t. \]
(ignoring financial assets \( b_t \) as in last lecture...)
As in MIU model without endogenous labor, households maximizes discounted lifetime utility subject to the budget constraint.
- Again, dynamic programming method is used.
- Note, however, that since \( l_t \) is a choice variable, it is inappropriate to treat available resources as the state variable at period \( t \). Instead, state variables will therefore be \( k_{t-1} \) and
\[ a_t \equiv \tau_t + m_{t-1} / (1 + \pi_t). \]
The optimization is then characterized by the value function
\[ V(a_t, k_{t-1}) = \max \mathbb{E}_t \{ u(c_t, m_t, l_t) + \beta V(a_{t+1}, k_t) \} \]
where the maximization is over \( c, m, k, \) and \( l \) subject to the budget constraint and the definition of the state variable \( a_t. \) \( \mathbb{E}_t \) is the rational expectations operator.

One substitutes the constraint and definition so as to eliminate \( k_t \) and \( a_{t+1} \) and get an unconstrained maximization problem

First-order condition with respect to \( c_t: \)
\[ u_c(c_t, m_t, l_t) = \mathbb{E}_t \beta V_k(a_{t+1}, k_t) \quad (2.51') \]
(as \( \partial a_{t+1}/\partial c_t = 0 \) by the definition of \( a \)). Usual interpretation: Marginal gain of consumption must equal the expected marginal loss in terms of lower capital in next period

First-order condition with respect to \( m_t: \)
\[ u_m(c_t, m_t, l_t) + \beta \mathbb{E}_t V_a(a_{t+1}, k_t) \frac{1}{1 + \pi_{t+1}} = \beta \mathbb{E}_t V_k(a_{t+1}, k_t) \quad (2.53') \]
Usual interpretation: Marginal gain in terms of current utility and expected next period monetary wealth must equal the expected marginal loss in terms of lower capital in next period

First-order condition with respect to \( l_t: \)
\[ u_l(c_t, m_t, l_t) = \mathbb{E}_t \beta V_k(a_{t+1}, k_t) f_a(k_{t-1}, 1 - l_t, z_t) \quad (2.54') \]
Marginal gain of leisure is equated to the marginal cost, which is the value loss from less next-period capital, times the marginal product of labor (the real wage)

Mathematical digression “not for lecturing”, but for reading:
Elimination of the value function

- We know that optimum will be characterized by optimal values of \( c_t, m_t, \) and \( l_t \) as functions of the state variables. Call these functions
\[ c_t = c(a_t, k_{t-1}), \quad m_t = m(a_t, k_{t-1}), \quad l_t = l(a_t, k_{t-1}) \]
- The value function is thus by definition given as
\[ V(a_t, k_{t-1}) = u(c(a_t, k_{t-1}), m(a_t, k_{t-1}), l(a_t, k_{t-1})) + \beta \mathbb{E}_t V(a_{t+1}, k_t) \]
This holds for all \( a_t, k_{t-1} \) so we have
\[
V_a(a_t, k_{t-1}) = u_c(c(a_t, k_{t-1}), m(a_t, k_{t-1}), l(a_t, k_{t-1})) c_t(a_t, k_{t-1}) \\
+ u_m(c(a_t, k_{t-1}), m(a_t, k_{t-1}), l(a_t, k_{t-1})) m_t(a_t, k_{t-1}) \\
+ u_l(c(a_t, k_{t-1}), m(a_t, k_{t-1}), l(a_t, k_{t-1})) l_t(a_t, k_{t-1}) \\
+ \beta \mathbb{E}_t V_a(a_{t+1}, k_t) \frac{\partial a_{t+1}}{\partial a_t} + \beta \mathbb{E}_t V_k(a_{t+1}, k_t) \frac{\partial k_t}{\partial a_t} \\
= \beta \mathbb{E}_t V_k(a_{t+1}, k_t) \\
\]
where last equality follows from the fact that when \( c_t = c(a_t, k_{t-1}), m_t = m(a_t, k_{t-1}), l_t = l(a_t, k_{t-1}), \) it follows that
\[
\frac{\partial a_{t+1}}{\partial a_t} = \frac{1}{1 + \pi_{t+1}} m_a(a_t, k_{t-1}), \\
\frac{\partial k_t}{\partial a_t} = 1 - c_a(a_t, k_{t-1}) - m_a(a_t, k_{t-1}) - f_a(k_{t-1}, 1 - l_t, z_t) l_a(a_t, k_{t-1}), \\
\]
such that all the terms in front of \( c_a(a_t, k_{t-1}), m_a(a_t, k_{t-1}) \) and \( l_a(a_t, k_{t-1}) \) are zero. I.e., at an optimum, the marginal value of changing \( c, m, \) or \( l \) must be zero.

Likewise we get
\[ V_k(a_t, k_{t-1}) = \beta \mathbb{E}_t V_k(a_{t+1}, k_t) [f_k(k_{t-1}, 1 - l_t, z_t) + 1 - \delta] \]
So, by the first-order condition guiding $c$:

$$u_c(c_t, m_t, l_t) = V_a(a_t, k_{t-1})$$

We then get the corresponding relationship for guiding money choice similar to simple MIU model:

$$u_m(c_t, m_t, l_t) + \beta E_t u_c(c_{t+1}, m_{t+1}, l_{t+1}) \frac{1}{1 + \pi_{t+1}} = \beta E_t V_k(a_{t+1}, k_t)$$

$$u_c(c_t, m_t, l_t) = u_c(c_t, m_t, l_t) (*)$$

Also, the condition guiding consumption can be modified by use of

$$V_k(a_t, k_{t-1}) = \beta E_t V_k(a_{t+1}, k_t) [f_k(k_{t-1}, n_t, z_t) + 1 - \delta]$$

$$u_c(c_t, m_t, l_t) = \beta E_t \beta E_{t+1} V_k(a_{t+2}, k_{t+1}) [f_k(k_{t+1}, 1 - l_{t+1}, z_{t+1}) + 1 - \delta]$$

and using that $V_a(a_t, k_{t-1}) = \beta E_t V_k(a_{t+1}, k_t)$, gives $V_a(a_{t+1}, k_t) = \beta E_t V_k(a_{t+1}, k_{t+1})$ and thus

$$u_c(c_t, m_t, l_t) = \beta E_t E_{t+1} V_a(a_{t+1}, k_t) [f_k(k_{t+1}, 1 - l_{t+1}, z_{t+1}) + 1 - \delta]$$

$$= \beta E_t R_t u_c(c_{t+1}, m_{t+1}, l_{t+1})$$

with $R_t \equiv f_k(k_{t+1}, 1 - l_{t+1}, z_{t+1}) + 1 - \delta$. I.e., ($**$) is the “modified Keynes-Ramsey rule”

Finally, we get the condition for the choice of $l_t$, which becomes

$$u_l(c_t, m_t, l_t) = u_c(c_t, m_t, l_t) f_n(k_{t-1}, 1 - l_t, z_t).$$

Hence, the equations ($*$), ($**$) and ($***) together with the budget constraint, provide solutions for the paths of $c$, $m$, $l$, and $k$.

End of mathematical digression “not for lecturing”

Particular functional forms of utility and production functions

- Model is solved by numerical methods under assumptions about particular functional forms for utility and production function

Utility (as before):

$$u(c_t, m_t, l_t) = \left(\frac{ac_t^{1-a} + (1-a)m_t^{1-b}}{1-a} \right)^{\frac{1}{1-a}} + \psi \frac{l_t^{1-\eta}}{1-\eta}$$

Production function, Cobb-Douglas:

$$y_t = k_{t-1}^\alpha n_t^{1-\alpha} e^{z_t}, \quad 0 < \alpha < 1$$

- Steady-state solution: Note that the real interest rate,

$$R_t = f_k(k_t, 1 - l_{t+1}, z_{t+1}) + 1 - \delta$$

then becomes

$$R_t = \alpha E_t k_{t-1}^{\alpha-1} (1 - l_{t+1})^{1-\alpha} e^{z_{t+1}} + 1 - \delta$$

$$= \alpha \frac{E_t y_{t+1}}{k_t} + 1 - \delta.$$  

By the steady-state condition $R^{ss} = 1/\beta$, this only determines the ratio $y^{ss}/k^{ss}$

This ratio will be independent of monetary factors, but the levels $y^{ss}$, $k^{ss}$, $e^{ss}$ may not, if superneutrality fails so that $l^{ss}$ will be affected.
**Dynamic effects of money and technology shocks**

- To assess the quantitative effects of money and technology shocks, the model is calibrated and simulated.

  Calibration: Assign empirically plausible values the parameters of the model.

  Simulation:
  - Perform a linearization of the model’s dynamic equations (everything is expressed as percentage deviations from steady state);
  - solve this system by numerical methods (various simulation programs are available on the internet);
  - create artificial time series data from the system.

- From the artificial data, one evaluates the properties of the model in terms of:
  - Standard deviations of various relevant variables, and their s.d. relative to output
  - Correlation coefficients of various variables with output
  - Impulse response patterns of variables when shocks hit

**Main results** (when $b > \Phi$; implying $u_{cm} > 0$)

- Steady-state non-superneutrality is of the form of: Higher $\theta$ → lower output
- If money shocks, $\varphi_t$—shocks, shall play a role, persistence in money growth is necessary ($\gamma > 0$ is needed). Then, the shock will affects expected next-period inflation, and thus — through the Fisher equation — period $t$ nominal interest rate. Real money holdings change and the consumption-leisure trade-off is altered.
- The effects of money shocks on labor and output are stronger the more persistence in money growth, but the effects are quantitatively very small.
- If technology shocks are met with procyclical money growth, output is more stable. The magnitude, however, is modest.
- Main effects of money shocks are on inflation and nominal interest rates.
- Positive money shocks lead to higher nominal interest rates.
  In contrast with usual IS/LM story (where a liquidity effect is present: nominal rates fall to increase money demand). Reason is flexible prices in the MIU model (contrary to the sticky price IS/LM model).
  * Prices adjust instantaneously so as to reduce real money supply, matching the fall in demand resulting from higher nominal interest rates.
Summary

• The MIU framework provides a setting in which the welfare costs of inflation can be assessed, and where the optimal long-run inflation rate can be determined

• This, in turn, is equivalent of determining the “optimal quantity of money”

• The stochastic, dynamic model without the superneutrality property can be used to assess the importance of monetary shocks for economic fluctuations

• In the calibrated, MIU model with endogenous labor, money matters for business cycle fluctuations, but not very much

• This is one indication that flexible-price models may be ill-suited for analysis of monetary phenomena in the short run

Plan for next lectures

Tuesday, February 19

1. Shopping-time models

2. Cash-in-Advance Models (certainty)

   Literature: Walsh (2003, Chapter 3, pp. 95-111)

Thursday, February 21

1. Cash-in-Advance models (stochastic)

2. Money and real costs of transactions

   Literature: Walsh (2003, Chapter 3, pp.126-131)