This set contains four pages (beginning with this page)
All questions must be answered
In the evaluation, the three main questions will be weighted equally

Henrik Jensen
Department of Economics, University of Copenhagen
Spring 2008

## QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In the Obstfeld-Rogoff two-country model with one-period sticky prices, a permanent, unanticipated increase in the Home country's nominal money supply has no effects on Foreign output.

(ii) Within the inflation-bias model of Barro and Gordon, it is always beneficial to appoint a Rogoff-conservative central banker even though this creates more output stability.

(iii) In the Lucas "islands" model, an unanticipated aggregate money shock has no real effects as agents have rational expectations.

(iv) In the basic New-Keynesian model of the closed economy, the central bank should refrain from policies that affect inflation expectations as this will worsen the inflation-output gap trade off.

## QUESTION 2:

**Strict inflation targeting and nominal interest rate rules**

Consider the following model for output and inflation determination in a closed economy:

$$y_t = \theta y_{t-1} - \sigma \left( i_{t-1} - \mathrm{E}_{t-1}\pi_t \right) + u_t, \qquad 0 < \theta < 1, \quad \sigma > 0, \tag{1}$$

$$\pi_t = \pi_{t-1} + \kappa y_t + \eta_t, \qquad \kappa > 0, \tag{2}$$

where $y_t$ is log of output in period $t$, $i_t$ is the nominal interest rate (the monetary policy instrument), $\pi_t$ is the inflation rate, $u_t$ and $\eta_t$ are independent, mean-zero, serially uncorrelated shocks. $\mathrm{E}_j$ is the rational expectations operator conditional on information up to and including period $j$. It is assumed that $\sigma\kappa < 1$.

(i) Discuss equations (1) and (2), with emphasis on the monetary transmission mechanism and the stability properties in absence of policy intervention (only a verbal discussion is required).

The objective of the central bank is to conduct monetary policy so as to maximize

$$U = -\frac{1}{2}\mathrm{E}_t \sum_{j=1}^{\infty} \beta^j \pi_{t+j}^2, \qquad 0 < \beta < 1.$$

(ii) Find the optimal interest rate rule for $i_t$ as a function of $\pi_t$ and $y_t$. (Hint: Treat $\mathrm{E}_t y_{t+1} \equiv y_{t+1} - u_{t+1}$ as the policy instrument, and solve the maximization problem by dynamic programming treating $\pi_t$ as the state variable. That is, find the optimal policy as $\mathrm{E}_t y_{t+1} = B\pi_t$, where $B$ is a parameter to be found, and use (1) and (2) to derive the associated nominal interest rate.)

(iii) Comment on the coefficient on $\pi_t$ in the optimal interest rate rule, with special emphasis on how its value affects the stability properties of the model.

(iv) Discuss how the coefficients on $\pi_t$ and $y_t$ in the optimal interest rate rule depend on the underlying parameters of the model. Discuss in particular whether the parameters reveal anything about the strict inflation-targeting preferences of the central bank.

## QUESTION 3:

### Employment and a flex-price money-in-the-utility-function model

Consider an infinite-horizon economy in discrete time, where the utility of the representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t, n_t), \qquad 0 < \beta < 1, \tag{1}$$

with

$$u(c_t, m_t, n_t) \equiv b \ln c_t + (1-b)\left(m^F \ln m_t - m_t\right) + \Psi \ln(1-n_t),$$
$$0 < b < 1, \quad \Psi > 0, \quad m^F > 0.$$

Agents maximize utility subject to the budget constraint

$$c_t + k_t + m_t = f(k_{t-1}, n_t) + \tau_t + (1-\delta) k_{t-1} + \frac{1}{1+\pi_t} m_{t-1}, \qquad 0 < \delta < 1, \tag{2}$$

where $c_t$ is consumption, $m_t$ is real money balances at the end of period $t$, $n_t$ is labor supply, $k_{t-1}$ is physical capital, $\tau_t$ are monetary transfers from the government, and $\pi_t$ is the inflation rate. Function $f$ is defined as

$$f(k_{t-1}, n_t) = k_{t-1}^{\alpha} n_t^{1-\alpha}, \qquad 0 < \alpha < 1.$$

(i) Derive the relevant first-order conditions for optimal behavior. For this purpose set up the value function $V(a_t, k_{t-1}) = \max_{c_t, m_t, n_t} \{u(c_t, m_t, n_t) + \beta V(a_{t+1}, k_t)\}$ where

$$a_t \equiv \tau_t + \frac{1}{1+\pi_t} m_{t-1}.$$

(ii) Interpret the first-order conditions intuitively, and show that they can be combined (along with the expressions for the partial derivatives of the value function) into

$$u_c(c_t, m_t, n_t) = \beta R_t u_c(c_{t+1}, m_{t+1}, n_{t+1}), \tag{3}$$

$$\frac{u_m(c_t, m_t, n_t)}{u_c(c_t, m_t, n_t)} = \frac{i_t}{1+i_t}, \tag{4}$$

$$-u_n(c_t, m_t, n_t) = u_c(c_t, m_t, n_t) f_n(k_{t-1}, n_t), \tag{5}$$

where $R_t \equiv f_k(k_{t-1}, n_t) + 1 - \delta = (1+i_t)/(1+\pi_{t+1})$ is the gross real interest rate, with $i_t$ being the nominal interest rate.

(iii) Using the functional forms for $u$ and $f$, examine the properties of the steady state using (3), (4), and (5) together with the national account identity $c^{ss} = {k^{ss}}^{\alpha} {n^{ss}}^{1-\alpha} - \delta k^{ss}$ (where superscript "$ss$" denote a steady-state value). Assess in particular whether monetary policy can affect employment or not, and explain the economic reasons.

(iv) Characterize the monetary policy that maximizes the utility of the representative agent and find the corresponding optimal steady-state real balances. Explain the result intuitively.