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## MONETARY ECONOMICS: MACRO ASPECTS

### SUGGESTED ANSWERS TO JUNE 16 EXAM

#### QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

- (i) In the Obstfeld-Rogoff two-country model with one-period sticky prices, a permanent, unanticipated increase in the Home country's nominal money supply has no effects on Foreign output.

**FALSE.** The short-run effects of the money shock is to depreciate the nominal exchange rate, causing demand to switch towards Home goods. This reduces Foreign output. On the other hand, the increase in aggregate consumption in both countries lead to production increases in both countries. The short-run net effect on Foreign output is therefor ambiguous. The long-run effect is positive, as wealth is redistributed from the Foreign to the Home economy, implying that the Foreign country is running a trade surplus to finance interest payments to the Home country. (The answer, "TRUE", is therefore only considered correct if it is explained to refer to the degenerate case in the short run where parameters are configured precisely such that the two opposing effects are equal.)

- (ii) Within the inflation-bias model of Barro and Gordon, it is always beneficial to appoint a Rogoff-conservative central banker even though this creates more output stability.

**TRUE.** Choosing how conservative the central should be involves a trade off between worse stabilization policies and better average inflation performance. But even if shock stabilization is very important (say because the shock variance is huge), it is always optimal with some degree of conservatism. The reason is that in absence of conservatism, shocks are stabilized optimally in the Barro and Gordon model, while inflation is inefficiently low. Introducing a small degree of conservatism therefor only involves a second-order loss in terms of poorer stabilization, but a first-order welfare gain in terms of lower inflation bias.

- (iii) In the Lucas “islands” model, an unanticipated aggregate money shock has no real effects as agents have rational expectations.

**FALSE.** It is indeed the unanticipated nature of the shock that causes the confusion, which ultimately causes real effects. With asymmetric information, agents on each island observe the money shock, but do not know whether it is an aggregate shock (in which case they should do nothing) or a local shock (in which case they should change labor supply). They solve a signal extraction problem, where they rationally put some weight to the possibility that it is a local shock. As alle agents do the same, aggregate labor supply and output change under rational expectations.

- (iv) In the basic New-Keynesian model of the closed economy, the central bank should refrain from policies that affect inflation expectations as this will worsen the inflation-output gap trade off.

**FALSE.** The ability to affect inflation expectations creates an additional channel for affecting inflation besides the change in the output gap caused by interest rate changes. The central bank can thus obtain a more favorable inflation-output gap trade off, as it can reduce inflation at a lower reduction in the output gap, if it can affect inflation expectations downwards. This, however, requires the credibility of commitment to a path of policies. Under discretionary policy, such commitment is absent, and the New Keynesian model provides another example of benefits from commitment.

## QUESTION 2:

### Strict inflation targeting and nominal interest rate rules

Consider the following model for output and inflation determination in a closed economy:

$$y_t = \theta y_{t-1} - \sigma (i_{t-1} - \mathbb{E}_{t-1} \pi_t) + u_t, \quad 0 < \theta < 1, \quad \sigma > 0, \quad (1)$$

$$\pi_t = \pi_{t-1} + \kappa y_t + \eta_t, \quad \kappa > 0, \quad (2)$$

where  $y_t$  is log of output in period  $t$ ,  $i_t$  is the nominal interest rate (the monetary policy instrument),  $\pi_t$  is the inflation rate,  $u_t$  and  $\eta_t$  are independent, mean-zero, serially uncorrelated shocks.  $\mathbb{E}_j$  is the rational expectations operator conditional on information up to and including period  $j$ . It is assumed that  $\sigma\kappa < 1$ .

- (i) Discuss equations (1) and (2), with emphasis on the monetary transmission mechanism and the stability properties in absence of policy intervention (only a verbal discussion is required).

**Main points are that there are lagged effects in both IS and Phillips curve. Moreover, policy takes effect with a one-period lag. In absence of policy intervention, the model is unstable, as, e.g., a positive demand shock will increase demand, subsequently inflation, subsequently lower the real interest rate, and then further expand output, increase inflation, and so on.**

The objective of the central bank is to conduct monetary policy so as to maximize

$$U = -\frac{1}{2} \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j \pi_{t+j}^2, \quad 0 < \beta < 1.$$

- (ii) Find the optimal interest rate rule for  $i_t$  as a function of  $\pi_t$  and  $y_t$ . (Hint: Treat  $\mathbb{E}_t y_{t+1} \equiv y_{t+1} - u_{t+1}$  as the policy instrument, and solve the maximization problem by dynamic programming treating  $\pi_t$  as the state variable. That is, find the optimal policy as  $\mathbb{E}_t y_{t+1} = B\pi_t$ , where  $B$  is a parameter to be found, and use (1) and (2) to derive the associated nominal interest rate.)

**Using the hint, the relevant value function becomes**

$$\begin{aligned} v(\pi_t) &= \max_{\mathbb{E}_t y_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{2} (\pi_t + \kappa y_{t+1} + \eta_{t+1})^2 + \beta v(\pi_t + \kappa y_{t+1} + \eta_{t+1}) \right\}, \\ &= \max_{\mathbb{E}_t y_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{2} (\pi_t + \kappa [\mathbb{E}_t y_{t+1} + u_{t+1}] + \eta_{t+1})^2 + \beta v(\pi_t + \kappa [\mathbb{E}_t y_{t+1} + u_{t+1}] + \eta_{t+1}) \right\}. \end{aligned}$$

The first-order condition is

$$\begin{aligned}
& -\mathbf{E}_t \kappa (\pi_t + \kappa [\mathbf{E}_t y_{t+1} + u_{t+1}] + \eta_{t+1}) \\
& + \mathbf{E}_t \beta \kappa v' (\pi_t + \kappa [\mathbf{E}_t y_{t+1} + u_{t+1}] + \eta_{t+1}) \\
& = 0, \\
& -\mathbf{E}_t \kappa (\pi_t + \kappa \mathbf{E}_t y_{t+1}) + \mathbf{E}_t \beta \kappa v' (\pi_t + \kappa \mathbf{E}_t y_{t+1}) = 0, \\
& -(\pi_t + \kappa \mathbf{E}_t y_{t+1}) + \beta v' (\pi_t + \kappa \mathbf{E}_t y_{t+1}) = 0.
\end{aligned}$$

Using the Envelope Theorem one gets:

$$v'(\pi_t) = -(\pi_t + \kappa \mathbf{E}_t y_{t+1}) + \beta v'(\pi_t + \kappa \mathbf{E}_t y_{t+1}).$$

So,  $v'(\pi_t) = 0$ . Hence,  $\mathbf{E}_t y_{t+1} = -\frac{1}{\kappa} \pi_t$ , showing that  $B = -\kappa^{-1}$ . We also have that

$$\begin{aligned}
\mathbf{E}_t y_{t+1} &= \theta y_t - \sigma (i_t - \mathbf{E}_t [\pi_t + \kappa y_{t+1}]), \\
\mathbf{E}_t y_{t+1} (1 - \sigma \kappa) &= \theta y_t - \sigma (i_t - \mathbf{E}_t \pi_t),
\end{aligned}$$

$$\begin{aligned}
\mathbf{E}_t y_{t+1} (1 - \sigma \kappa) &= \theta y_t - \sigma (i_t - \mathbf{E}_t \pi_t) \\
\mathbf{E}_t y_{t+1} &= \frac{\theta}{(1 - \sigma \kappa)} y_t - \frac{\sigma}{(1 - \sigma \kappa)} (i_t - \mathbf{E}_t \pi_t).
\end{aligned}$$

So, the interest rate rule follows from

$$\begin{aligned}
-\frac{1}{\kappa} \pi_t &= \frac{\theta}{(1 - \sigma \kappa)} y_t - \frac{\sigma}{(1 - \sigma \kappa)} (i_t - \mathbf{E}_t \pi_t), \\
-\frac{(1 - \sigma \kappa)}{\kappa} \pi_t &= \theta y_t - \sigma (i_t - \pi_t), \\
\sigma i_t &= \left[ \sigma + \frac{(1 - \sigma \kappa)}{\kappa} \right] \pi_t + \theta y_t,
\end{aligned}$$

as

$$i_t = \left[ 1 + \frac{(1 - \sigma \kappa)}{\sigma \kappa} \right] \pi_t + \frac{\theta}{\sigma} y_t.$$

- (iii) Comment on the coefficient on  $\pi_t$  in the optimal interest rate rule, with special emphasis on how its value affects the stability properties of the model.

The main issue is that the coefficient is greater than one. Hence, it is an active Taylor-type rule, such that any rise in inflation is met by a larger increase in the nominal interest rate. This increases the real interest rate, and will serve to contract output and thus reduce inflation. Hence, it serves a stabilizing role.

- (iv) Discuss how the coefficients on  $\pi_t$  and  $y_t$  in the optimal interest rate rule depend on the underlying parameters of the model. Discuss in particular whether the parameters reveal anything about the strict inflation-targeting preferences of the central bank.

It can be seen that the structural parameters  $\sigma$  and  $\kappa$  reduce the inflation coefficient. This is because when these values are lower, a smaller nominal interest rate response is needed to stabilize inflation (as demand is more sensitive and inflation is more sensitive to demand). Furthermore it is observed that output increases will lead to nominal interest rate changes, even though the central bank is conducting strict inflation targeting. The reason being that output changes provides information about inflation one period ahead (as long as there is output inertia; i.e., when  $\theta > 0$ ). Hence, the parameter values, and the variables in the interest rate rule, tell nothing about the preferences of the central bank. From the curriculum, it is known that a model with a flexible inflation-targeting bank yields the same form of the optimal rule.

### QUESTION 3:

#### Employment and a flex-price money-in-the-utility-function model

Consider an infinite-horizon economy in discrete time, where the utility of the representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t, n_t), \quad 0 < \beta < 1, \quad (1)$$

with

$$u(c_t, m_t, n_t) \equiv b \ln c_t + (1 - b) (m^F \ln m_t - m_t) + \Psi \ln(1 - n_t),$$

$$0 < b < 1, \quad \Psi > 0, \quad m^F > 0.$$

Agents maximize utility subject to the budget constraint

$$c_t + k_t + m_t = f(k_{t-1}, n_t) + \tau_t + (1 - \delta) k_{t-1} + \frac{1}{1 + \pi_t} m_{t-1}, \quad 0 < \delta < 1, \quad (2)$$

where  $c_t$  is consumption,  $m_t$  is real money balances at the end of period  $t$ ,  $n_t$  is labor supply,  $k_{t-1}$  is physical capital,  $\tau_t$  are monetary transfers from the government, and  $\pi_t$  is the inflation rate. Function  $f$  is defined as

$$f(k_{t-1}, n_t) = k_{t-1}^\alpha n_t^{1-\alpha}, \quad 0 < \alpha < 1.$$

- (i) Derive the relevant first-order conditions for optimal behavior. For this purpose set up the value function  $V(a_t, k_{t-1}) = \max_{c_t, m_t, n_t} \{u(c_t, m_t, n_t) + \beta V(a_{t+1}, k_t)\}$  where

$$a_t \equiv \tau_t + \frac{1}{1 + \pi_t} m_{t-1}.$$

**Inserting the definition of  $a_t$  and using  $k_t$  as given by (2) in the suggested value function, the maximization leads to the following three first-order conditions:**

$$u_c(c_t, m_t, n_t) - \beta V_k(a_{t+1}, k_t) = 0, \quad (*)$$

$$u_m(c_t, m_t, n_t) + \beta V_a(a_{t+1}, k_t) \frac{1}{1 + \pi_{t+1}} - \beta V_k(a_{t+1}, k_t) = 0, \quad (**)$$

$$u_n(c_t, m_t, n_t) + \beta V_k(a_{t+1}, k_t) f_n(k_{t-1}, n_t). \quad (***)$$

- (ii) Interpret the first-order conditions intuitively, and show that they can be combined (along with the expressions for the partial derivatives of the value function) into

$$u_c(c_t, m_t, n_t) = \beta R_t u_c(c_{t+1}, m_{t+1}, n_{t+1}), \quad (3)$$

$$\frac{u_m(c_t, m_t, n_t)}{u_c(c_t, m_t, n_t)} = \frac{i_t}{1 + i_t}, \quad (4)$$

$$-u_n(c_t, m_t, n_t) = u_c(c_t, m_t, n_t) f_n(k_{t-1}, n_t), \quad (5)$$

where  $R_t \equiv f_k(k_t, n_{t+1}) + 1 - \delta = (1 + i_t) / (1 + \pi_{t+1})$  is the gross real interest rate,<sup>1</sup> with  $i_t$  being the nominal interest rate.

**Interpretations are the usual ones in terms of marginal gains equalling**

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<sup>1</sup>Note that in the question, there is a typo in the definition of  $R_t$  as it involves  $f_k(k_{t-1}, n_t)$  and not  $f_k(k_t, n_{t+1})$ . This is not considered a big typo: A real interest rate in period  $t$  should reflect the marginal product of period  $t$  capital — not past capital; see, e.g., Walsh, 2003, p. 50 eq. (2.13), p. 82 eq. (2.61). The presence of the typo is, however, taken into account in the evaluation of the answers.

marginal losses. The partial derivatives of the value function gives, applying the Envelope Theorem:

$$V_a(a_t, k_{t-1}) = \beta V_k(a_{t+1}, k_t)$$

$$V_k(a_t, k_{t-1}) = \beta V_k(a_{t+1}, k_t) [f_k(k_{t-1}, n_t) + 1 - \delta]$$

The last line becomes

$$V_k(a_t, k_{t-1}) = \beta R_{t-1} V_k(a_{t+1}, k_t).$$

Using this forwarded one period with (\*), immediately gives (3). Dividing (\*\*) through with  $u_c$  or  $\beta V_k(a_{t+1}, k_t)$  yields

$$\frac{u_m(c_t, m_t, n_t)}{u_c(c_t, m_t, n_t)} + \frac{V_a(a_{t+1}, k_t)}{V_k(a_{t+1}, k_t)} \frac{1}{1 + \pi_{t+1}} - 1 = 0,$$

which gives

$$\frac{u_m(c_t, m_t, n_t)}{u_c(c_t, m_t, n_t)} + \frac{\beta V_k(a_{t+2}, k_{t+1})}{V_k(a_{t+1}, k_t)} \frac{1}{1 + \pi_{t+1}} - 1 = 0,$$

and thus

$$\begin{aligned} \frac{u_m(c_t, m_t, n_t)}{u_c(c_t, m_t, n_t)} + \frac{1}{R_t} \frac{1}{1 + \pi_{t+1}} - 1 &= 0, \\ \frac{u_m(c_t, m_t, n_t)}{u_c(c_t, m_t, n_t)} + \frac{1 + \pi_{t+1}}{1 + i_t} \frac{1}{1 + \pi_{t+1}} - 1 &= 0, \\ \frac{u_m(c_t, m_t, n_t)}{u_c(c_t, m_t, n_t)} - \frac{i_t}{1 + i_t} &= 0, \end{aligned}$$

which gives (4). Finally, (\*\*\*) immediately gives (5).

- (iii) Using the functional forms for  $u$  and  $f$ , examine the properties of the steady state using (3), (4), and (5) together with the national account identity  $c^{ss} = k^{ss\alpha} n^{ss^{1-\alpha}} - \delta k^{ss}$  (where superscript “ss” denote a steady-state value). Assess in particular whether monetary policy can affect employment or not, and explain the economic reasons.

The three equations (3), (4) and (5) in steady state, and with the particular functional forms, read

$$b \frac{1}{c^{ss}} = \beta R^{ss} b \frac{1}{c^{ss}}, \quad (3')$$

$$\frac{c^{ss} (1 - b) \left[ \frac{m^F}{m^{ss}} - 1 \right]}{b} = \frac{i^{ss}}{1 + i^{ss}}, \quad (4')$$

$$\Psi \frac{1}{1 - n^{ss}} = b \frac{1}{c^{ss}} (1 - \alpha) \left( \frac{k^{ss}}{n^{ss}} \right)^\alpha. \quad (5')$$

From (3') it follows that

$$\frac{1}{\beta} = \alpha \left( \frac{k^{ss}}{n^{ss}} \right)^{\alpha-1} + 1 - \delta.$$

We thus immediately see that this equation together with (5') and the national account identity determine  $k^{ss}$ ,  $c^{ss}$  and  $n^{ss}$  independently of monetary factors. The reason is that monetary policy affects  $m^{ss}$ , but as that does not affect the marginal utility of consumption, it does not affect the labor supply decision, and hence output and capital. Only  $m^{ss}$  is affected by monetary policy.

- (iv) Characterize the monetary policy that maximizes the utility of the representative agent and find the corresponding optimal steady-state real balances. Explain the result intuitively.

The optimal policy will be one that, given policy cannot affect consumption and labor, maximizes utility of money holdings. With this utility function it happens at

$$\begin{aligned} u_m &= (1 - b) \left[ \frac{m^F}{m^{ss}} - 1 \right] = 0, \\ m^{ss} &= m^F. \end{aligned}$$

I.e., at a policy that secures steady-state money balances equal to  $m^F$ , which can be interpreted as the Friedman optimal quantity of money. Indeed it is achieved by a monetary policy that implements the Friedman rule,  $i^{ss} = 0$  (as this gives  $u_m = 0$ .)