This set contains four pages (beginning with this page)
All questions must be answered
In the evaluation, the three main questions will be weighted equally

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QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In the simple New-Keynesian model with monopolistic competition and sticky prices, a monetary policy implementing the Friedman rule is optimal as it eliminates any relative demand distortions.

(ii) Under a nominal interest-rate targeting procedure, monetary policymaking performed without knowledge of the realizations of current shocks can be improved by using the money stock as an intermediate target whenever money-market shocks are predominant in the economy.

(iii) A country's nominal interest rate policy was for a period shown to follow a Taylor-type rule like $i_t = 1.5\pi_t + 0.5x_t$, where i_t is the nominal interest rate, π_t is the inflation rate and x_t is the output gap. In a subsequent period, where a new central bank governor took office, monetary policy was characterized by $i_t = 2.5\pi_t + 0.5x_t$. As this was the only structural change in the economy, the new central bank governor had the same preferences for inflation and output stability as the old one.

(iv) In a simple money-in-the-utility-function model, superneutrality of money only fails when money shocks create unanticipated inflation.

QUESTION 2:

Monetary policy and a "conservative" central banker

Consider the following model of inflation determination in a closed economy:

$$\pi_t = \mathrm{E}_{t-1}\pi_t + \kappa x_t + \varepsilon_t, \qquad \kappa > 0, \tag{1}$$

where π_t is inflation, x_t is the output gap and ε_t is a mean-zero, serially uncorrelated shock with variance σ^2. E_{t-1} is the rational expectations operator conditional upon all information up to and including period $t-1$. The central bank is assumed to affect aggregate demand through monetary policy, and for simplicity x_t is taken to be the instrument of monetary policy. The aim of monetary policy is to maximize

$$V = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\lambda\left(x_t - k\right)^2 + \pi_t^2\right], \qquad k > 0, \quad \lambda > 0, \quad 0 < \beta < 1. \tag{2}$$

(i) Discuss (1) and (2) with focus on the underlying economic mechanisms, and derive the optimal time-consistent outcomes for output and inflation [Hint: Maximize (2) w.r.t. x_t subject to (1), which is a sequence of one-period problems, taking as given $\mathrm{E}_{t-1}\pi_t$; from the first-order condition derive $\mathrm{E}_{t-1}\pi_t$ and the solutions]. What is the inefficiency of the solution? Explain.

(ii) Society now delegates monetary policymaking to a "conservative" central banker with a utility function given by

$$V^c = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\lambda^c\left(x_t - k\right)^2 + \pi_t^2\right], \qquad \lambda > \lambda^c > 0. \tag{3}$$

Show formally how the time-consistent outcomes change relative to those derived in (i)? Will delegation of this form always be beneficial?

Assume now that (1) is replaced by

$$\pi_t = \beta\mathrm{E}_t\pi_{t+1} + \kappa x_t + \varepsilon_t. \tag{4}$$

(iii) Derive the optimal time-consistent outcomes for output and inflation [Hint: Maximize (2) w.r.t. x_t subject to (4), which is a sequence of one-period problems, taking as given $\mathrm{E}_t\pi_{t+1}$; use the first-order condition together with (4) and derive π_t and thus x_t.] Discuss the solution, and point out similarities and differences with the solution when equation (1) applies.

(iv) Discuss whether delegation to a conservative central banker is beneficial when (4) applies.

QUESTION 3:

Investment under a cash-in-advance constraint

Assume a model of a closed economy formulated in discrete time, where representative individuals have utility functions

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1, \tag{1}$$

with

$$u(c_t) \equiv \frac{(c_t)^{1-\sigma} - 1}{1-\sigma}, \qquad \sigma > 0,$$

and budget constraints

$$f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + \frac{1}{1+\pi_t} m_{t-1} = c_t + k_t + m_t, \tag{2}$$

where c_t is consumption, m_t is real money balances at the end of period t, k_{t-1} is physical capital at the end of period $t-1$, τ_t are monetary transfers by the government, $0 < \delta < 1$ is capital's rate of depreciation and π_t is the inflation rate. The function f is defined as $f(k_{t-1}) \equiv k_{t-1}^{\alpha} f(k_{t-1}) \equiv k_{t-1}^{\alpha}$, $0 < \alpha < 1$.

Purchases of consumption goods, as well as investment in physical capital, are subject to a cash-in-advance constraint. This is modelled as

$$c_t + k_t - (1-\delta) k_{t-1} \leq \tau_t + \frac{1}{1+\pi_t} m_{t-1}. \tag{3}$$

(i) Discuss the model given by (1), (2) and (3).

(ii) Derive the relevant first-order conditions for optimal individual behavior, For this purpose, use the value function

$$\begin{aligned} V(k_{t-1}, m_{t-1}) &= \max\{u(c_t) + \beta V(k_t, m_t), \\ &\quad -\mu_t [c_t + k_t - (1-\delta) k_{t-1} - \tau_t - (1/(1+\pi_t)) m_{t-1}]\} \end{aligned}$$

where μ_t is the multiplier on (3), and where the maximization is over c_t, m_t and k_t and subject to (2). [Hint: Simplify the problem by using (2) to substitute out k_t in the value function]

(iii) Interpret the first-order conditions and show that they (along with the expressions for the partial derivatives of the value function derived using the Envelope Theorem) can be combined into the following steady-state relationships:

$$(c^{ss})^{-\sigma} = \beta V_k(k^{ss}, m^{ss}), \tag{4}$$

$$V_k(k^{ss}, m^{ss}) = \beta V_k(k^{ss}, m^{ss})(1-\delta) + \beta V_m(k^{ss}, m^{ss})\,\alpha (k^{ss})^{\alpha-1}, \tag{5}$$

$$V_m(k^{ss}, m^{ss}) = \beta V_k(k^{ss}, m^{ss}) \frac{1}{1+\pi^{ss}} \tag{6}$$

where superscript "ss" denotes steady-state values.

(iv) By use of (5) and (6), derive the steady-state value of k, and show formally whether or not the model exhibits superneutrality. Explain the result and discuss the characteristics of the optimal rate of inflation.