This set contains four pages (beginning with this page)
All questions must be answered
In the evaluation, the three main questions will be weighted equally

Henrik Jensen
Department of Economics, University of Copenhagen
Spring 2009

MONETARY ECONOMICS: MACRO ASPECTS

SOLUTIONS TO JUNE 15 EXAM

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In the simple New-Keynesian model with monopolistic competition and sticky prices, a monetary policy implementing the Friedman rule is optimal as it eliminates any relative demand distortions.

A False. In the New-Keynesian model with sticky prices, inflation causes relative price changes as some firms can change their price, while others cannot. Therefore, a monetary policy implementing the Friedman rule will not be optimal as deflation would also cause relative price changes. Only zero inflation is optimal.

(ii) Under a nominal interest-rate targeting procedure, monetary policymaking performed without knowledge of the realizations of current shocks can be improved by using the money stock as an intermediate target whenever money-market shocks are predominant in the economy.

A False. When money-market shocks are predominant, the movements in the observable money stock will be relatively uninformative about shocks that affects output and inflation. Hence, adjusting the interest rate in response to money stock movements will not improve monetary policy.

(iii) A country's nominal interest rate policy was for a period shown to follow a Taylor-type rule like $i_t = 1.5\pi_t + 0.5x_t$, where i_t is the nominal interest rate, π_t is the inflation rate and x_t is the output gap. In a subsequent period, where a new central bank governor took office, monetary policy was characterized by $i_t = 2.5\pi_t + 0.5x_t$. As this was the only structural change in the economy, the new central bank governor had the same preferences for inflation and output stability as the old one.

A False. As the *only* structural change in the economy is a change in governor, a change in measured monetary policy must be due to a change in priorities in monetary policymaking, i.e., a change in preferences.

(iv) In a simple money-in-the-utility-function model, superneutrality of money only fails when money shocks create unanticipated inflation.

A False. For superneutrality to fail the real money stock must change in response to shocks. This happens when the nominal interest rate changes, thereby altering the opportunity cost of holding money, From the Fisher relationship, the nominal rate is related to expected inflation: $i_t = r_t + \mathrm{E}_t\pi_{t+1}$. Hence unexpected movements in inflation have no effects on the nominal interest rate in the simple MIU model.

QUESTION 2:

Monetary policy and a "conservative" central banker

Consider the following model of inflation determination in a closed economy:

$$\pi_t = \mathrm{E}_{t-1}\pi_t + \kappa x_t + \varepsilon_t, \qquad \kappa > 0, \tag{1}$$

where π_t is inflation, x_t is the output gap and ε_t is a mean-zero, serially uncorrelated shock with variance σ^2. E_{t-1} is the rational expectations operator conditional upon all information up to and including period $t-1$. The central bank is assumed to affect aggregate demand through monetary policy, and for simplicity x_t is taken to be the instrument of monetary policy. The aim of monetary policy is to maximize

$$V = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\lambda\left(x_t - k\right)^2 + \pi_t^2\right], \qquad k > 0, \quad \lambda > 0, \quad 0 < \beta < 1. \tag{2}$$

(i) Discuss (1) and (2) with focus on the underlying economic mechanisms, and derive the optimal time-consistent outcomes for output and inflation [Hint: Maximize (2) w.r.t. x_t subject to (1), which is a sequence of one-period problems, taking as given $\mathrm{E}_{t-1}\pi_t$; from the first-order condition derive $\mathrm{E}_{t-1}\pi_t$ and the solutions]. What is the inefficiency of the solution? Explain.

A Equation (1) is a conventional expectations augmented Phillips curve, which can be rationalized by e.g., presence of one-period nominal wage contracts or Lucas-style asymmetric information about local versus aggregate shocks. Equation (2) shows that the central bank dislikes fluctuations in the output gap around a target k, and fluctuations in inflation around a target value of zero. A positive target value for the output gap can be rationalized by the fact that the natural rate of output is considered inefficiently low (say, due to imperfect competition).

In determining the optimal time-consistent outcomes for output and inflation, substitute (1) and (2) and obtain the relevant first-order condition as:

$$-\lambda\left(x_t - k\right) - \kappa\left(\mathrm{E}_{t-1}\pi_t + \kappa x_t + \varepsilon_t\right) = 0.$$

Taking period $t-1$ expectations one gets

$$-\lambda\left(\mathrm{E}_{t-1}x_t - k\right) - \kappa\left(\mathrm{E}_{t-1}\pi_t + \kappa\mathrm{E}_{t-1}x_t + \mathrm{E}_{t-1}\varepsilon_t\right) = 0.$$

Using that (since ε_t is mean zero) $\mathrm{E}_{t-1}x_t = 0$ by (1), one gets inflation expectations from this expression as

$$\mathrm{E}_{t-1}\pi_t = \frac{\lambda}{\kappa}k.$$

Inserting this back into the first order condition yields

$$-\lambda\left(x_t - k\right) - \kappa\left(\frac{\lambda}{\kappa}k + \kappa x_t + \varepsilon_t\right) = 0,$$

and thus

$$x_t = -\frac{\kappa}{\lambda + \kappa^2}\varepsilon_t.$$

This is inserted into (1) to get inflation as

$$\pi_t = \frac{\lambda}{\kappa}k + \frac{\lambda}{\lambda + \kappa^2}\varepsilon_t.$$

These solutions demonstrate that the economy will suffer from an inflation bias when $k > 0$. Since, the central bank is expected to aim at pushing output above the natural rate, inflation expectations will go up to a point where the inflationary consequences of expansive monetary policy is too costly. In equilibrium inflation is excessive and average output is at the natural rate. The shock ε_t, however, is efficiently "spread out" between inflation and output to a degree determined by the relative preference for output versus inflation stability (λ)

(ii) Society now delegates monetary policymaking to a "conservative" central banker with a utility function given by

$$V^c = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\lambda^c\left(x_t - k\right)^2 + \pi_t^2\right], \qquad \lambda > \lambda^c > 0. \tag{3}$$

Show formally how the time-consistent outcomes change relative to those derived in (i)? Will delegation of this form always be beneficial?

A The solutions for output and inflation become

$$\begin{aligned} x_t &= -\frac{\kappa}{\lambda^c + \kappa^2}\varepsilon_t \\ \pi_t &= \frac{\lambda^c}{\kappa}k + \frac{\lambda^c}{\lambda^c + \kappa^2}\varepsilon_t \end{aligned}$$

Since appointing a conservative central banker implies that less emphasis will be put on attaining the output gap target of $k > 0$, inflation expectations will go down, and the economy will end up in a situation with a lower inflation bias; cf. $\frac{\lambda^c}{\kappa}k < \frac{\lambda}{\kappa}k$. This is the benefit of the conservative central banker. The loss, however, is that the stabilization of the ε-shock becomes inefficient, as output will become too volatile (as $\frac{\kappa}{\lambda^c+\kappa^2} > \frac{\kappa}{\lambda+\kappa^2}$), and inflation too stable (as $\frac{\lambda^c}{\lambda^c+\kappa^2} < \frac{\lambda}{\lambda+\kappa^2}$). Some degree of "conservativeness" is, however, always beneficial. The reason is that starting from a situation of $\lambda^c = \lambda$, lowering λ^c will cause a gain in terms of a lower inflation bias which is of first order, while the loss in terms of distorted shock stabilization is of second order. Hence, some λ^c, $0 < \lambda^c < \lambda$ is always optimal.

Assume now that (1) is replaced by

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + \varepsilon_t. \tag{4}$$

(iii) Derive the optimal time-consistent outcomes for output and inflation [Hint: Maximize (2) w.r.t. x_t subject to (4), which is a sequence of one-period problems, taking as given $\mathrm{E}_t \pi_{t+1}$; use the first-order condition together with (4) and derive π_t and thus x_t.] Discuss the solution, and point out similarities and differences with the solution when equation (1) applies.

A The relevant first-order condition is [where (4) has been substituted into (2)]:

$$-\lambda (x_t - k) - \kappa \pi_t = 0.$$

This is inserted into (4):

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \kappa [(\kappa/\lambda) \pi_t - k] + \varepsilon_t,$$

which becomes

$$\pi_t = \frac{\lambda \beta}{\lambda + \kappa^2} \mathrm{E}_t \pi_{t+1} + \frac{\lambda \kappa}{\lambda + \kappa^2} k + \frac{\lambda}{\lambda + \kappa^2} \varepsilon_t.$$

This is a first-order expectational difference equation with one unstable root; so there is a unique non-explosive solution to π_t. Conjecture a solution of the following format:[1]

$$\pi_t = Y + X \varepsilon_t,$$

where Y and X are the undetermined coefficients to be determined. Forward the conjecture one period, and take period t expectations:

$$\pi_{t+1} = Y + X \varepsilon_{t+1},$$

[1] It is sufficient to derive the solution for the case of $k = 0$, and verbally argue that there will be an inflation bias (i.e., $Y > 0$) when $k > 0$ applies.

$$\mathrm{E}_t\pi_{t+1} = Y + X\mathrm{E}_t\varepsilon_{t+1},$$

$$\mathrm{E}_t\pi_{t+1} = Y,$$

(as $\mathrm{E}_t\varepsilon_{t+1} = 0$ since ε is serially uncorrelated). Insert this into the difference equation:

$$\pi_t = \frac{\lambda\beta}{\lambda+\kappa^2}Y + \frac{\lambda\kappa}{\lambda+\kappa^2}k + \frac{\lambda}{\lambda+\kappa^2}\varepsilon_t$$

$$\pi_t = \frac{\lambda\beta Y + \lambda\kappa k}{\lambda+\kappa^2} + \frac{\lambda}{\lambda+\kappa^2}\varepsilon_t$$

This verifies the form of the conjecture, and identifies the coefficients by the equations

$$Y = \frac{\lambda\beta Y + \lambda\kappa k}{\lambda+\kappa^2} \qquad X = \frac{\lambda}{\lambda+\kappa^2}$$

Y therefore follows as

$$Y = \frac{\lambda\kappa}{\lambda(1-\beta)+\kappa^2}k,$$

Hence,

$$\pi_t = \frac{\lambda\kappa}{\lambda(1-\beta)+\kappa^2}k + \frac{\lambda}{\lambda+\kappa^2}\varepsilon_t.$$

Output is found from (4) as

$$\frac{\lambda\kappa}{\lambda(1-\beta)+\kappa^2}k + \frac{\lambda}{\lambda+\kappa^2}\varepsilon_t = \beta\frac{\lambda\kappa}{\lambda(1-\beta)+\kappa^2}k + \kappa x_t + \varepsilon_t,$$

$$\frac{\lambda}{\lambda(1-\beta)+\kappa^2}k + \frac{\lambda/\kappa}{\lambda+\kappa^2}\varepsilon_t = \beta\frac{\lambda}{\lambda(1-\beta)+\kappa^2}k + x_t + \frac{1}{\kappa}\varepsilon_t,$$

so

$$x_t = \left[\frac{\lambda/\kappa}{\lambda+\kappa^2} - \frac{1}{\kappa}\right]\varepsilon_t + \frac{\lambda(1-\beta)}{\lambda(1-\beta)+\kappa^2}k,$$

$$x_t = \left[\frac{\lambda}{\kappa(\lambda+\kappa^2)} - \frac{(\lambda+\kappa^2)}{\kappa(\lambda+\kappa^2)}\right]\varepsilon_t + \frac{\lambda(1-\beta)}{\lambda(1-\beta)+\kappa^2}k,$$

$$x_t = -\frac{\kappa}{\lambda+\kappa^2}\varepsilon_t + \frac{\lambda(1-\beta)}{\lambda(1-\beta)+\kappa^2}k.$$

The social inefficiencies of this solution are two-fold. First, there is an inflation bias when $k > 0$, and for the same reasons when (1) applies (a great answer notes that with (4) average inflation has a permanent output effect, as (4) does not depict a vertical long-run Phillips curve). In addition, there is inefficient stabilization of the ε-shock. The central bank could improve inflation stabilization, if it could commit to a continuation of contractive policies following a temporary positive realization of ε. The reason is that this will dampen inflation expectations and thus current inflation; i.e., it would improve the inflation-output gap trade off. However, such a commitment is not time consistent, as the central banker—when the shock has passed—has no incentives to contract.

(iv) Discuss whether delegation to a conservative central banker is beneficial when (4) applies.

A A conservative central banker will beneficial, as the inflation bias will be reduced. The inefficient stabilization policy, however, will not be improved by conservativeness in this special case where the shock does not persist into the future. In that case, there is no future inflation to signal contractive behavior about. In other words, conservativeness cannot be a means of affecting expectations about future stabilization policy, when shocks are temporary.

QUESTION 3:

Investment under a cash-in-advance constraint

Assume a model of a closed economy formulated in discrete time, where representative individuals have utility functions

$$U = \sum_{t=0}^{\infty} \beta^t u\left(c_t\right), \qquad 0 < \beta < 1, \tag{1}$$

with

$$u\left(c_t\right) \equiv \frac{\left(c_t\right)^{1-\sigma} - 1}{1-\sigma}, \qquad \sigma > 0,$$

and budget constraints

$$f\left(k_{t-1}\right) + \tau_t + \left(1-\delta\right) k_{t-1} + \frac{1}{1+\pi_t} m_{t-1} = c_t + k_t + m_t, \tag{2}$$

where c_t is consumption, m_t is real money balances at the end of period t, k_{t-1} is physical capital at the end of period $t-1$, τ_t are monetary transfers by the government,

$0 < \delta < 1$ is capital's rate of depreciation and π_t is the inflation rate. The function f is defined as $f(k_{t-1}) \equiv k_{t-1}^{\alpha}$, $0 < \alpha < 1$.

Purchases of consumption goods, as well as investment in physical capital, are subject to a cash-in-advance constraint. This is modelled as

$$c_t + k_t - (1-\delta) k_{t-1} \leq \tau_t + \frac{1}{1+\pi_t} m_{t-1}. \tag{3}$$

(i) Discuss the model given by (1), (2) and (3).

A The discussion can be relatively brief, with main focus on (3) and its deviation from the standard expression from the curriculum without investment.

(ii) Derive the relevant first-order conditions for optimal individual behavior, For this purpose, use the value function

$$\begin{aligned} V(k_{t-1}, m_{t-1}) &= \max \{u(c_t) + \beta V(k_t, m_t)\ , \\ &\quad -\mu_t [c_t + k_t - (1-\delta) k_{t-1} - \tau_t - (1/(1+\pi_t)) m_{t-1}]\} \end{aligned}$$

where μ_t is the multiplier on (3), and where the maximization is over c_t, m_t and k_t and subject to (2). [Hint: Simplify the problem by using (2) to substitute out k_t in the value function]

A Do the substitution and one gets:

$$\begin{aligned} V(k_{t-1}, m_{t-1}) &= \max \left\{ u(c_t) + \beta V\left(f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + \frac{1}{1+\pi_t} m_{t-1} - c_t - m_t, m_t \right) \right. \\ &\quad \left. -\mu_t [f(k_{t-1}) - m_t] \right\} \end{aligned}$$

The first-order conditions are therefore:

$$u'(c_t) = \beta V_k(k_t, m_t)$$

$$-\beta V_k(k_t, m_t) + \beta V_m(k_t, m_t) + \mu_t = 0$$

The value function derivatives are, after using the Envelope theorem:

$$V_k(k_{t-1}, m_{t-1}) = \beta V_k(k_t, m_t)[f'(k_{t-1}) + 1 - \delta] - \mu_t f'(k_{t-1})$$

$$V_m(k_{t-1}, m_{t-1}) = \beta V_k(k_t, m_t) \frac{1}{1+\pi_t}$$

(iii) Interpret the first-order conditions and show that they (along with the expressions for the partial derivatives of the value function derived using the Envelope Theorem) can be combined into the following steady-state relationships:

$$\begin{aligned} \left(c^{ss}\right)^{-\sigma} &= \beta V_k\left(k^{ss}, m^{ss}\right), & (4) \\ V_k\left(k^{ss}, m^{ss}\right) &= \beta V_k\left(k^{ss}, m^{ss}\right)(1-\delta) + \beta V_m\left(k^{ss}, m^{ss}\right)\alpha\left(k^{ss}\right)^{\alpha-1}, & (5) \\ V_m\left(k^{ss}, m^{ss}\right) &= \beta V_k\left(k^{ss}, m^{ss}\right)\frac{1}{1+\pi^{ss}} & (6) \end{aligned}$$

where superscript "ss" denotes steady-state values.

A Eliminate first $\mu_t > 0$:

$$\begin{aligned} V_k\left(k_{t-1}, m_{t-1}\right) &= \beta V_k\left(k_t, m_t\right)\left[f'\left(k_{t-1}\right)+1-\delta\right] - \mu_t f'\left(k_{t-1}\right) \\ &= \beta V_k\left(k_t, m_t\right)\left[f'\left(k_{t-1}\right)+1-\delta\right] - \left[\beta V_k\left(k_t, m_t\right) - \beta V_m\left(k_t, m_t\right)\right] f'\left(k_{t-1}\right) \\ &= \beta V_k\left(k_t, m_t\right)(1-\delta) + \beta V_m\left(k_t, m_t\right) f'\left(k_{t-1}\right) \end{aligned}$$

Therefore, in steady state

$$u'(c) = \beta V_k\left(k^{ss}, m^{ss}\right)$$

$$V_k\left(k^{ss}, m^{ss}\right) = \beta V_k\left(k^{ss}, m^{ss}\right)(1-\delta) + \beta V_m\left(k^{ss}, m^{ss}\right) f'\left(k^{ss}\right)$$

$$V_m\left(k^{ss}, m^{ss}\right) = \beta V_k\left(k^{ss}, m^{ss}\right)\frac{1}{1+\pi^{ss}}$$

which by use of the functional forms for utility and production gives the desired expressions.

(iv) By use of (5) and (6), derive the steady-state value of k, and show formally whether or not the model exhibits superneutrality. Explain the result and discuss the characteristics of the optimal rate of inflation.

A (5) and (6) become:

$$V_k\left(k^{ss}, m^{ss}\right) = \beta V_k\left(k^{ss}, m^{ss}\right)(1-\delta) + \beta V_m\left(k^{ss}, m^{ss}\right) f'\left(k^{ss}\right)$$

$$V_m\left(k^{ss}, m^{ss}\right)\frac{1+\pi^{ss}}{\beta} = V_k\left(k^{ss}, m^{ss}\right)$$

Thus,

$$V_k\left(k^{ss}, m^{ss}\right) = \beta V_k\left(k^{ss}, m^{ss}\right)(1-\delta) + \beta V_m\left(k^{ss}, m^{ss}\right) f'\left(k^{ss}\right)$$

becomes

$$V_m\left(k^{ss},m^{ss}\right)\frac{1+\pi^{ss}}{\beta}=\beta V_m\left(k^{ss},m^{ss}\right)\frac{1+\pi^{ss}}{\beta}\left(1-\delta\right)+\beta V_m\left(k^{ss},m^{ss}\right)f'\left(k^{ss}\right),$$

leading to

$$\frac{1+\pi^{ss}}{\beta}=\left(1+\pi^{ss}\right)\left(1-\delta\right)+\beta f'\left(k^{ss}\right)$$

$$\frac{\left(1+\pi^{ss}\right)\left[\frac{1}{\beta}-1+\delta\right]}{\beta}=f'\left(k^{ss}\right)$$

With the given production function this is:

$$\frac{\left(1+\pi^{ss}\right)\left[\frac{1}{\beta}-1+\delta\right]}{\beta}=\alpha\left(k^{ss}\right)^{a-1}$$

$$\frac{\left(1+\pi^{ss}\right)\left[\frac{1}{\beta}-1+\delta\right]}{\alpha\beta}=\left(k^{ss}\right)^{a-1}$$

Hence, in steady state:

$$k^{ss}=\left[\frac{\alpha\beta}{\left(1+\pi^{ss}\right)\left[\frac{1}{\beta}-1+\delta\right]}\right]^{\frac{1}{1-\alpha}},$$

It is seen that higher inflation reduces the capital stock. Superneutrality thus fails (in comparison with the model where only consumption is subject to a CIA constraint). The reason is that steady-state inflation raises the steady-state nominal interest rate, and thus the opportunity cost of holding money. As money must be used for investment by (3), capital formation is depressed and steady-state capital becomes lower. The optimal rate of inflation is therefore the one that makes the nominal interest rate zero; i.e., a monetary policy that implements the Friedman rule.

It is excellent (but not necessary) to formally show that this happens when $\mu=0$ and thus when

$$\frac{1}{\beta}=f'\left(k^{ss}\right)+1-\delta;$$

i.e., when the gross marginal product of capital net of depreciation equals the subjective gross rate of interest as in the case without a CIA constraint on investment. With the particular production function this is equivalent of

$$\frac{1}{\beta}-1+\delta=\alpha\left(k^{ss}\right)^{a-1}.$$

From above, this is achieved when

$$\frac{1+\pi^{ss}}{\beta}=1,$$

and thus

$$\pi^{ss}=\beta-1$$

which indeed states that the gross nominal interest rate should be one, which implies a rate of deflation equal to the subjective real rate of interest.