This set contains four pages (beginning with this page)
All questions must be answered
Questions 1 and 2 each weigh 25 % while question 3 weighs 50 %. These weights, however, are only indicative for the overall evaluation.

Henrik Jensen
Department of Economics, University of Copenhagen
Spring 2011

## QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) A shopping-time model of money demand provides a foundation for the Cash-in-Advance approach.

(ii) In the simple New-Keynesian model with price rigidities only, absence of any exogenous fluctuations in firms' desired markup, implies that the central bank can achieve the efficient allocation when an appropriate labor subsidy is in place.

(iii) In the Barro and Gordon model where the monetary policymaker's utility function is $U = -(\lambda/2)(y-k)^2 - (1/2)\pi^2,\ k > 0$, where $y$ is output given by $y = \pi - \pi^e + \varepsilon$, and where $\pi$, $\pi^e$, and $\varepsilon$ are inflation, inflation expectations and a supply shock, respectively, a linear inflation contract of the form $t(\pi) = -C\pi$ where $C$ is some constant, will eliminate the inflation bias but distort shock stabilization.

# QUESTION 2:

**Cash in advance, labor supply and interest rates**

Consider an infinite-horizon economy in discrete time, where utility of the representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1 - n_t\right), \qquad 0 < \beta < 1, \tag{1}$$

with

$$u\left(c_t, 1 - n_t\right) \equiv \frac{\left(c_t\right)^{1-\sigma}}{1-\sigma} + \Psi \frac{\left(1 - n_t\right)^{1-\eta}}{1-\eta}, \qquad \eta, \sigma, \Psi > 0$$

where $c_t$ is consumption and $n_t$ is the fraction of time spent working. Purchases of consumption goods are subject to a cash-in-advance constraint

$$c_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau_t - b_t, \tag{2}$$

where $m_{t-1}$ is real money balances at the end of period $t$, $\pi_t$ is the inflation rate, $\tau_t$ are real monetary transfers from the government, and $b_t$ are real bonds traded on financial markets before the goods market open.

Agents maximize utility subject to (2) and the budget constraint

$$f\left(k_{t-1}, n_t\right) + (1-\delta) k_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t + i_t b_t = c_t + k_t + m_t, \qquad 0 < \delta < 1, \tag{3}$$

where $k_{t-1}$ is physical capital, $i_t$ is the nominal interest rate and function $f$ is given by $f\left(k_{t-1}, n_t\right) = y_t = k_{t-1}^{\alpha} n_t^{1-\alpha}$, with $y_t$ denoting output.

(i) Derive the relevant first-order conditions for optimal behavior [Hint: Set up the value function $V\left(k_{t-1}, b_{t-1}, m_{t-1}\right) = \max\left\{u\left(c_t, 1-n_t\right) + \beta V\left(k_t, b_t, m_t\right)\right\}$ and eliminate $k_t$ by the budget constraint, and maximize over $c$, $m$, $n$ and $b$ subject to (2)—let $\mu_t$ denote the Lagrange multiplier on (2)]. Interpret the first-order conditions intuitively.

(ii) Show that the Envelope theorem along with the first-order conditions lead to the following conditions

$$\frac{\beta V_k\left(k_{t+1}, b_{t+1}, m_{t+1}\right) + \mu_{t+1}}{1+\pi_{t+1}} = V_k\left(k_t, b_t, m_t\right), \tag{4}$$

$$V_k\left(k_t, b_t, m_t\right) = \beta\left[\alpha\left(y_{t+1}/k_t\right) - \delta\right] V_k\left(k_{t+1}, b_{t+1}, m_{t+1}\right), \tag{5}$$

$$i_t = \frac{\mu_t}{\beta V_k\left(k_t, b_t, m_t\right)}. \tag{6}$$

Furthermore, discuss whether superneutrality holds in steady state.

(iii) Show that in steady state, the consumption-leisure choice is determined by

$$\frac{\left(c^{ss}\right)^{-\sigma}}{\Psi\left(1-n^{ss}\right)^{-\eta}} = \frac{1+i^{ss}}{(1-\alpha)\left(y^{ss}/n^{ss}\right)},$$

and discuss if and why the nominal interest rate affects steady-state labour supply.

## QUESTION 3:

**Monetary policymaking and cost shocks**

Consider the following log-linear model of a closed economy:

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n\right), \qquad \sigma > 0, & (1) \\ \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \qquad 0 < \beta < 1, \quad \kappa > 0, & (2) \end{aligned}$$

where $x_t$ is the output gap, $i_t$ is the nominal interest rate (the monetary policy instrument), $\pi_t$ is goods price inflation, $r_t^n$ is the natural rate of interest, which is assumed to be a mean-zero, serially uncorrelated shock, and $u_t$ is a mean-zero serially uncorrelated "cost" shock. $\mathrm{E}_t$ is the rational expectations operator conditional on all information up to and including period $t$.

(i) Discuss the micro-foundations behind equations (1) and (2).

(ii) The monetary authority wants to minimize the loss function

$$L = \frac{1}{2}\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda x_t^2 + \pi_t^2\right], \qquad \lambda > 0. \tag{3}$$

Discuss the micro-economic rationale for this loss function.

(iii) Assume that $r_t^n = 0$ for all $t$. Show that the optimal values of $x_t$ and $\pi_t$ under discretionary policymaking are

$$\begin{aligned} x_t &= -\frac{\kappa}{\kappa^2 + \lambda} e_t, \\ \pi_t &= \frac{\lambda}{\kappa^2 + \lambda} e_t. \end{aligned}$$

Discuss.

(iv) Continue to assume that $r_t^n = 0$ for all $t$. Now assume that the monetary policymaker follows a rule for nominal interest-rate setting given as

$$i_t = \phi \pi_t, \qquad \phi > 1. \tag{4}$$

Derive the solutions for $x_t$ and $\pi_t$ for the system (1), (2) and (4). [Hint: Conjecture that the solutions are linear functions of the period's cost shock]. Discuss the differences between these solutions and the ones obtained under discretionary policymaking. Can the monetary policy rule (4) be parameterized such that it will "deliver" the outcomes under discretionary policymaking?

(v) Discuss how the introduction of shocks to the natural rate of interest, $r_t^n \neq 0$, may potentially alter the answer to (iv).

(vi) Will an ability to conduct optimal policy under commitment be advantageous in this setting? Will the answer depend on whether $e_t$ is a persistent shock or not? Discuss.