This set contains four pages (beginning with this page)
All questions must be answered
Questions 1 and 2 each weigh 25 % while question 3 weighs 50 %. These weights, however, are only indicative for the overall evaluation.

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MONETARY ECONOMICS: MACRO ASPECTS
SOLUTIONS TO AUGUST 19 EXAM

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) A shopping-time model of money demand provides a foundation for the Cash-in-Advance approach.

A False. A shopping-time model formalises how money can reduce time spent on purchases, and therefore motivates money in the utility function as a proxy for reduced shopping time.

(ii) In the simple New-Keynesian model with price rigidities only, absence of any exogenous fluctuations in firms' desired markup, implies that the central bank can achieve the efficient allocation when an appropriate labor subsidy is in place.

A True. Under these conditions a monetary policy of stable prices will induce all firms to hire the efficient aggregate amount of workers. All firms will produce the same under price stability (as no relative prices change) whereby any output dispersion caused by price rigidities are absent.

(iii) In the Barro and Gordon model where the monetary policymaker's utility function is $U = -(\lambda/2)(y-k)^2 - (1/2)\pi^2$, $k > 0$, where y is output given by $y = \pi - \pi^e + \varepsilon$, and where π, π^e, and ε are inflation, inflation expectations and a supply shock, respectively, a linear inflation contract of the form $t(\pi) = -C\pi$ where C is some constant, will eliminate the inflation bias but distort shock stabilization.

A False. The contract indeed eliminates the inflation bias by proper choice of C, but as it does not change the relative weight on output and inflation fluctuations, stabilization policy will not be distorted (unlike, e.g., in the case of a Rogoff-conservative central bank). One can show that the first-order condition under discretionary policymaking is $-\lambda(\pi - \pi^e + \varepsilon - k) - \pi - C = 0$. Inflation expectations will under a mean-zero supply shock be $\pi = \lambda k - C$. Hence, $C = \lambda k$ eliminates the inflation bias, but retains efficient stabilization policy $\pi = -[\lambda/(1+\lambda)]\varepsilon_t$.

QUESTION 2:

Cash in advance, labor supply and interest rates

Consider an infinite-horizon economy in discrete time, where utility of the representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t), \qquad 0 < \beta < 1, \tag{1}$$

with

$$u(c_t, 1-n_t) \equiv \frac{(c_t)^{1-\sigma}}{1-\sigma} + \Psi \frac{(1-n_t)^{1-\eta}}{1-\eta}, \qquad \eta, \sigma, \Psi > 0$$

where c_t is consumption and n_t is the fraction of time spent working. Purchases of consumption goods are subject to a cash-in-advance constraint

$$c_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau_t - b_t, \tag{2}$$

where m_{t-1} is real money balances at the end of period t, π_t is the inflation rate, τ_t are real monetary transfers from the government, and b_t are real bonds traded on financial markets before the goods market open.

Agents maximize utility subject to (2) and the budget constraint

$$f\left(k_{t-1}, n_t\right) + (1-\delta)\, k_{t-1} + \frac{m_{t-1}}{1+\pi_t} + \tau_t + i_t b_t = c_t + k_t + m_t, \qquad 0<\delta<1, \tag{3}$$

where k_{t-1} is physical capital, i_t is the nominal interest rate and function f is given by $f\left(k_{t-1}, n_t\right) = y_t = k_{t-1}^{\alpha} n_t^{1-\alpha}$, with y_t denoting output.

(i) Derive the relevant first-order conditions for optimal behavior [Hint: Set up the value function $V\left(k_{t-1}, b_{t-1}, m_{t-1}\right) = \max\left\{u\left(c_t, 1-n_t\right) + \beta V\left(k_t, b_t, m_t\right)\right\}$ and eliminate k_t by the budget constraint, and maximize over c, m, n and b subject to (2)—let μ_t denote the Lagrange multiplier on (2)]. Interpret the first-order conditions intuitively.

A With the function V as defined by the hint, the first-order conditions are

$$u_c\left(c_t, 1-n_t\right) - \beta V_k\left(k_t, b_t, m_t\right) - \mu_t = 0, \tag{*}$$

$$-\beta V_k\left(k_t, b_t, m_t\right) + \beta V_m\left(k_t, b_t, m_t\right) = 0, \tag{**}$$

$$-u_n\left(c_t, 1-n_t\right) + \beta V_k\left(k_t, b_t, m_t\right) f_n\left(k_{t-1}, n_t\right) = 0, \tag{***}$$

$$\beta V_b\left(k_t, b_t, m_t\right) + \beta V_k\left(k_t, b_t, m_t\right) i_t - \mu_t = 0. \tag{****}$$

The first condition equates the marginal gain of consumption with the marginal losses in terms of less savings in capital and the "liquidity" cost in case of a binding CIA constraint. The second equates the marginal loss of real balances in terms of less savings in capital with the marginal gain per se. The third condition equates the marginal loss of leisure with the marginal gain of more work (the real wage times the marginal value of savings in capital). The fourth condition equates the marginal gain of bond savings (per se and in terms of interest) with the marginal loss in terms of lost liquidity services.

(ii) Show that the Envelope theorem along with the first-order conditions lead to the following conditions[1]

$$\frac{\beta V_k\left(k_{t+1}, b_{t+1}, m_{t+1}\right) + \mu_{t+1}}{1+\pi_{t+1}} = V_k\left(k_t, b_t, m_t\right), \tag{4}$$

$$V_k\left(k_t, b_t, m_t\right) = \beta\left[\alpha\left(y_{t+1}/k_t\right) + 1 - \delta\right] V_k\left(k_{t+1}, b_{t+1}, m_{t+1}\right), \tag{5}$$

[1] Note that in the assignment $-\delta$ in (5) should read $1-\delta$. This, however, is considered a minor typo, and one that does not affect the important results in the excercise.

$$i_t = \frac{\mu_t}{\beta V_k (k_t, b_t, m_t)}. \tag{6}$$

Furthermore, discuss whether superneutrality holds in steady state.

A The envelope theorem gives

$$\begin{aligned} V_k (k_{t-1}, b_{t-1}, m_{t-1}) &= \beta V_k (k_t, b_t, m_t) [f_k (k_{t-1}, n_t) + (1-\delta)], & (*****) \\ V_b (k_{t-1}, b_{t-1}, m_{t-1}) &= 0, & (******) \\ V_m (k_{t-1}, b_{t-1}, m_{t-1}) &= \beta V_k (k_t, b_t, m_t) \frac{1}{1+\pi_t} + \mu_t \frac{1}{1+\pi_t}. & (*******) \end{aligned}$$

Forwarding (*******) one period and combining it with (**) gives (4). Forwarding (*****) one period and using the functional form of f gives (5). Combining (******) with (****) gives (6).

(iii) Show that in steady state, the consumption-leisure choice is determined by

$$\frac{(c^{ss})^{-\sigma}}{\Psi (1-n^{ss})^{-\eta}} = \frac{1+i^{ss}}{(1-\alpha)(y^{ss}/n^{ss})},$$

and discuss if and why the nominal interest rate affects steady-state labour supply.

A Combining (*) and (***) with the particular functional forms of u and f gives

$$\begin{aligned} \frac{(c^{ss})^{-\sigma}}{\Psi (1-n^{ss})^{-\eta}} &= \frac{\beta V_k (k^{ss}, b^{ss}, m^{ss}) + \mu^{ss}}{\beta V_k (k^{ss}, b^{ss}, m^{ss}) (1-\alpha)(y^{ss}/n^{ss})}, \\ &= \frac{1 + \mu^{ss}/\beta V_k (k^{ss}, b^{ss}, m^{ss})}{(1-\alpha)(y^{ss}/n^{ss})}, \\ &= \frac{1+i^{ss}}{(1-\alpha)(y^{ss}/n^{ss})}. \end{aligned}$$

where the last equality follows from (6). It follows that superneutraliy does not hold, as different nominal interest rates (and thus different monetary policies) alter the consumption leisure trade-off. A higher nominal interest rate implies that consumption becomes more "expensive" relative to leisure causing agents to substitute away from consumption and supply less labor.

QUESTION 3:

Monetary policymaking and cost shocks

Consider the following log-linear model of a closed economy:

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1} \left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n \right), \qquad \sigma > 0, \tag{1}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \qquad 0 < \beta < 1, \quad \kappa > 0, \tag{2}$$

where x_t is the output gap, i_t is the nominal interest rate (the monetary policy instrument), π_t is goods price inflation, r_t^n is the natural rate of interest, which is assumed to be a mean-zero, serially uncorrelated shock, and u_t is a mean-zero serially uncorrelated "cost" shock. E_t is the rational expectations operator conditional on all information up to and including period t.

(i) Discuss the micro-foundations behind equations (1) and (2).

A Here is should be mentioned that (1), a dynamic IS curve, is derived from a log-linearization of consumers' consumption-Euler equations: A higher real interest rate, $i_t -$ $\mathrm{E}_t \pi_{t+1}$, make consumers increase future consumption relative to current. Equation (2), a New-Keynesian Phillips Curve, is derived from the optimal price-setting decisions of monopolistically competitive firms that operate under price stickiness. Prices are set as a markup over marginal costs, and as the output gap is proportional to marginal costs, it enters (2) positively. The more price rigidity (e.g., the lower a probability of price adjustment under a Calvo price-setting scheme), the smaller is κ. Expected future prices are central for price determination, as firms are forward looking, since they acknowledge that the price set today may be effective for some periods.

(ii) The monetary authority wants to minimize the loss function

$$L = \frac{1}{2} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda x_t^2 + \pi_t^2 \right], \qquad \lambda > 0. \tag{3}$$

Discuss the micro-economic rationale for this loss function.

A This type of loss function can be derived as the second-order Taylor approximation to (the negative of) the representative household's utility function. Price rigidity causes losses from aggregate mark-ups being different from the desired markup, and under the Calvo-price structure, staggering cause inefficient dispersion of consumption of various goods. The quadratic terms in (3) reflect the costs from these fluctuations. Inflation is proportional to the inefficient goods dispersion, and output gap fluctuations are proportional to the fluctuations in the mark-up gap (that causes inefficient fluctuations in consumption and labor).

(iii) Assume that $r_t^n = 0$ for all t. Show that the optimal values of x_t and π_t under discretionary policymaking are

$$\begin{aligned} x_t &= -\frac{\kappa}{\kappa^2 + \lambda} e_t, \\ \pi_t &= \frac{\lambda}{\kappa^2 + \lambda} e_t. \end{aligned}$$

Discuss.

A Under discretionary policymaking, the authority optimizes on a period-by-period basis, essentially making it a static problem. Hence, it solves, treating x_t as the policy instrument,

$$\min_{x_t} \lambda x_t^2 + \pi_t^2 \quad \text{s.t.} \quad \pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

taking as given inflation expectations. The first-order condition is

$$\lambda x_t + \kappa \pi_t = 0.$$

Using this to substitute out x_t in (2) gives the difference equation

$$\begin{aligned} \pi_t &= \beta \mathrm{E}_t \pi_{t+1} - \left(\kappa^2/\lambda\right) \pi_t + e_t \\ \pi_t \left(1 + \kappa^2/\lambda\right) &= \beta \mathrm{E}_t \pi_{t+1} + e_t. \end{aligned}$$

Since $\mathrm{E}_t e_{t+1} = 0$, we immediately get

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda} e_t,$$

and by the first-order condition,

$$x_t = -\frac{\kappa}{\kappa^2 + \lambda} e_t$$

as desired. This solution shows how the authority reduces the inflationary impact of a cost shock by contracting the output gap (and it can be noted that the lower is λ, the stronger is this inflation stabilization as the authority cares less for the output gap relative to inflation).

(iv) Continue to assume that $r_t^n = 0$ for all t. Now assume that the monetary policymaker follows a rule for nominal interest-rate setting given as

$$i_t = \phi \pi_t, \qquad \phi > 1. \tag{4}$$

Derive the solutions for x_t and π_t for the system (1), (2) and (4). [Hint: Conjecture that the solutions are linear functions of the period's cost shock]. Discuss the differences between these solutions and the ones obtained under discretionary policymaking. Can the monetary policy rule (4) be parameterized such that it will "deliver" the outcomes under discretionary policymaking?

A Using the hint one conjectures that inflation and the output gap are given by

$$\begin{aligned} \pi_t &= A_\pi e_t, \\ x_t &= A_x e_t, \end{aligned}$$

which imply that $\mathrm{E}_t \pi_{t+1} = \mathrm{E}_t x_{t+1} = 0$. Using these in (1) and (2) gives

$$\begin{aligned} A_x e_t &= -\sigma^{-1} \phi A_\pi e_t, \\ A_\pi e_t &= \kappa A_x e_t + e_t, \end{aligned}$$

which identifies the unknown coefficients by

$$\begin{aligned} A_x &= -\sigma^{-1} \phi A_\pi, \\ A_\pi &= \kappa A_x + 1. \end{aligned}$$

Hence,

$$A_x = -\sigma^{-1} \phi \left(\kappa A_x + 1 \right),$$

and thus

$$A_x = -\frac{\sigma^{-1} \phi}{1 + \sigma^{-1} \kappa \phi},$$

and, consequently

$$A_\pi = \frac{1}{1 + \sigma^{-1} \kappa \phi}.$$

The solutions are therefore

$$\begin{aligned} \pi_t &= \frac{1}{1 + \sigma^{-1} \kappa \phi} e_t, \\ x_t &= -\frac{\sigma^{-1} \phi}{1 + \sigma^{-1} \kappa \phi} e_t. \end{aligned}$$

One sees again that the inflationary cost shock is dampenened when $\phi > 1$, at the cost of output gap instability. The higher is ϕ, the more stable will inflation be relative to the output gap.

The easiest way of checking whether a properly designed policy rule can deliver the outcomes under discretionary policymaking is to see whether ϕ can be chosen so the first-order condition is satisfied. Rewritten, this reads

$$\frac{\pi_t}{x_t} = -\frac{\lambda}{\kappa}.$$

(Note that this condition can be recovered from question (iii) without having computed it.) Insert the solutions under the policy rule, and one gets

$$-\frac{1}{\sigma^{-1}\phi} = -\frac{\lambda}{\kappa},$$

or,

$$\phi = \frac{\kappa}{\sigma^{-1}\lambda}.$$

Hence, with a value of this policy rule parameter, (4) will reproduce policy under discretion (here it should be noted that $\kappa / (\sigma^{-1}\lambda) > 1$ should be satisfied in order for the equilibrium to be unique).

(v) Discuss how the introduction of shocks to the natural rate of interest, $r_t^n \neq 0$, may potentially alter the answer to (iv).

A With fluctuations in the natural rate of interest, the result that a policy rule can reproduce the discretionary policy breaks down. Under discretionary policymaking, any variation in the natural rate of interest can be perfectly offset by proper movements in the nominal interest rate; i.e., the shock does not pose a trade off for policy. With a policy rule like (4), this can only be achieved if $\phi \to \infty$ (as this creates full inflation stabilization and full output stabilization in the face of shocks to the natural rate of interest). Otherwise, the policy rule cannot reproduce optimal policy.

(vi) Will an ability to conduct optimal policy under commitment be advantageous in this setting? Will the answer depend on whether e_t is a persistent shock or not? Discuss.

A Commitment involves an ability to affect market expectations to the policy-maker's advantage. This will never be disadvantageous. Even in the case of no serial correlation in e_t, such an ability will be beneficial. In the model, it implies that a cost shock should be met by less contraction, but a persistent contraction, as this will dampen inflation expectations and thereby help stabilize inflation without too big output costs.