Monetary Policy Effectiveness in a Dynamic AS/AD Model with Sticky Wages*

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Version 1.0, April 12, 2012

*Teaching note for M.Sc. course on "Monetary Economics: Macro Aspects". The note builds on, and extends, a model laid out in Walsh (2010), as well as my earlier teaching material on this subject. I thank Christian Groth and Carl E. Walsh for comments and discussions on earlier versions.

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Preface

This note deals with monetary policy effectiveness in a New-Classical model with micro-foudations. The model is a simple money-in-the-utility function model with one-period nominal wage rigidity. The aggregate supply curve of the model is akin to the "Lucas supply" function, which is a defining element of New-Classical economics. This research program, focusing on rational expectations, gave rise to the famous "Policy Ineffectiveness Proposition," stating that systematic components in policy design had no influence on real activity. This note shows how systematic components of policymaking affect the variability output, thereby emphasizing that even within early rational-expectations models of this type, there is real impact of systematic policy intervention.

Henrik Jensen, April 2012

1 Introduction

Consider the log-linearized, simplified money-in-the utility function model with flexible prices as presented in Walsh (2010) p. 227.[1] All lower-case letters denote log-deviations from steady state, except interest rates which are absolute deviations of respective rates from their steady-state values. See Walsh (2010, Section 2.7.1) for the derivation of the full version of the model.

Output, y_t, is produced by labor, n_t, according to a Cobb-Douglas technology:

$$y_t = (1-\alpha)\, n_t + e_t, \qquad 0 < \alpha < 1, \tag{6.1}$$

where e_t is a mean-zero supply shock assumed to be serially uncorrelated (this makes a distinction between the supply shock e_t and the shock $\varepsilon_t \equiv e_t - \mathrm{E}_{t-1} e_t$ defined in Walsh, 2010, irrelevant). All output is consumed in equilibrium:

$$y_t = c_t, \tag{6.2}$$

where c_t is consumption. Firms hire labor up to the point where the marginal product of labor equals the real wage, such that labor demand is characterized by

$$y_t - n_t = w_t - p_t, \tag{6.3}$$

where w_t is the nominal wage and p_t is the price level. Consumers choose consumption over time so as to satisfy the conventional Keynes-Ramsey rule, which in logs can be written as

$$\Phi \mathrm{E}_t\,(c_{t+1} - c_t) - r_t = 0, \qquad \Phi > 0, \tag{6.4}$$

where r_t is the real interest rate and Φ is the coefficient of relative risk aversion in consumption, which in this setting equals the inverse of the intertemporal rate of substitution. E_t is the rational-expectations operator conditional on all information up until, and including, period t. Consumers choose to supply labor such that the marginal rate of substitution between leisure and consumption equals the real wage; in logs this is

$$\eta \left(\frac{n^{ss}}{1-n^{ss}}\right) n_t + \Phi c_t = w_t - p_t, \qquad \eta > 0, \tag{6.5}$$

where n^{ss} is the steady-state value of labor, and η is the coefficient of relative risk aversion in leisure, which in this setting can be interpreted as the inverse (Frisch) elasticity of labor

[1] In order to facilitate the comparison with Walsh (2010), I retain his equation numbering when adequate.

supply. With flexible wages, as well as prices, (6.1)–(6.5) determine y_t, c_t, n_t, r_t and $w_t - p_t$. Monetary policy will have no role for output determination.

Money demand is given by

$$m_t - p_t = c_t - \left(\frac{1}{bi^{ss}}\right) i_t, \qquad b = \Phi > 0, \tag{6.6}$$

where m_t is the nominal money supply, i_t is the nominal interest rate, i^{ss} is the steady-state nominal interest rate, and b is a utility function parameter determining the interest-rate elasticity of money demand. This version of the model assumes $b = \Phi$, so as to focus on the simple version with separable utility of consumption, real money holdings and leisure. (The parameters b and Φ will, however, be used independently throughout so as to highlight their different roles.) The Fisher equation links the nominal interest rate to the real rate and expected inflation:

$$i_t = r_t + \mathrm{E}_t p_{t+1} - p_t. \tag{6.7}$$

Finally, the model is closed by a specification of the process for the money supply:

$$m_t = \rho_m m_{t-1} + s_t, \qquad 0 \leq |\rho_m| \leq 1, \tag{6.8}$$

where the shock s_t has mean zero and is serially uncorrelated.

To introduce a role for monetary policy, some nominal stickiness is assumed. Specifically, it is stipulated that nominal wages are fixed for one period, and are determined the period before according to:

$$\begin{aligned} w_t &= \mathrm{E}_{t-1} p_t + \mathrm{E}_{t-1} y_t - \mathrm{E}_{t-1} n_t, \qquad (1) \\ &= \mathrm{E}_{t-1} p_t + \mathrm{E}_{t-1} \omega_t^*. \end{aligned}$$

I.e., the expected real wage is set so as to match the expected marginal product of labor, labelled $\mathrm{E}_{t-1}\omega_t^*$. With this wage-setting rule, equation (6.5) becomes redundant. Also, equation (6.8) is replaced by the slightly more general expression

$$m_t = \mu + \rho_m m_{t-1} + s_t.$$

where μ is some constant. Note that in Appendix 6.5, Walsh (2010) considers the special case of $m_t = m_{t-1} + s_t$. This simplification unintentionally hides the effectiveness of systematic monetary policy as will be clear below.[2]

We now turn to the solution of the model under sticky wages.

[2] He acknowledges this in Footnote 6 on p. 230.

2 Solving the model

2.1 Fleshing out an AS/AD structure

It is appropriate to make some simplifications of the presented system of equations. Here, it will be easy to substitute out c_t by y_t and r_t by $r_t = i_t - (\mathrm{E}_t p_{t+1} - p_t)$. Also, one can immediately substitute the nominal wage rule, (1), into (6.3). Then, a simpler system is:

$$\begin{aligned} y_t &= (1-\alpha)\, n_t + e_t, & (2)\\ y_t - n_t &= \mathrm{E}_{t-1} p_t + \mathrm{E}_{t-1} y_t - \mathrm{E}_{t-1} n_t - p_t, & (3)\\ \Phi \mathrm{E}_t\,(y_{t+1} - y_t) &= i_t - (\mathrm{E}_t p_{t+1} - p_t)\,, & (4)\\ m_t - p_t &= y_t - \left(\frac{1}{bi^{ss}}\right) i_t, & (5)\\ m_t &= \mu + \rho_m m_{t-1} + s_t. & (6) \end{aligned}$$

This system of five equations will determine the rational-expectations solutions for the five variables y_t, n_t, i_t, p_t, and m_t, given the shocks e_t and s_t and given m_{t-1}.

Now we proceed as in Walsh (2010, Appendix 6.5) by splitting up the system into the aggregate supply and demand sides. Equations (2) and (3) constitute the supply side, and give supply of output and employment as function of prices, whereas equations (4), (5) and (6) give output demand and the nominal interest rate (via money demand) as functions of prices. These aggregate supply and demand schedules then provide the equilibrium output and price level.

Let us start with the supply side. From (3) we get

$$n_t = \mathrm{E}_{t-1} n_t + y_t - \mathrm{E}_{t-1} y_t + p_t - \mathrm{E}_{t-1} p_t,$$

which inserted into (2) gives

$$y_t = (1-\alpha)\,(\mathrm{E}_{t-1} n_t + y_t - \mathrm{E}_{t-1} y_t + p_t - \mathrm{E}_{t-1} p_t) + e_t,$$

and therefore

$$\alpha y_t = (1-\alpha)\,(\mathrm{E}_{t-1} n_t - \mathrm{E}_{t-1} y_t + p_t - \mathrm{E}_{t-1} p_t) + e_t.$$

Take period-$t-1$ expectations on both sides of (2) to get (remembering that $\mathrm{E}_{t-1} e_t = 0$ by assumption):

$$\mathrm{E}_{t-1} y_t = (1-\alpha)\, \mathrm{E}_{t-1} n_t,$$

and substitute out $(1-\alpha)\mathrm{E}_{t-1} n_t$ above:

$$\begin{aligned} \alpha y_t &= \mathrm{E}_{t-1} y_t + (1-\alpha)\,(-\mathrm{E}_{t-1} y_t + p_t - \mathrm{E}_{t-1} p_t) + e_t,\\ \alpha y_t &= \alpha \mathrm{E}_{t-1} y_t + (1-\alpha)\,(p_t - \mathrm{E}_{t-1} p_t) + e_t, \end{aligned}$$

thereby providing the output equation

$$y_t = \mathrm{E}_{t-1}y_t + \frac{1-\alpha}{\alpha}(p_t - \mathrm{E}_{t-1}p_t) + \frac{1}{\alpha}e_t,$$

or,

$$y_t = \mathrm{E}_{t-1}y_t + a(p_t - \mathrm{E}_{t-1}p_t) + (1+a)e_t, \qquad a \equiv (1-\alpha)/\alpha > 0.$$

As $\mathrm{E}_{t-1}e_t = 0$, it follows that $\mathrm{E}_{t-1}y_t = 0$ (as $\mathrm{E}_{t-1}n_t = 0$).[3] We therefore have:

$$y_t = a(p_t - \mathrm{E}_{t-1}p_t) + (1+a)e_t. \tag{7}$$

This aggregate supply schedule is a version of the Lucas supply curve that has been central in much macroeconomics since the 1970s.

Now consider the demand side. Equations (4) and (5) can immediately be rewritten as a dynamic IS curve and a standard LM curve:

$$y_t = \mathrm{E}_t y_{t+1} - \frac{1}{\Phi}(i_t - [\mathrm{E}_t p_{t+1} - p_t]), \tag{8}$$

$$m_t = p_t + y_t - \left(\frac{1}{bi^{ss}}\right) i_t. \tag{9}$$

Following the approach in Walsh, we rewrite the money demand equation in terms of expected inflation and expected output growth using the dynamic IS curve:

$$m_t = p_t + y_t - \frac{1}{bi^{ss}}(\mathrm{E}_t p_{t+1} - p_t) - \frac{\Phi}{bi^{ss}}(\mathrm{E}_t y_{t+1} - y_t).$$

Using the exogenous process for the money supply, we get a characterization of the aggregate demand side in terms of output and prices:

$$\mu + \rho_m m_{t-1} + s_t = p_t + y_t - \frac{1}{bi^{ss}}(\mathrm{E}_t p_{t+1} - p_t) - \frac{\Phi}{bi^{ss}}(\mathrm{E}_t y_{t+1} - y_t). \tag{10}$$

Equation (10) indeed characterizes, for given expectations and money supply, a negative relationship between prices and output. The reasons are partly the standard one from the IS/LM story: Higher prices will increase nominal money demand. To secure money market equilibrium, the nominal interest rate will increase, which depresses demand.

[3] This follows from the underlying model when the real wage "target" ω_t^* from (1) is consistent with the flexible-wage employment level; i.e., the real wage that equates labor demand with labor supply. This results in the following expression for employment (see Walsh, 2010, p. 228):

$$n_t^* = \frac{1-\Phi}{1+\eta\left(\frac{n^{ss}}{1-n^{ss}}\right) + (1-\alpha)(\Phi-1)} e_t.$$

This is derived by combining (6.1), 6.2), (6.3) and (6.5).

In this intertemporal model, an additional channel is present. For given period-$t+1$ price expectations, a higher price in period t reduces inflation expectations, $\mathrm{E}_t p_{t+1} - p_t$, and increases the real interest rate. This causes consumers to substitute demand from now to later—for given expectations about future demand, period-t demand decreases. The same will happen if, for example, expectations about future prices go down. This "expected inflation"–channel is normally not presented in standard IS/LM expositions, where there is no distinction between nominal and real interest rates in the IS relationship. The distinction, arising naturally from the underlying micro-founded model, is crucial for the results on policy effectiveness as will be clear below.

2.2 Applying the method of undetermined coefficients

With (7) and (10) we have a two-equation rational expectations model for y_t and p_t. This is solved by the method of undetermined coefficients. For this purpose we conjecture, or guess, a solution in terms of undetermined coefficients. In this model, we conjecture the following solution for prices:

$$p_t = \gamma_0 \mu + \gamma_1 m_{t-1} + \gamma_2 s_t + \gamma_3 e_t. \tag{11}$$

Knowing what to conjecture is not always obvious, but in linear rational expectations models, it is safe to make a conjecture that is a linear function of shocks, constants and state variables. In a moment we will verify the that this form of the conjecture is indeed consistent with a solution of the model, and we can derive the values of γ_0, γ_1, γ_2, and γ_3 as function of the model's parameters.

Now, insert y_t and $\mathrm{E}_t y_{t+1}$ as given by (7) into (10) so as to obtain an expression in prices only:

$$\begin{aligned} & \mu + \rho_m m_{t-1} + s_t \\ = \; & p_t + a\left[p_t - \mathrm{E}_{t-1} p_t\right] + (1+a)\, e_t - \frac{1}{bi^{ss}}\left(\mathrm{E}_t p_{t+1} - p_t\right) \\ & - \frac{\Phi}{bi^{ss}}\left(\mathrm{E}_t\left[a\left\{p_{t+1} - \mathrm{E}_t p_{t+1}\right\} + \{1+a\}\, e_{t+1}\right] - \left[a\left\{p_t - \mathrm{E}_{t-1} p_t\right\} + \{1+a\}\, e_t\right]\right). \end{aligned}$$

We readily see that

$$\mathrm{E}_t\left[a\left\{p_{t+1} - \mathrm{E}_t p_{t+1}\right\} + \{1+a\}\, e_{t+1}\right] = \mathrm{E}_t y_{t+1} = 0,$$

as $\mathrm{E}_t e_{t+1} = 0$ by assumption, and as $\mathrm{E}_t\left[a\left\{p_{t+1} - \mathrm{E}_t p_{t+1}\right\}\right] = 0$. Hence, we have

$$\begin{aligned} \mu + \rho_m m_{t-1} + s_t \;=\; & p_t + a\left[p_t - \mathrm{E}_{t-1} p_t\right] + (1+a)\, e_t - \frac{1}{bi^{ss}}\left(\mathrm{E}_t p_{t+1} - p_t\right) \\ & + \frac{\Phi}{bi^{ss}}\left(a\left[p_t - \mathrm{E}_{t-1} p_t\right] + [1+a]\, e_t\right). \end{aligned} \tag{12}$$

as a rational-expectations equation determining prices. This is what we need, as we have made a conjecture for the solution for prices. We can therefore apply that the conjecture, (11), implies

$$\begin{aligned} p_t - \mathrm{E}_{t-1} p_t &= \gamma_2 s_t + \gamma_3 e_t, \\ \mathrm{E}_t p_{t+1} - p_t &= \gamma_1 m_t - \gamma_1 m_{t-1} - \gamma_2 s_t - \gamma_3 e_t, \end{aligned}$$

as $\mathrm{E}_{t-1} e_t = \mathrm{E}_{t-1} s_t = 0$ for all t. Using this in (12) then gives

$$\begin{aligned} \mu + \rho_m m_{t-1} + s_t &= \gamma_0 \mu + \gamma_1 m_{t-1} + \gamma_2 s_t + \gamma_3 e_t \\ &\quad + a\left[\gamma_2 s_t + \gamma_3 e_t\right] + (1+a)\, e_t - \frac{1}{bi^{ss}} \left(\gamma_1 m_t - \gamma_1 m_{t-1} - \gamma_2 s_t - \gamma_3 e_t\right) \\ &\quad + \frac{\Phi}{bi^{ss}} \left(a\left[\gamma_2 s_t + \gamma_3 e_t\right] + [1+a]\, e_t\right), \end{aligned} \tag{13}$$

Equation (13) is a linear equation involving a constant, the shocks, and past and current money supply. The latter is eliminated by use of (6), such that we finally get:

$$\begin{aligned} &\mu + \rho_m m_{t-1} + s_t \\ = \; &\gamma_0 \mu + \gamma_1 m_{t-1} + \gamma_2 s_t + \gamma_3 e_t \\ &+ a\left[\gamma_2 s_t + \gamma_3 e_t\right] + (1+a)\, e_t - \frac{1}{bi^{ss}} \left(\gamma_1 \left[\mu + \rho_m m_{t-1} + s_t\right] - \gamma_1 m_{t-1} - \gamma_2 s_t - \gamma_3 e_t\right) \\ &+ \frac{\Phi}{bi^{ss}} \left(a\left[\gamma_2 s_t + \gamma_3 e_t\right] + [1+a]\, e_t\right). \end{aligned} \tag{14}$$

Equation (14) verifies the conjecture that in a rational expectations equilibrium satisfying the model's equations, prices are a linear function of a constant, the shocks, and the past period's money supply.

We have not solved for the price level yet, but looking closer at (14), we will (hopefully) realize that the solution is right in front of us. Equation (14) must hold for *any* values of μ, s_t, e_t and m_{t-1}. So, differentiating the left- and right-hand sides of (14) with respect to these variables in turn, provides exactly four equations that will give the solutions for the four unknown coefficients. Thus, we obtain the rational expectations solution for p_t of the form conjectured. The four equations are:

$$1 = \gamma_0 - \frac{\gamma_1}{bi^{ss}}, \tag{15}$$

$$1 = \gamma_2 + a\gamma_2 - \frac{1}{bi^{ss}} \left(\gamma_1 - \gamma_2\right) + \frac{\Phi}{bi^{ss}} a\gamma_2, \tag{16}$$

$$0 = \gamma_3 + a\gamma_3 + 1 + a + \frac{1}{bi^{ss}} \gamma_3 + \frac{\Phi}{bi^{ss}} \left(a\gamma_3 + 1 + a\right), \tag{17}$$

$$\rho_m = \gamma_1 - \frac{1}{bi^{ss}} \left(\gamma_1 \rho_m - \gamma_1\right). \tag{18}$$

Notice that (18) gives the solution for γ_1, (17) gives the solution for γ_3, (16) gives the solution for γ_2 given γ_1, and (15) gives the solution for γ_0 given γ_1.

From (18) we get

$$\gamma_1 = \frac{bi^{ss}\rho_m}{1 + bi^{ss} - \rho_m}, \tag{19}$$

and from (17) we recover

$$\gamma_3 = -\frac{(1+a)\left(bi^{ss} + \Phi\right)}{1 + bi^{ss} + a\left(bi^{ss} + \Phi\right)}. \tag{20}$$

Using (19) in (16) gives

$$\begin{aligned}
1 &= \gamma_2 + a\gamma_2 + \frac{1}{bi^{ss}}\gamma_2 + \frac{\Phi}{bi^{ss}}a\gamma_2 - \frac{\rho_m}{1 + bi^{ss} - \rho_m}, \\
bi^{ss} + \frac{bi^{ss}\rho_m}{1 + bi^{ss} - \rho_m} &= \left(1 + bi^{ss} + a\left[bi^{ss} + \Phi\right]\right)\gamma_2, \\
\frac{bi^{ss}\left(1 + bi^{ss}\right)}{1 + bi^{ss} - \rho_m} &= \left(1 + bi^{ss} + a\left[bi^{ss} + \Phi\right]\right)\gamma_2,
\end{aligned}$$

$$\gamma_2 = \frac{bi^{ss}\left(1 + bi^{ss}\right)}{\left(1 + bi^{ss} - \rho_m\right)\left(1 + bi^{ss} + a\left[bi^{ss} + \Phi\right]\right)}, \tag{21}$$

and using (19) in (15) gives

$$1 = \gamma_0 - \frac{\rho_m}{1 + bi^{ss} - \rho_m},$$

$$\gamma_0 = \frac{1 + bi^{ss}}{1 + bi^{ss} - \rho_m}. \tag{22}$$

Hence, the solution for p_t is

$$\begin{aligned}
p_t &= \frac{1 + bi^{ss}}{1 + bi^{ss} - \rho_m}\mu + \frac{bi^{ss}\rho_m}{1 + bi^{ss} - \rho_m}m_{t-1} \\
&\quad + \frac{bi^{ss}\left(1 + bi^{ss}\right)}{\left(1 + bi^{ss} - \rho_m\right)\left(1 + bi^{ss} + a\left[bi^{ss} + \Phi\right]\right)}s_t - \frac{(1+a)\left(bi^{ss} + \Phi\right)}{1 + bi^{ss} + a\left(bi^{ss} + \Phi\right)}e_t.
\end{aligned} \tag{23}$$

We can immediately see that

$$\mathrm{E}_{t-1}p_t = \frac{1 + bi^{ss}}{1 + bi^{ss} - \rho_m}\mu + \frac{bi^{ss}\rho_m}{1 + bi^{ss} - \rho_m}m_{t-1},$$

and insert these solutions for p_t and $\mathrm{E}_{t-1}p_t$ into (7) so as to get the solution for y_t:

$$\begin{aligned}
y_t &= a\left[\frac{bi^{ss}\left(1 + bi^{ss}\right)}{\left(1 + bi^{ss} - \rho_m\right)\left(1 + bi^{ss} + a\left[bi^{ss} + \Phi\right]\right)}s_t - \frac{(1+a)\left(bi^{ss} + \Phi\right)}{1 + bi^{ss} + a\left(bi^{ss} + \Phi\right)}e_t\right] + (1+a)e_t, \\
&= \frac{abi^{ss}\left(1 + bi^{ss}\right)}{\left(1 + bi^{ss} - \rho_m\right)\left(1 + bi^{ss} + a\left[bi^{ss} + \Phi\right]\right)}s_t \\
&\quad + \frac{1 + bi^{ss} + a\left(bi^{ss} + \Phi\right) - a\left(bi^{ss} + \Phi\right)}{1 + bi^{ss} + a\left(bi^{ss} + \Phi\right)}(1+a)e_t,
\end{aligned}$$

$$y_t = \frac{abi^{ss}\left(1 + bi^{ss}\right)}{\left(1 + bi^{ss} - \rho_m\right)\left(1 + bi^{ss} + a\left[bi^{ss} + \Phi\right]\right)}s_t + \frac{\left(1 + bi^{ss}\right)(1+a)}{1 + bi^{ss} + a\left(bi^{ss} + \Phi\right)}e_t. \tag{24}$$

With the solutions for p_t and y_t, we can find the equilibrium nominal interest rate from the dynamic IS curve (8),

$$y_t = \mathrm{E}_t y_{t+1} - \Phi^{-1}\left(i_t - \left[\mathrm{E}_t p_{t+1} - p_t\right]\right)$$

since it with use of (23) and (24) becomes

$$\begin{aligned}
& \frac{abi^{ss}\left(1+bi^{ss}\right)}{\left(1+bi^{ss}-\rho_m\right)\left(1+bi^{ss}+a\left[bi^{ss}+\Phi\right]\right)}s_t + \frac{\left(1+bi^{ss}\right)(1+a)}{1+bi^{ss}+a\left(bi^{ss}+\Phi\right)}e_t \\
= \; & 0 - \Phi^{-1} i_t \\
& +\Phi^{-1}\left\{\frac{1+bi^{ss}}{1+bi^{ss}-\rho_m}\mu + \frac{bi^{ss}\rho_m}{1+bi^{ss}-\rho_m}m_t\right\} \\
& -\Phi^{-1}\left\{\frac{1+bi^{ss}}{1+bi^{ss}-\rho_m}\mu + \frac{bi^{ss}\gamma}{1+bi^{ss}-\gamma}m_{t-1}\right. \\
& \left. + \frac{bi^{ss}\left(1+bi^{ss}\right)}{\left(1+bi^{ss}-\rho_m\right)\left(1+bi^{ss}+a\left[bi^{ss}+\Phi\right]\right)}s_t - \frac{(1+a)\left(bi^{ss}+\Phi\right)}{1+bi^{ss}+a\left(bi^{ss}+\Phi\right)}e_t\right\},
\end{aligned}$$

$$\begin{aligned}
\Phi^{-1} i_t \; = \; & -\frac{abi^{ss}\left(1+bi^{ss}\right)}{\left(1+bi^{ss}-\rho_m\right)\left(1+bi^{ss}+a\left[bi^{ss}+\Phi\right]\right)}s_t - \frac{\left(1+bi^{ss}\right)(1+a)}{1+bi^{ss}+a\left(bi^{ss}+\Phi\right)}e_t \\
& +\Phi^{-1}\frac{bi^{ss}\rho_m}{1+bi^{ss}-\rho_m}\left(\mu+\rho_m m_{t-1}+s_t\right) \\
& -\Phi^{-1}\left\{\frac{bi^{ss}\rho_m}{1+bi^{ss}-\rho_m}m_{t-1} + \frac{bi^{ss}\left(1+bi^{ss}\right)}{\left(1+bi^{ss}-\rho_m\right)\left(1+bi^{ss}+a\left[bi^{ss}+\Phi\right]\right)}s_t\right. \\
& \left. - \frac{(1+a)\left(bi^{ss}+\Phi\right)}{1+bi^{ss}+a\left(bi^{ss}+\Phi\right)}e_t\right\},
\end{aligned}$$

where in the second equation, the exogenous process for m_t has been inserted. This equation is further simplified as

$$\begin{aligned}
i_t \; = \; & -\frac{\Phi abi^{ss}\left(1+bi^{ss}\right)}{\left(1+bi^{ss}-\rho_m\right)\left(1+bi^{ss}+a\left[bi^{ss}+\Phi\right]\right)}s_t - \frac{\Phi\left(1+bi^{ss}\right)(1+a)}{1+bi^{ss}+a\left(bi^{ss}+\Phi\right)}e_t \\
& +\frac{bi^{ss}\rho_m}{1+bi^{ss}-\rho_m}\left(\mu+\rho_m m_{t-1}+s_t\right) \\
& -\frac{bi^{ss}\rho_m}{1+bi^{ss}-\rho_m}m_{t-1} - \frac{bi^{ss}\left(1+bi^{ss}\right)}{\left(1+bi^{ss}-\rho_m\right)\left(1+bi^{ss}+a\left[bi^{ss}+\Phi\right]\right)}s_t \\
& +\frac{(1+a)\left(bi^{ss}+\Phi\right)}{1+bi^{ss}+a\left(bi^{ss}+\Phi\right)}e_t, \\
= \; & \frac{bi^{ss}\rho_m}{1+bi^{ss}-\rho_m}\mu + \left[\frac{bi^{ss}\rho_m}{1+bi^{ss}-\rho_m} - \frac{bi^{ss}}{1+bi^{ss}-\rho_m}\right]\rho_m m_{t-1} \\
& -\frac{bi^{ss}}{\left(1+bi^{ss}-\rho_m\right)}\left[\frac{\Phi a\left(1+bi^{ss}\right)}{1+bi^{ss}+a\left(bi^{ss}+\Phi\right)} + \frac{bi^{ss}\left(1+bi^{ss}\right)}{1+bi^{ss}+a\left(bi^{ss}+\Phi\right)} - \rho_m\right]s_t \\
& -\frac{(\Phi-1)\left(1+bi^{ss}\right)(1+a)}{1+bi^{ss}+a\left(bi^{ss}+\Phi\right)}e_t,
\end{aligned}$$

leading to

$$i_t = \frac{bi^{ss}\rho_m}{1+bi^{ss}-\rho_m}\mu - \frac{bi^{ss}(1-\rho_m)\rho_m}{1+bi^{ss}-\rho_m}m_{t-1} - \frac{bi^{ss}}{(1+bi^{ss}-\rho_m)}\left[\frac{(1+bi^{ss})[1+a\Phi]}{1+bi^{ss}+a(bi^{ss}+\Phi)} - \rho_m\right]s_t - \frac{(\Phi-1)(1+bi^{ss})(1+a)}{1+bi^{ss}+a(bi^{ss}+\Phi)}e_t. \quad (25)$$

3 The economics of the solution and the effectiveness of monetary policy

We now have the solutions for output, prices and the nominal interest rate from (24), (23) and (25), respectively. These are stated again for convenience, and then the properties of the solution are thoroughly discussed with focus on the role of the money supply process parameter ρ_m for the transmission of shocks.

$$y_t = \frac{abi^{ss}(1+bi^{ss})}{(1+bi^{ss}-\rho_m)(1+bi^{ss}+a[bi^{ss}+\Phi])}s_t + \frac{(1+bi^{ss})(1+a)}{1+bi^{ss}+a(bi^{ss}+\Phi)}e_t.$$

$$p_t = \frac{1+bi^{ss}}{1+bi^{ss}-\rho_m}\mu + \frac{bi^{ss}\rho_m}{1+bi^{ss}-\rho_m}m_{t-1} + \frac{bi^{ss}(1+bi^{ss})}{(1+bi^{ss}-\rho_m)(1+bi^{ss}+a[bi^{ss}+\Phi])}s_t - \frac{(1+a)(bi^{ss}+\Phi)}{1+bi^{ss}+a(bi^{ss}+\Phi)}e_t.$$

$$i_t = \frac{bi^{ss}\rho_m}{1+bi^{ss}-\rho_m}\mu - \frac{bi^{ss}(1-\rho_m)\rho_m}{1+bi^{ss}-\rho_m}m_{t-1} - \frac{bi^{ss}}{1+bi^{ss}-\rho_m}\left[\frac{(1+bi^{ss})[1+a\Phi]}{1+bi^{ss}+a(bi^{ss}+\Phi)} - \rho_m\right]s_t - \frac{(\Phi-1)(1+bi^{ss})(1+a)}{1+bi^{ss}+a(bi^{ss}+\Phi)}e_t.$$

3.1 Supply shocks

Consider a positive supply shock, $e_t > 0$. Inspection of the AS and AD schedules, (7) and (10), reveals that insofar $\mathrm{E}_t y_{t+1}$ and $\mathrm{E}_t p_{t+1}$ are unaffected, these schedules determine the output and price response directly. As the shock is white noise, we immediately have that $\mathrm{E}_t y_{t+1} = 0$. Moreover, expected future prices will also be unaffected be the supply shock. This follows as $\mathrm{E}_t p_{t+1} = \gamma_0\mu + \gamma_1 m_t$ by (11), and because the process for m_t is independent of the supply shock.

It is thus convenient to frame the discussion of the economy's output and price response in terms of AS and AD schedules in a (y_t,p_t) space, and then trace out the nominal interest rate effects by the IS/LM schedules underlying the AD schedule in a (y_t,i_t) space. A graphical representation is provided by Figure 1.[4] The supply shock moves the AS curve to the right, i.e., from AS_0 to AS_1, such that the new equilibrium will be at point A. This involves an increase in output from y_0 to y_1 and lower prices. This can also be confirmed by inspection of (24) and (23), respectively.

The lower price level will matter for the IS and LM curves. For a given nominal interest rate, a lower price level increases the real money supply and moves the LM curve to the right, from LM_0 to LM_1, which puts downward pressure on the nominal interest rate so as to clear the money market. This would be the only effect of lower prices in a standard IS/LM model. But in this dynamic model, a lower price level also plays a role by lowering the real interest rate since expected inflation, $\mathrm{E}_t p_{t+1} - p_t$, increases. This increases demand, i.e., moves the IS curve from IS_0 to IS_1, and puts upward pressure on the nominal interest rate.

This "expected inflation" effect thus make the impact of the shock stronger—point C, which corresponds to point A in the AS/AD graph of Figure 1, involves higher output than point B, which is the intersection point of the IS and LM curves when the IS curve does not move. A further implication is that the effect on the nominal interest rate is ambiguous. I will depend on whether the "IS-" or the "LM- effect" is the strongest. When $\Phi > 1$, the real interest-rate channel is rather weak (the elasticity of intertemporal substitution is relatively small), and the IS curve "moves less" than the LM curve. This is the case depicted in Figure 1. In that case, the nominal interest rate will fall when a positive supply shock hits. This can be seen mathematically by inspection of (25); the opposite, of course, holds for $\Phi < 1$.

Note that the money supply process parameter ρ_m, a systematic part of monetary policymaking in the model, has no effect on how the supply shock is transmitted. Any change in this parameter will therefore not influence output fluctuations arising from supply shocks. The crucial factor behind this result is that $\mathrm{E}_t p_{t+1}$ is unaffected, when m_t is. This will be evident, when we turn to the response of the economy to the shock that does affect the process for the money supply, and thereby $\mathrm{E}_t p_{t+1}$, namely s_t.

3.2 Monetary policy shocks

Consider a positive nominal money supply shock, $s_t > 0$. In a standard, static IS/LM model with fixed prices, this will increase output and lower the nominal interest rate as the real money supply is increased. In this model, the equilibrium response will be different

[4] In this, and the other figures, it should be noted that the subscripts do not represent periods, but are merely used to distinguish different values.

as p_t is endogenous, and—of particular interest in this dynamic context—because $E_t p_{t+1}$ is affected as $E_t p_{t+1} = \gamma_0 \mu + \gamma_1 m_t$. I.e., the "expected inflation" effect working through the real interest rate will play a crucial role.

Is is instructive to split up the analysis up two parts. First, consider the special case of $\rho_m = 0$. This case implies that the money supply shock will not affect the next period's money supply or expected prices. The latter can be confirmed analytically, as $\gamma_1 = 0$ applies when $\rho_m = 0$; cf. (19). With the shock, the LM curve moves to the right. This is depicted in Figure 2 as the move from LM_0 to LM_1. In absence of any effects on the real interest rate, i.e., any price effects, this would increase output from y_0 to y_1, and the economy would end in point A. In the associated AS/AD graph, the AD curve would move from AD_0 to AD_1. The increase in aggregate demand will increase output and put upward pressure on prices, and the increase in p_t will dampen the increase in the real money supply—moving the LM curve back to LM_2—as well as put upward pressure on the real interest rate—moving the IS curve from IS_0 to IS_2. Both movements serve to dampen the output increase, and output becomes y_2 (and the associated demand curve is AD_2). So, for a given $E_t p_{t+1}$ the effect of $s_t > 0$ is an increase in output and prices and a fall in the nominal interest rate. The positive impact on prices, not present in standard IS/LM analysis, dampens the effect on output partly through the contractive effects of the associated decrease in expected inflation.

Now consider the case of $\rho_m > 0$. The shock to nominal money in period t under consideration then affects the nominal money supply in period $t+1$ positively, and thus period-$t+1$ prices and the period$-t$ expectations of these. Indeed, as $\gamma_1 > 0$ when $\rho_m > 0$, $E_t p_{t+1}$ will increase, which exerts upward pressure on expected inflation. The effect of the shock will therefore we very different from the case of $\rho_m = 0$. The case is shown in Figure 3. Point A is the same point as in Figure 2. I.e., the point where output has increased to y_1 due to the increase in the money supply for given prices. As in the previous case, this increases demand and puts upward pressure on current prices. This effect will, as in the case of $\rho_m = 0$, reduce the real money stock and reduce inflation expectations moving both the IS and LM curves inwards.

Importantly, when $\rho_m > 0$, expectations about the future feed back and affect *current* variables in this rational expectations framework. And the higher is ρ_m, the stronger is the current impact of the expected future prices. Indeed, if ρ_m is sufficiently high, the increase $E_t p_{t+1}$ stimulates demand through the downward pressure on the real interest rate, pushing the IS curve outwards. In Figure 3, the IS curve settles at IS_2—i.e., the expansive effects of higher $E_t p_{t+1}$ dominates the contractive effects of higher p_t. The equilibrium will be characterized by point B, with an output level y_2 which is higher than the output response when $\rho_m = 0$.

Hence, the size of ρ_m, a systematic component of monetary policy, has implications for the transmission of the shock onto real variables in the model. Specifically, a higher ρ_m implies that the effects of the money-supply shock on output and prices become stronger (analytically it is seen from the coefficients to s_t in (24) and (23), which are increasing in ρ_m). In addition, the magnitude of expected future price effects may be so strong that the associated upward pressure on the nominal interest rate may dominate the fall in the nominal rate found in conventional static models. See (25), where it is seen that a high ρ_m may turn the coefficient on s_t positive.[5]

While this model is merely an example of policy effectiveness under rational expectations, it has generality for the role of stabilization policies in such settings. A shock that persists into the future (either by itself of through a policy response), affects expectations about future variables, which may have current effects. Policy may then be designed so as to affect expectations about future variables appropriately. In this model, for example, a policy rule that stabilizes output as much as possible would be (6) with $\rho_m \rightarrow -1$; i.e., any positive money supply shock should be reversed in the following period. In that case, $\mathrm{E}_t p_{t+1}$ would fall, putting upward pressure on the real interest rate, and thus downward pressure on aggregate demand. See (24) where the coefficient on s_t is minimized when ρ_m is a small as possible (while still retaining stationarity of the money supply).

4 General policy implications and discussion of some early literature

The implication of the previous discussion is that under rational expectations, monetary policy is generally not ineffective. Systematic components of the monetary policy rule do not affect average output, but it can affect its variability. This model is just one example of this violation of the policy ineffectiveness proposition.

Interestingly, the early—and very influential—literature on the policy limitations under rational expectations generally did not acknowledge this at all. Papers like Sargent and Wallace (1975), Barro (1976), Woglom (1979), McCallum (1980), Canzoneri *et al.* (1983) were very focused on presenting Lucas-style aggregate supply equations satisfying the natural-rate hypothesis. They therefore simplified the specification of the demand side such that in their models, systematic monetary policy rule parameters had no influence

[5]When demand is highly real interest-rate elastic, $\Phi < 1$, this occurs when $\rho_m \rightarrow 1$.

on output determination. These simplifications included either an *ad hoc* definition of the real interest rate that failed to account for period-t expectations of future prices, or a specification of aggregate demand that eliminated any real interest rate effects by replacing the IS/LM equations by a quantity equation $m_t = p_t + y_t$.

As evident by this model, such simplifications are way too strong when it comes to making statements about policy ineffectiveness. Ineffectiveness fails when there are money supply shocks. Moreover, if the process for the money supply rule is further amended by, e.g., inclusion of the supply shock and/or proper responses to output, the policy ineffectiveness proposition fails more generally. In this model, *any* policy rule that will affect price expectations, i.e., $\mathrm{E}_t p_{t+1}$, will affect the real interest rate and thereby aggregate demand and output in the short run.

Sargent (1973) models the real interest rate as a function of expected prices, and his rational-expectations solution derived in the article's appendix reveal upon inspection that policy rule parameters do affect output determination. To the best of my reading, he does not make comment of that fact. Dotsey and King (1983), on the other hand, did note how a feedback policy could affect future prices and thereby current prices and output volatility. Their model is a Lucas-style imperfect information model, and the computation of rational expectations equilibria under different feedback policies is complicated as the coefficients of the model are functions of the variances of underlying shocks and thus to what extent they are stabilized. They are able to show, however, that a money supply feedback rule that abstains from responding to past period's money supply reduces output variance (in this model, it would correspond to setting $\rho_m = 0$), as does a countercyclical response to past period's output.[6]

Groth (1997) also demonstrates how a countercyclical money supply rule stabilizes output. He lets m_t be a negative function of y_{t-1} in a model very similar to the one of this exercise. This means that a shock (e.g., a supply shock) that increases output in period t, is expected to be followed by a lower money stock in period $t+1$; $\mathrm{E}_t p_{t+1}$ go down, the real interest rate increases and the expansive effects of the shock are reduced. So, by appropriate policy design, fluctuation due to supply shocks can also be reduced by systematic monetary policy.

The introduction of rational expectations into macroeconomics was by many seen as paving the "death" for demand-oriented public policies (and still is for some), as the

[6] As they can only demonstrate it for a particular case, they write in their Footnote 12 (p. 370): "*We do not attach a great deal of importance to the particular example chosen, as it was selected for analytical convenience.*"

policy ineffectiveness proposition was considered central. The upshot of this analysis is, ironically, that it is indeed the rational expectations about future prices that create a scope for policy effectiveness in an otherwise standard New-Classical model framework.

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Figures

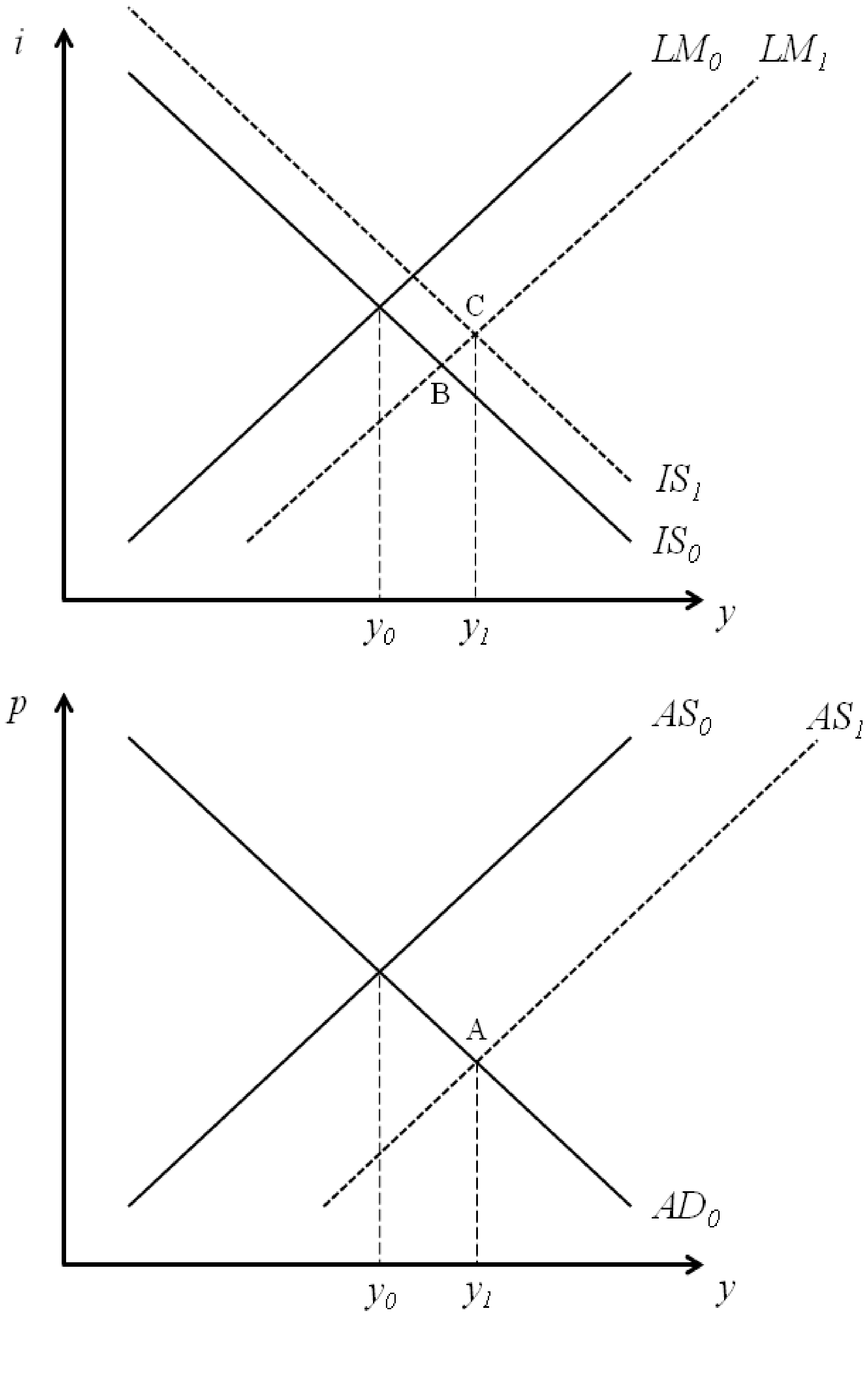

Figure 1: Effects of a positive supply shock, $e > 0$

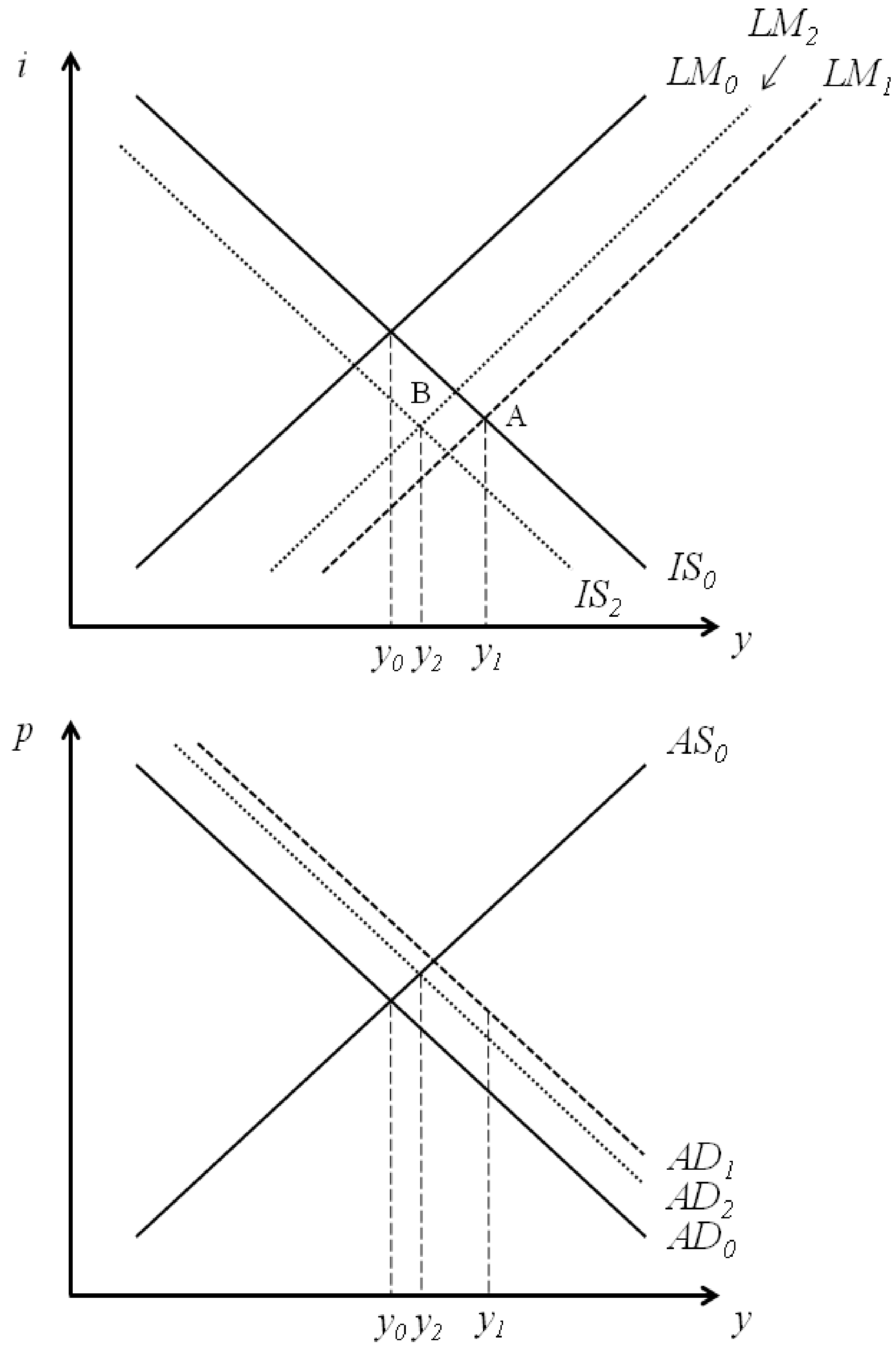

Figure 2: Effects of a positive money supply shock, $s > 0$. The case of $\rho_m = 0$.

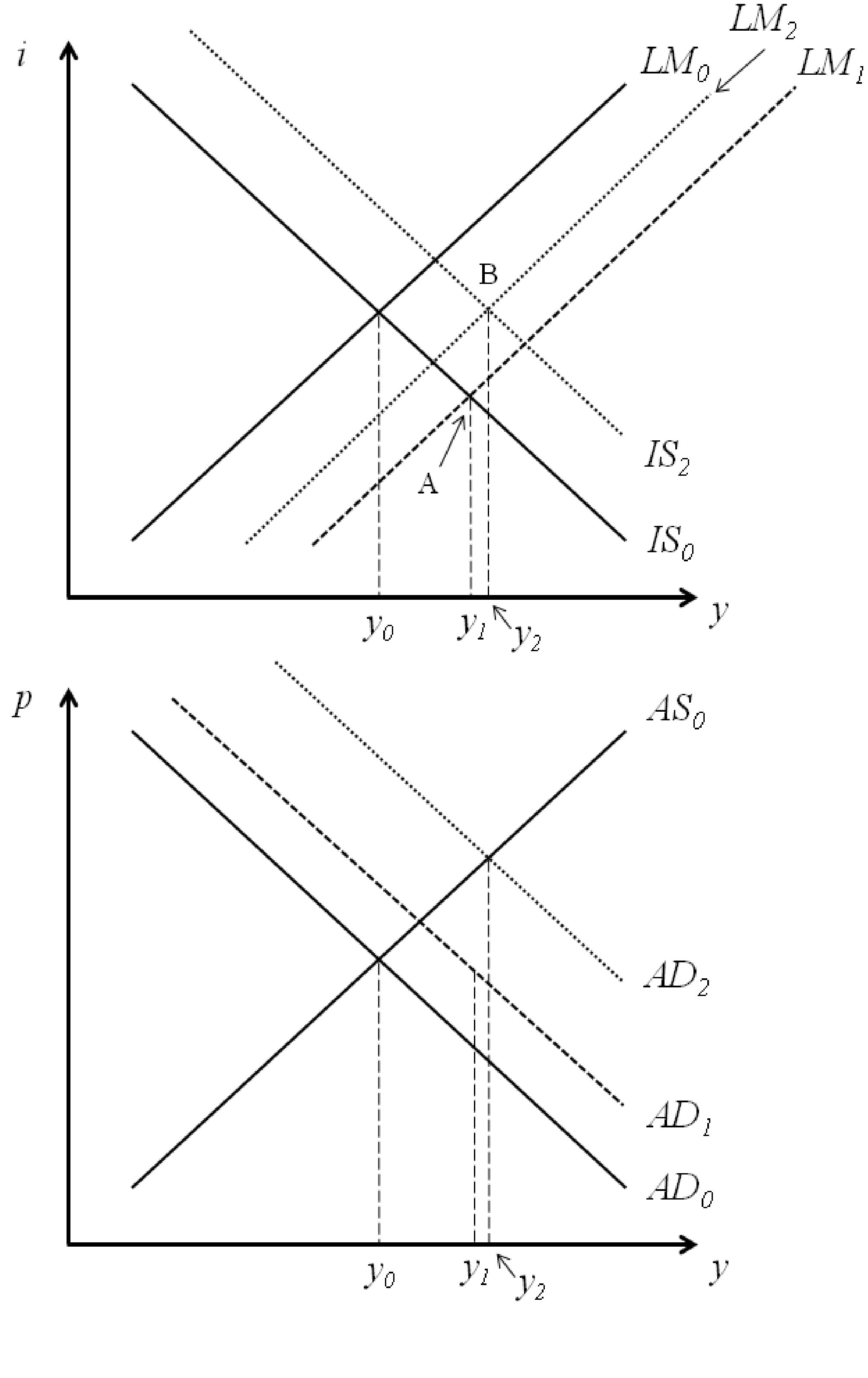

Figure 3: Effects of a positive money supply shock, $s > 0$. The case of $\rho_m > 0$.