This set contains three pages (beginning with this page)
All questions must be answered
Questions 1 and 2 each weigh 25 % while question 3 weighs 50 %. These weights, however, are only indicative for the overall evaluation.

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QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) The Taylor Principle in monetary policy requires that the central bank raises the interest rate when output increases above its natural rate and lowers the interest rate when output falls below the natural rate.

(ii) Under a nominal interest-rate operating procedure, it is never optimal to take movements in the nominal money supply into consideration when setting the interest rate.

(iii) In the simple New-Keynesian Phillips curve where only prices are sticky, inflation depends positively on current marginal costs and thereby negatively on the natural rate of output.

QUESTION 2:

Monetary policy with a "cash-in-advance" constraint
Consider an economy formulated in discrete time, where the utility of a representative agent is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1, \tag{1}$$

where c_t is real consumption and $u' > 0$, $u'' < 0$. The agent faces the budget constraint

$$\begin{aligned} \omega_t &\equiv f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + \frac{m_{t-1} + (1+i_{t-1}) b_{t-1}}{1+\pi_t} \\ &= c_t + k_t + m_t + b_t, \end{aligned} \tag{2}$$

where k_{t-1} is real capital at the end of period $t-1$, f is a production function where $f' > 0$, $f'' < 0$, τ_t denotes real monetary transfers from the government, $0 < \delta < 1$

is the rate of depreciation of capital, m_{t-1} denotes real money holdings at the end of period $t-1$, i_{t-1} is the nominal interest rate on bonds (denoted b_{t-1} in real terms), and π_t is the rate of inflation.

The agent also faces the following cash-in-advance constraint on consumption:

$$c_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau_t. \tag{3}$$

(i) Examine the optimal choices of consumption, capital and real money holdings. For that purpose, show first that the budget constraint (2) can be rewritten as

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + (1-\delta) k_t + \frac{m_t}{1+\pi_{t+1}} + R_t (\omega_t - c_t - k_t - m_t),$$

with $R_t \equiv (1+i_t)/(1+\pi_{t+1})$ being the real interest rate. Use that the agent's optimization problem can be characterized by

$$V(\omega_t, m_{t-1}) = \max \left\{ u(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t \left(c_t - \frac{m_{t-1}}{1+\pi_t} - \tau_t \right) \right\},$$

where maximization is over c, k, and m, and where μ_t is the multiplier on (3). Then derive and interpret these necessary optimality conditions:

$$u_c(c_t) = \beta R_t V_\omega(\omega_{t+1}, m_t) + \mu_t,$$

$$\beta V_\omega(\omega_{t+1}, m_t) [f_k(k_t) + 1 - \delta] = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

and show that by use of the Envelope theorem one finds

$$V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

$$V_m(\omega_t, m_{t-1}) = \mu_t \frac{1}{1+\pi_t}.$$

(ii) Let $\lambda_t \equiv V_\omega(\omega_t, m_{t-1})$, and use the results from (i), to obtain an expression for the nominal interest rate, i_t, as a function of μ_{t+1} and λ_{t+1}. Explain this relationship with focus on the role of a binding or non-binding cash-in-advance constraint.

(iii) Show formally that monetary policy—here different rates of inflation—has no real effects in steady state. Explain the result. Discuss which variables, on the other hand, will be affected by different long-run inflation rates.

QUESTION 3:

Monetary policy trade offs and commitment policies

Consider the following log-linear "New-Keynesian" model:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \qquad 0 < \beta < 1, \quad \kappa > 0, \tag{1}$$

$$e_t = \rho_u e_{t-1} + \varepsilon_t, \qquad 0 \le \rho_u < 1, \tag{2}$$

where π_t is goods price inflation, x_t is the output gap, and e_t is a "cost-push" shock, which is assumed to given by the autoregressive process (2), where ε_t is a mean-zero, serially uncorrelated shock. E_t is the rational expectations operator conditional on all information up to and including period t.

(i) Discuss the micro foundations behind equation (1).

(ii) Assume that the monetary authority wants to maximize the utility function

$$U = -\frac{1}{2}\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda x_t^2 + \pi_t^2\right], \qquad \lambda > 0. \tag{3}$$

Discuss the economic foundations for this utility function.

(iii) It is assumed that the authority can commit to policies of the form

$$x_t = \psi_x e_t, \qquad \pi_t = \psi_\pi e_t. \tag{4}$$

Find the optimal values of ψ_x and ψ_π. For this purpose use (2) to show that utlity can be written as a function of ψ_x alone:

$$U = -\frac{1}{2}\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t e_t^2 \left[\lambda \psi_x^2 + \left(\frac{1 + \kappa \psi_x}{1 - \beta \rho_u}\right)^2\right]$$

(iv) Under discretionary policymaking, the solutions for the output gap and inflation are given as

$$x_t = -\frac{\kappa}{\kappa^2 + \lambda\left(1 - \beta \rho_u\right)} e_t,$$

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda\left(1 - \beta \rho_u\right)} e_t.$$

Compare how inflation responds to the cost-push shock under the particular commitment policy and discretion. Focus on the relevance of $\rho_u = 0$ versus the case of $\rho_u > 0$ for the comparison.

(v) Is commitment of the form (4) always advantageous? Explain.

(vi) Can macroeconomic outcomes be improved relative to those arising under (4)? Explain.