Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) An exogenous nominal interest rate in the simple New-Keynesian model results in infinitely many stable equilibria.

(ii) In the flexible-price, money-in-the-utility function model with endogenous labor supply, shocks to the nominal money supply have large employment effects and small effects on the nominal interest rate.

(iii) In models of monetary financing of public spending, revenue from seigniorage may be the same at different inflation rates.

QUESTION 2:

Consider a New-Keynesian model of inflation determination:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{1}$$

where π_t is inflation, $0 < \beta < 1$ is a discount factor, E_t is the rational expectations operator, $\kappa > 0$ is a parameter, x_t is the output gap, and e_t is a "cost-push" shock that follows the process

$$e_t = \rho e_{t-1} + \varepsilon_t, \qquad 0 < \rho < 1,$$

where ε_t is a mean-zero i.i.d. disturbance.

It is assumed that the monetary authority controls x_t and has the utility function

$$U = -\frac{\lambda}{2} x_t^2 - \frac{1}{2} \pi_t^2, \qquad \lambda > 0. \tag{2}$$

(i) Show that under discretionary policymaking, optimal policy is characterized by

$$-\lambda x_t = \kappa \pi_t. \tag{3}$$

Explain the result intuitively, and describe (in words) how inflation and the output gap will respond to a positive "cost-push" shock.

(ii) Assume now that the policymaker can commit to a policy rule of the form:

$$x_t^c = -\omega e_t, \tag{4}$$

where ω is a policy-rule parameter and superscript "c" indicates commitment. Find the optimal relationship between x_t^c and π_t^c. [Hint: Combine (4) with (1) to show that $\pi_t^c = [\kappa/(1-\beta\rho)]\, x_t^c + [1/(1-\beta\rho)]\, e_t$ and maximize U, expressed in terms of x_t^c, w.r.t. x_t^c.]

(iii) Discuss, based on the result of (ii), whether appointing a "conservative" policymaker, one characterized by $\lambda^c < \lambda$, is beneficial when commitment is not possible. Comment in particular on whether $\rho > 0$ is crucial.

QUESTION 3:

Consider an infinite-horizon economy in discrete time, where the utility of the representative agent is given by

$$U = \sum_{i=0}^{\infty} \beta^i \left[\ln c_{t+i} + \ln\left(1 - n_{t+i}\right)\right], \qquad 0 < \beta < 1, \tag{1}$$

where c_t is consumption in period t, and n_t is employment. The economy is characterized by flexible prices and perfect competition in the goods and labor markets. Agents have perfect foresight and face the budget constraint

$$c_t + b_t + m_t \leq y_t + \frac{1 + i_{t-1}}{1 + \pi_t} b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} + \tau_t, \tag{2}$$

where y_t is real output, b_{t-1} denotes real government bond holdings at the end of period $t-1$, i_{t-1} is the nominal interest rate, π_t is the inflation rate, m_{t-1} is real money holdings, and τ_t denotes real government transfers. Output is produced with labor as only input:

$$y_t = n_t^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{3}$$

Purchases of consumption goods are subject to a cash-in-advance constraint:

$$c_t \leq \frac{m_{t-1}}{1 + \pi_t} + \tau_t. \tag{4}$$

(i) Find the relevant first-order conditions characterizing the optimal choices of c_t, n_t, and m_t, and interpret them intuitively. [Hint: Use dynamic programming and express the value as a function of the state variables b_{t-1} and m_{t-1}. I.e., the Bellman equation becomes

$$V\left(b_{t-1}, m_{t-1}\right) = \max_{c_t, n_t, m_t} \left\{ \begin{array}{c} \ln c_t + \ln\left(1 - n_t\right) + \beta V\left(b_t, m_t\right) \\ -\mu_t \left[c_t - \dfrac{m_{t-1}}{1 + \pi_t} - \tau_t \right] \end{array} \right\},$$

where b_t can be substituted out by (2), using (3), and where μ_t is the multiplier on (4).]

(ii) Use the envelope theorem to eliminate the partial derivatives of the value function, define $\lambda_t \equiv \beta V_b\left(b_t, m_t\right)$, where V_b denotes $\partial V\left(b_t, m_t\right)/\partial b_t$, and show that the steady state can be characterized by

$$\begin{aligned} 1/c^{ss} &= \lambda^{ss}\left(1 + i^{ss}\right), \\ 1/\left(1 - n^{ss}\right) &= \lambda^{ss}\left(1 - \alpha\right)\left(n^{ss}\right)^{-\alpha}, \\ \beta^{-1} &= \frac{1 + i^{ss}}{1 + \pi^{ss}}, \end{aligned}$$

where superscript "ss" denotes steady-state values. Derive steady-state employment as a function of the nominal interest rate. [Hint: Use $y_t = c_t$.] Explain.

(iii) Derive the monetary policy that provides the utility-maximizing solution for employment in steady state,

$$n^{umax} = \frac{1 - \alpha}{2 - \alpha}.$$

Explain.