Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

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MONETARY POLICY
SUGGESTED SOLUTIONS TO AUGUST 24 EXAM, 2017

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) An exogenous nominal interest rate in the simple New-Keynesian model results in infinitely many stable equilibria.

A **True**. When the nominal interest rate is set in such a "passive" fashion, self-fulfilling bursts of inflation and the output gap are possible. E.g., when inflation expectations increase for no reason, the real interest rate decrease which stimulate the output gap, and thus inflation. The economy will subsequently gradually return to the steady state of zero inflation and no output gap. As these expectations-driven bursts can be of any size (and also negative) there are infinitely many stable equilibrium combinations of inflation and the output gap. In technical terms, the dynamics feature a saddle path as the model will have one stable and one unstable root. To secure determinacy, there should be two unstable roots, which could be achieved by an policy rule like an active Taylor-type rule.

(ii) In the flexible-price, money-in-the-utility function model with endogenous labor supply, shocks to the nominal money supply have large employment effects and small effects on the nominal interest rate.

A **False.** In the simulations we have seen, the reverse is true. Even in the case where the utility function is designed to give monetary policy an effect on the labor supply, the effect is of very small magnitude. Instead, higher money growth translates into higher inflation and a higher nominal interest rate, of substantial magnitude. Hence, the model behaves, by and large, in the short run as in the long run.

(iii) In models of monetary financing of public spending, revenue from seigniorage may be the same at different inflation rates.

A **True.** The point is that a "Laffer curve" effect is present. Inflation can be seen as a tax on the private sector's money holdings. At low inflation rates, higher inflation raises revenue, but as inflation becomes sufficiently high, money demand falls. This fall is sufficiently strong to imply that an increase in inflation will reduce revenue; the reason being that the "tax base", money holdings, fall by more. The resulting non-monotonic relationship between inflation and revenues, implies that it is possible that different inflation rates (say, a "high" and a "low") can generate the same revenue.

QUESTION 2:

Consider a New-Keynesian model of inflation determination:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{1}$$

where π_t is inflation, $0 < \beta < 1$ is a discount factor, E_t is the rational expectations operator, $\kappa > 0$ is a parameter, x_t is the output gap, and e_t is a "cost-push" shock that follows the process

$$e_t = \rho e_{t-1} + \varepsilon_t, \qquad 0 < \rho < 1,$$

where ε_t is a mean-zero i.i.d. disturbance.

It is assumed that the monetary authority controls x_t and has the utility function

$$U = -\frac{\lambda}{2} x_t^2 - \frac{1}{2} \pi_t^2, \qquad \lambda > 0. \tag{2}$$

(i) Show that under discretionary policymaking, optimal policy is characterized by

$$-\lambda x_t = \kappa \pi_t. \tag{3}$$

Explain the result intuitively, and describe (in words) how inflation and the output gap will respond to a positive "cost-push" shock.

A Under discretion, expectations cannot be affected by policy, so maximizing

$$-\frac{1}{2}\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\lambda x_{t+i}^2 + \pi_{t+i}^2\right], \qquad 0 < \beta < 1$$

w.r.t. x_t subject to (2) is equivalent of maximizing

$$-\frac{\lambda}{2} x_t^2 - \frac{1}{2}\pi_t^2 + F_t$$

w.r.t. x_t subject to

$$\pi_t = \kappa x_t + f_t$$

taking as given F_t and f_t. This immediately provides the first-order condition:

$$-\lambda x_t = \kappa \pi_t$$

It describes a "leaning against the wind" policy. If inflationary pressures arise due to a positive "cost-push" shock, the policymaker should contract output ($x_t < 0$) such that the marginal cost of lower output equals the marginal gain of reducing inflation.

(ii) Assume now that the policymaker can commit to a policy rule of the form:

$$x_t^c = -\omega e_t, \tag{4}$$

where ω is a policy-rule parameter and superscript "c" indicates commitment. Find the optimal relationship between x_t^c and π_t^c. [Hint: Combine (4) with (1) to show that $\pi_t^c = [\kappa/(1-\beta\rho)]\, x_t^c + [1/(1-\beta\rho)]\, e_t$ and maximize U, expressed in terms of x_t^c, w.r.t. x_t^c.]

A Use the hint. Inflation follows from the Phillips curve (1), together with the policy rule (4), as:

$$\begin{aligned}
\pi_t^c &= \beta \mathrm{E}_t \pi_{t+1}^c + \kappa x_t^c + e_t, \\
&= \beta \mathrm{E}_t \pi_{t+1}^c - \kappa \omega e_t + e_t.
\end{aligned}$$

Solving forward:

$$\begin{aligned}
\pi_t^c &= \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[-\kappa \omega e_{t+i} + e_{t+i}\right], \\
&= \sum_{i=0}^{\infty} \beta^i \left[-\kappa \omega + 1\right] \rho_t^i e_t, \\
&= \frac{1-\kappa\omega}{1-\beta\rho} e_t,
\end{aligned}$$

or,

$$\begin{aligned}
\pi_t^c &= -\frac{\kappa}{1-\beta\rho}\omega e_t + \frac{1}{1-\beta\rho} e_t, \\
&= \frac{\kappa}{1-\beta\rho} x_t^c + \frac{1}{1-\beta\rho} e_t,
\end{aligned}$$

where the last line follows from the definition of the policy rule. Then maximize

$$-\frac{1}{2}\left[\lambda \left(x_t^c\right)^2 + \left(\pi_t^c\right)^2\right]$$

w.r.t. x_t^c subject to the expression for π_t^c. This gives the first-order condition:

$$\lambda x_t^c + \frac{\kappa}{1-\beta\rho}\pi_t^c = 0.$$

Or written like in the discretionary case:

$$-\lambda x_t^c = \frac{\kappa}{1-\beta\rho}\pi_t^c.$$

(iii) Discuss, based on the result of (ii), whether appointing a “conservative” policymaker, one characterized by $\lambda^c < \lambda$, is beneficial when commitment is not possible. Comment in particular on whether $\rho > 0$ is crucial.

A One can rewrite the found relationship in (iii) as

$$-\lambda \left(1-\beta\rho\right) x_t^c = \kappa \pi_t^c,$$

or,

$$-\lambda^c x_t^c = \kappa \pi_t^c,$$

where

$$\lambda^c \equiv \lambda \left(1-\beta\rho\right) \leq \lambda.$$

So as long as $\rho > 0$, the solution features $\lambda^c < \lambda$, i.e., a *smaller weight on output* than in the social utility function. Hence, what is referred to as “conservatism”

is optimal as it mimics the commitment solution derived above. It is, however, crucial that $\rho > 0$, as this implies that a current shock has implications for the future. And in this model with forward-looking expectations, it is the ability to affect future expectations that is the benefit of commitment.
If $\rho = 0$, the future is not affected by current shocks and there is no need to try to affect future expectations by acting conservative. But if the shock persists, being conservative implies a tougher stance on future inflation, which helps stabilize current inflation better.

QUESTION 3:

Consider an infinite-horizon economy in discrete time, where the utility of the representative agent is given by

$$U = \sum_{i=0}^{\infty} \beta^i \left[\ln c_{t+i} + \ln\left(1 - n_{t+i}\right)\right], \qquad 0 < \beta < 1, \tag{1}$$

where c_t is consumption in period t, and n_t is employment. The economy is characterized by flexible prices and perfect competition in the goods and labor markets. Agents have perfect foresight and face the budget constraint

$$c_t + b_t + m_t \leq y_t + \frac{1 + i_{t-1}}{1 + \pi_t} b_{t-1} + \frac{m_{t-1}}{1 + \pi_t} + \tau_t, \tag{2}$$

where y_t is real output, b_{t-1} denotes real government bond holdings at the end of period $t-1$, i_{t-1} is the nominal interest rate, π_t is the inflation rate, m_{t-1} is real money holdings, and τ_t denotes real government transfers. Output is produced with labor as only input:

$$y_t = n_t^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{3}$$

Purchases of consumption goods are subject to a cash-in-advance constraint:

$$c_t \leq \frac{m_{t-1}}{1 + \pi_t} + \tau_t. \tag{4}$$

(i) Find the relevant first-order conditions characterizing the optimal choices of c_t, n_t, and m_t, and interpret them intuitively. [Hint: Use dynamic programming and express the value as a function of the state variables b_{t-1} and m_{t-1}. I.e., the Bellman equation becomes

$$V\left(b_{t-1}, m_{t-1}\right) = \max_{c_t, n_t, m_t} \left\{ \begin{array}{c} \ln c_t + \ln\left(1 - n_t\right) + \beta V\left(b_t, m_t\right) \\ -\mu_t \left[c_t - \dfrac{m_{t-1}}{1 + \pi_t} - \tau_t\right] \end{array} \right\},$$

where b_t can be substituted out by (2), using (3), and where μ_t is the multiplier on (4).]

A The relevant first-order conditions follow as

$$\begin{aligned} 1/c_t - \beta V_b(b_t, m_t) - \mu_t &= 0, \\ -1/(1-n_t) + \beta V_b(b_t, m_t)(1-\alpha) n_t^{-\alpha} &= 0, \\ \beta V_m(b_t, m_t) - \beta V_b(b_t, m_t) &= 0. \end{aligned}$$

All these are interpreted as marginal gains in terms of, respectively, current consumption, leisure and money, being equal to marginal losses in terms of lost future wealth and/or current liquidity costs (of consumption due to the cash-in-advance constraint).

(ii) Use the envelope theorem to eliminate the partial derivatives of the value function, define $\lambda_t \equiv \beta V_b(b_t, m_t)$, where V_b denotes $\partial V(b_t, m_t)/\partial b_t$, and show that the steady state can be characterized by

$$\begin{aligned} 1/c^{ss} &= \lambda^{ss}(1+i^{ss}), \\ 1/(1-n^{ss}) &= \lambda^{ss}(1-\alpha)(n^{ss})^{-\alpha}, \\ \beta^{-1} &= \frac{1+i^{ss}}{1+\pi^{ss}}, \end{aligned}$$

where superscript "ss" denotes steady-state values. Derive steady-state employment as a function of the nominal interest rate. [Hint: Use $y_t = c_t$.] Explain.

A The value function derivatives are, by application of the envelope theorem (implying that any effect of b_{t-1} and m_{t-1} on c_t, n_t and m_t cancel out by the first-order conditions), found as

$$\begin{aligned} V_b(b_{t-1}, m_{t-1}) &= \beta V_b(b_t, m_t)\frac{1+i_{t-1}}{1+\pi_t}, \\ V_m(b_{t-1}, m_{t-1}) &= \beta V_b(b_t, m_t)\frac{1}{1+\pi_t} + \mu_t \frac{1}{1+\pi_t}. \end{aligned}$$

Since $\lambda_t \equiv \beta V_b(b_t, m_t)$, the first of the value function derivatives can be written as

$$\lambda_t = \beta\lambda_{t+1}\frac{1+i_t}{1+\pi_{t+1}}.$$

The second can be rewritten as

$$\begin{aligned}
V_m(b_t, m_t) &= \beta V_b(b_{t+1}, m_{t+1}) \frac{1}{1+\pi_{t+1}} + \mu_{t+1} \frac{1}{1+\pi_{t+1}}, \\
V_b(b_t, m_t) &= \beta V_b(b_{t+1}, m_{t+1}) \frac{1}{1+\pi_{t+1}} + \mu_{t+1} \frac{1}{1+\pi_{t+1}}, \\
\beta V_b(b_t, m_t) &= \beta^2 V_b(b_{t+1}, m_{t+1}) \frac{1}{1+\pi_{t+1}} + \beta\mu_{t+1} \frac{1}{1+\pi_{t+1}}, \\
\lambda_t &= \beta \frac{\lambda_{t+1} + \mu_{t+1}}{1+\pi_{t+1}},
\end{aligned}$$

where the second line uses the third of the first-order conditions derived in (i). The first two first-order conditions is rewritten as

$$1/c_t - \lambda_t - \mu_t = 0,$$

$$-1/(1-n_t) + \lambda_t(1-\alpha) n_t^{-\alpha} = 0.$$

Hence, one has

$$\begin{aligned}
1/c_t - \lambda_t - \mu_t &= 0, \\
-1/(1-n_t) + \lambda_t(1-\alpha) n_t^{-\alpha} &= 0, \\
\lambda_t = \beta\lambda_{t+1} \frac{1+i_t}{1+\pi_{t+1}} &= \beta \frac{\lambda_{t+1} + \mu_{t+1}}{1+\pi_{t+1}},
\end{aligned}$$

which in steady state becomes:

$$\begin{aligned}
1/c^{ss} - \lambda^{ss} - \mu^{ss} &= 0, \\
-1/(1-n^{ss}) + \lambda(1-\alpha)(n^{ss})^{-\alpha} &= 0, \\
\beta^{-1} &= \frac{1+i^{ss}}{1+\pi^{ss}}, \\
\beta^{-1} &= \frac{1+\mu^{ss}/\lambda^{ss}}{1+\pi^{ss}},
\end{aligned}$$

This is readily reformulated since the last two conditions imply $\mu^{ss}/\lambda^{ss} = i^{ss}$, which is used to eliminate μ^{ss} in the first condition. This result in

$$\begin{aligned}
1/c^{ss} &= \lambda^{ss}(1+i^{ss}), \\
1/(1-n^{ss}) &= \lambda^{ss}(1-\alpha)(n^{ss})^{-\alpha}, \\
\beta^{-1} &= \frac{1+i^{ss}}{1+\pi^{ss}},
\end{aligned}$$

as required.

Combining the first two steady-state requirements, one can express employment and consumption as a function of i^{ss}:

$$c^{ss}/(1-n^{ss}) = \frac{(1-\alpha^{ss})(n^{ss})^{-\alpha}}{1+i^{ss}}.$$

Then use the hint to express consumption as a function of employment, $y^{ss} = (n^{ss})^{1-\alpha} = c^{ss}$. One then gets

$$\begin{aligned} \frac{(n^{ss})^{1-\alpha}}{1-n} &= \frac{(1-\alpha)(n^{ss})^{-\alpha}}{1+i^{ss}}, \\ \frac{n^{ss}}{1-n^{ss}} &= \frac{1-\alpha}{1+i^{ss}}, \end{aligned}$$

and thus

$$n^{ss} = \frac{1-\alpha}{i^{ss}+2-\alpha}.$$

One sees that employment is a decreasing function of the nominal interest rate. Monetary superneutrality fails in the model, as different inflation rates lead to different nominal interest rates, and thus different employment and output levels. The intuition is that consumption is "taxed" by the cash-in-advance constraint for positive nominal interest rates, while leisure is not. An increasing nominal interest rate thus makes consumption relatively more expensive than leisure, and agents substitute away from consumption and supply less labor.

(iii) Derive the monetary policy that provides the utility-maximizing solution for employment in steady state,

$$n^{umax} = \frac{1-\alpha}{2-\alpha}.$$

Explain.

A The optimal monetary policy is one that alleviates the above-mentioned distortionary nature of the cash-in-advance constraint when in binds. Here, this will be one that implements the Friedman rule. I.e., $i^{ss} = 0$. Hence, the optimal employment level becomes

$$n^{ss} = \frac{1-\alpha}{2-\alpha},$$

which indeed equals n^{umax} stated above.