Chapter 16
Investment and Asset Prices

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The previous chapter showed that private investment is the most volatile component of aggregate demand and that it is highly correlated with total output. Understanding the forces driving private investment is therefore crucial for understanding business cycles. In this chapter we present a theory of business investment as well as a theory of housing investment. This will give us an opportunity to study two of the most important asset markets in the economy: the stock market and the market for owner-occupied housing. As we shall see, there is a systematic link between stock prices and business investment, and a similar systematic impact of housing prices on housing investment. To understand investment, we must therefore study how asset prices are formed.

A glance at Figure 16.1 should make clear why we are interested in asset prices. The figure is constructed from data for the 16 most important industrial countries and shows the link between the evolution of housing prices and stock prices and the evolution of the so-called output gap, defined as the difference between actual GDP and the estimated level of GDP which would prevail if unemployment were always at its natural rate. *There is a close relationship between asset price fluctuations and output fluctuations and a clear tendency for stock price movements to lead movements in output.*

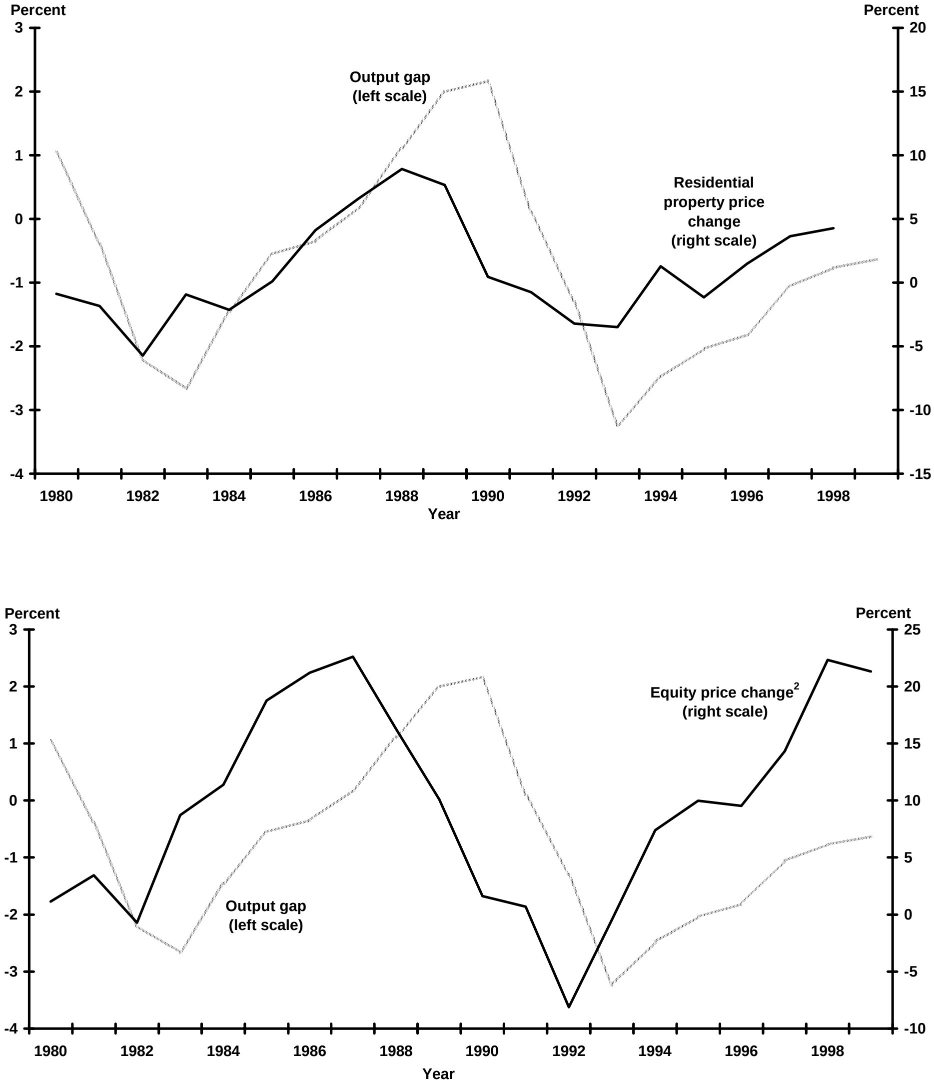

Figure 16.1: Output gap, real property prices, and real equity prices in industrial countries[1]

[1] Arithmetic average of the respective variables in 16 industrial countries, excluding Portugal prior to 1989 owing to lack of data.

[2] Three-year moving average

Source: IMF, World Economic Outlook, May 2000

Thus the figure suggests that an increase in stock prices or in housing prices will trigger an increase in economic activity, whereas a significant drop in asset prices may be a signal of a future economic downturn. As we will show in this chapter and the next one, Figure 16.1 reflects that higher asset prices tend to stimulate private consumption and investment. In particular, the present chapter will explain why higher stock prices tend to be followed by higher business investment, and why higher housing prices provide a boost to housing investment.

The basic idea underlying our theory of investment may be most easily illustrated by looking at the housing market. At any point in time there is a certain market price for houses of a given size and quality. This price may well exceed the cost of constructing a new house of a similar size and quality (the replacement cost). The more the market price exceeds the replacement cost, the more profitable it will be for construction firms to build and sell new houses. Hence we will observe a higher level of housing investment the greater the discrepancy between the market price and the replacement cost of housing. Note that the market price can deviate from the replacement cost for a long time, since it takes time for new construction to feed into a significant increase in the existing housing stock, and since it is time-consuming to shift economic resources into the construction industry if construction activity becomes more profitable due to a rise in the market price of housing.

For business investment a similar basic principle applies. The market price of the business assets owned by a corporation is reflected in the market price of the shares in the firm. The replacement cost of the firm's assets is given by the price at which it can acquire machinery etc. from its suppliers of capital goods. If the stock market value of the firm's assets is higher than their replacement cost, the firm can increase the wealth of its shareholders by purchasing additional capital goods, that is, by investing. The higher the stock price relative to the replacement cost, the greater is the incentive to invest, so the higher the level of investment will be. One might think that the firm would instantaneously adjust its capital stock so that any discrepancy between the stock market value and the replacement cost of its assets is immediately eliminated. However, this is not realistic, since in practice the firm will incur various costs of adjusting the capital stock, and these costs are likely to increase more than proportionally with the level of investment. Hence it will be more profitable to allow a gradual adjustment of the capital stock, and during this (potentially long) adjustment period the stock market value of the firm will deviate from the replacement cost of its assets.

In the sections below we will explain this theory of the link between asset prices and investment in more detail, starting with the theory of the stock market and business investment.

1 A Few Facts about the Stock Market

Figure 16.2 documents the well-known fact that stock prices are highly volatile and sometimes go through dramatic swings.

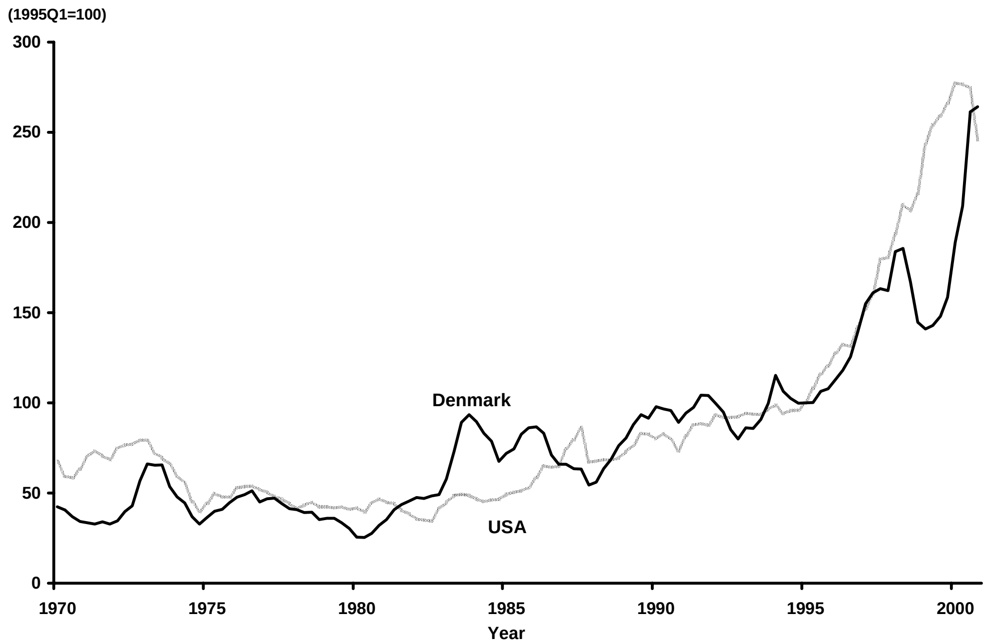

Figure 16.2: Real stock price index

Source: International Financial Statistics, IMF (2001b)

For example, on October 19, 1987 the U.S. Dow Jones index fell by 22.6% in a single day. This was even more apocalyptic than the notorious crash in the 'Black October' of 1929 when the Wall Street stock market dropped by 23% in the course of two days (however, the recovery of stock prices after October 1987 was much faster, so the macroeconomic effects of the crash of 1929 were much more serious). Figure 16.2 also illustrates the enormous stock market boom of the 1990s which was followed by a sharp downturn in the U.S. after the turn of the new century.

Mainly as a consequence of rising stock prices, the market value of outstanding shares (the 'stock market capitalization') as a percentage of GDP rose sharply during the 1990s, as shown in Figure 16.3. In this way stocks became a much more important component of total financial wealth.

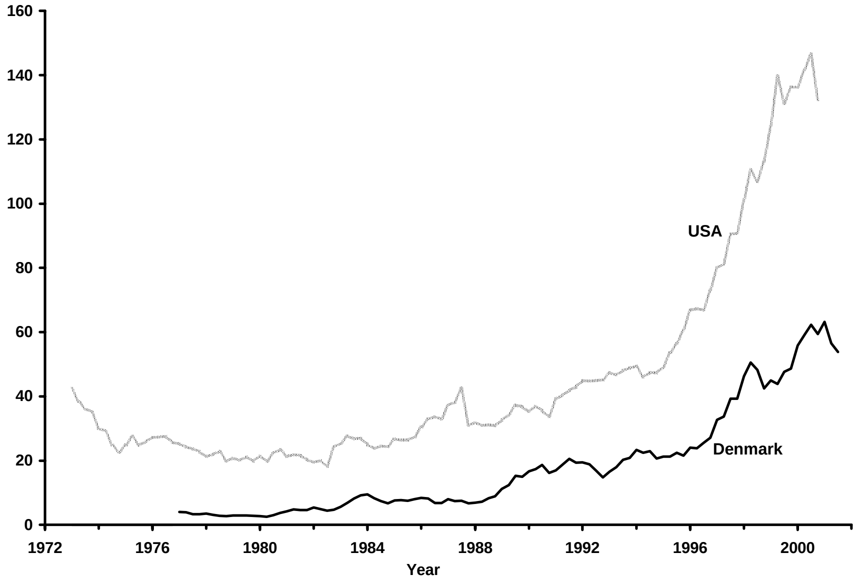

Figure 16.3: Stock market capitalization as a percentage of GDP

Source: Datastream

In many countries the booming stock market motivated a growing proportion of households to invest in shares, and half of the adult U.S. population now owns stocks, as indicated in Figure 16.4. In a country like Sweden, the corresponding proportion is about one third, whereas only about 15% of the adult population in Denmark hold shares.

The stock market is actually more important for households than Figure 16.4 might suggest, since the numbers in the figure only include households who are *direct* owners of stocks. Households also channel a large part of their savings through pension funds, life insurance companies and other financial intermediaries which in turn invest a substantial part of their funds in shares. Hence the performance of the stock market directly or indirectly determines the return to a large fraction of household savings. In this way stock market developments may determine when people feel they can afford to retire from the labour market or when they can afford to buy new consumer durables. Moreover, the evolution of stock prices may have an important impact on the level of output and employment, via its influence on consumption and business investment. In short, the stock market is important for individual consumers and for the macro economy, so it is worthwhile to invest some effort in understanding how it works, and how it affects investment decisions.

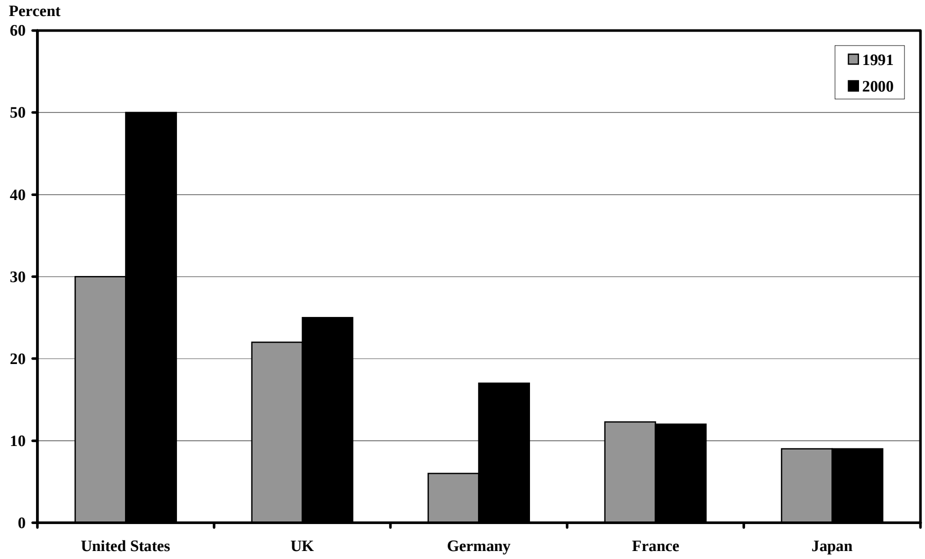

Figure 16.4: Percent of adult population owning stocks

Source: Hali Edison and Torsten Sløk: " Wealth Effects and the New Economy", IMF Working Paper, WP/01/77, 2001

2 The Price of Stock

The Value of a Firm and the Fundamental Stock Price

The starting point for our analysis is the assumption that business investment is guided by a desire to *maximize the wealth of the owners of the firm.* In modern western economies where the bulk of business activity is carried out by firms organized as joint stock companies (corporations), maximization of the wealth of the firm's owners is equivalent to maximization of the market value of the outstanding shares in the corporation. This is the reason for our focus on the stock market. However, our theory of investment will also be relevant for unincorporated firms or for corporations which are not quoted in an official stock exchange. As we shall see, our theory of the stock market implies that *the market value of shares equals the discounted value of the expected future cash flow from the firm to its owners*. But this is exactly how a rational outside investor would also value an unquoted or unincorporated firm if he were contemplating to buy or invest in such a firm. If the owner of an unincorporated firm wants to maximize the market value of his business assets, his investment behaviour will therefore be similar to the investment behaviour of a corporation whose shares are traded in a public stock exchange, as we shall explain in more detail later.

You may wonder why we assume that the objective of the firm is to maximize the *wealth* of its owners? The answer is that maximization of the current market value of the firm will also maximize the *consumption possibilities* of its owners. This will become clear in the next chapter where we show that a person's potential present and future consumption is constrained by the sum of his financial wealth and his current and discounted future labour income. Therefore, if a firm can change its operations so as to increase the market value of its assets, it will increase the financial wealth of its owners and enable them to increase their consumption either now or in the future. In both cases the owners will obviously be better off. In your basic micro course you may have learned that firms maximize their profits rather than the market value of their assets. Fortunately, there is no contradiction between these two goals. A firm that maximizes its discounted stream of profits over time will also maximize its market value, as we shall demonstrate below.

If a corporation plans its investment with the purpose of maximizing the market value of its shares, we must base our theory of investment on a theory of the value of the firm. Our starting point for such a theory is an *arbitrage condition* which says that the market value of the shares in the firm must adjust to ensure that the holding of shares is equally attractive as the holding of bonds. Suppose that, at the beginning of period t, the shareholders in the firm expect to receive a dividend D_t^e at the end of the period, and that they expect the market value of their shares at the start of period $t+1$ to be V_{t+1}^e. If V_t is the *actual* market value of shares in the firm at the beginning of period t, shareholders thus expect to earn a capital gain equal to $V_{t+1}^e - V_t$ during period t. Hence the total expected return on shareholding is $D_t^e + \left(V_{t+1}^e - V_t\right)$, composed of the expected dividend plus the expected capital gain.

In a capital market equilibrium this expected return must equal the 'required' return on shares. The required return consists of the opportunity cost of holding shares rather than bonds, plus an appropriate risk premium. If r is the market rate of interest on bonds (assumed for simplicity to be constant over time), the opportunity cost of shareholding is rV_t, since this is the interest income which the shareholder could have earned during period t if he had sold his shares at the inital market value V_t and invested the corresponding amount in bonds. Furthermore, since stock prices and dividends are generally more volatile than bond prices and interest payments, shares are a riskier investment than bonds. Because investors are risk averse, the expected rate of return on shares must therefore include a risk premium ε on top of the market interest rate to ensure that shareholding is considered just as attractive as the holding of bonds. Hence the total required return on the shares is $(r+\varepsilon)V_t$, and the arbitrage condition for capital market equilibrium may then be written as:

$$\overbrace{(r+\varepsilon)V_t}^{\text{required return}} = \underbrace{D_t^e + \overbrace{V_{t+1}^e - V_t}^{\substack{\text{expected}\\ \text{capital gain}}}}_{\substack{\text{total expected}\\ \text{return on shares}}}. \tag{1}$$

If the current market value V_t is so high that the required return on the left-hand side of (1) exceeds the expected return on the right-hand side, financial investors will sell off their shares in the firm in order to buy bonds, and the market value V_t will drop. On the other hand, if the current share price (and hence V_t) is so low that shares in the firm promise a total rate of return in excess of $r + \varepsilon$, investors will shift from bonds to shares, thereby driving up the current market value V_t. Hence the stock market can only be in equilibrium when the arbitrage condition (1) is met. Since investors derive utility from their *real* consumption possibilities, we are measuring all variables in equation (1) in *real* (inflation-adjusted) terms, so r is the real rate of interest.

We may rearrange (1) to get

$$V_t = \frac{D_t^e + V_{t+1}^e}{1 + r + \varepsilon}. \qquad (2)$$

This is a very important relationship in our analysis below. It says that the value of the firm at the beginning of any period equals the present value of that period's expected dividend plus the expected market value at the end of the period. We see that the rate at which future values are discounted includes the market interest rate i and the required risk premium ε.

As we have argued above, the firm will choose an investment plan which maximizes V_t. To characterize the firm's optimal investment policy we must therefore study how V_{t+1}^e and D_t^e and hence V_t depend on the firm's planned investment. This is the purpose of the following analysis.

Since arbitrage conditions similar to (2) must hold for all subsequent periods, rational financial investors will expect that future stock prices will satisfy the relationships

$$V_{t+1}^e = \frac{D_{t+1}^e + V_{t+2}^e}{1 + r + \varepsilon}, \qquad V_{t+2}^e = \frac{D_{t+2}^e + V_{t+3}^e}{1 + r + \varepsilon}, \quad \cdots \quad V_{t+n}^e = \frac{D_{t+n}^e + V_{t+n+1}^e}{1 + r + \varepsilon}, \quad \text{etc.} \qquad (3)$$

By successive substitutions of the expressions in (3) into (2), we find that

$$\begin{aligned} V_t &= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{V_{t+2}^e}{(1+r+\varepsilon)^2} \\ &= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + \frac{V_{t+3}^e}{(1+r+\varepsilon)^3} \\ &= \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + \cdots + \frac{V_{t+n}^e}{(1+r+\varepsilon)^n}. \end{aligned} \tag{4}$$

It is reasonable to assume that investors do not expect future real stock prices V_{t+n}^e to rise indefinitely at a rate faster than $r+\varepsilon$, for if the opposite were the case, the current stock price V_t would become infinitely high according to (4). Hence we assume that

$$\lim_{n\to\infty} \frac{V_{t+n}^e}{(1+r+\varepsilon)^n} = 0. \tag{5}$$

If we continue the successive substitutions indicated in (4) and use our assumption (5), we end up with:

$$V_t = \frac{D_t^e}{1+r+\varepsilon} + \frac{D_{t+1}^e}{(1+r+\varepsilon)^2} + \frac{D_{t+2}^e}{(1+r+\varepsilon)^3} + \cdots \tag{6}$$

Equation (6) is our first important result, stating that the market value of the shares in a firm equals the present discounted value of the expected future dividends paid out by the firm. This is sometimes referred to as the *fundamental share price*, because it is a price based on a 'fundamental', namely the firm's ability to generate future cash flows to its owners.

Note that there must be a close correlation between a firm's dividends and its profits, since the latter are the source of the former. This observation is the basis for our earlier claim that maximization of (the present value of) profits is roughly equivalent to maximization of market value.

Why are Stock Prices so Volatile?

As illustrated in Figure 16.2, stock prices fluctuate quite a lot. Equation (6) suggests three possible explanations for the observed volatility of stock prices: 1) Fluctuations in (the growth rate of) expected future real dividends, 2) Fluctuations in the real interest rate r, 3) Fluctuations in the required risk premium on shares, ε.

The great stock market boom of the 1990s seems to have been driven mainly by more optimistic expectations regarding future real dividends, as financial investors came to believe that the rapid innovations in information technology would create a 'new economy' characterized by a significantly higher real growth rate in output and business profits. Perhaps changes in the required risk premium on shares have also contributed to the recent turbulence on the stock market. There is some evidence that investors frequently change their attitude towards risk. This is illustrated by Figure 16.5 which plots recent values of a 'Risk Appetite Index' for the major industrial and emerging market economies, constructed by the International Monetary Fund.

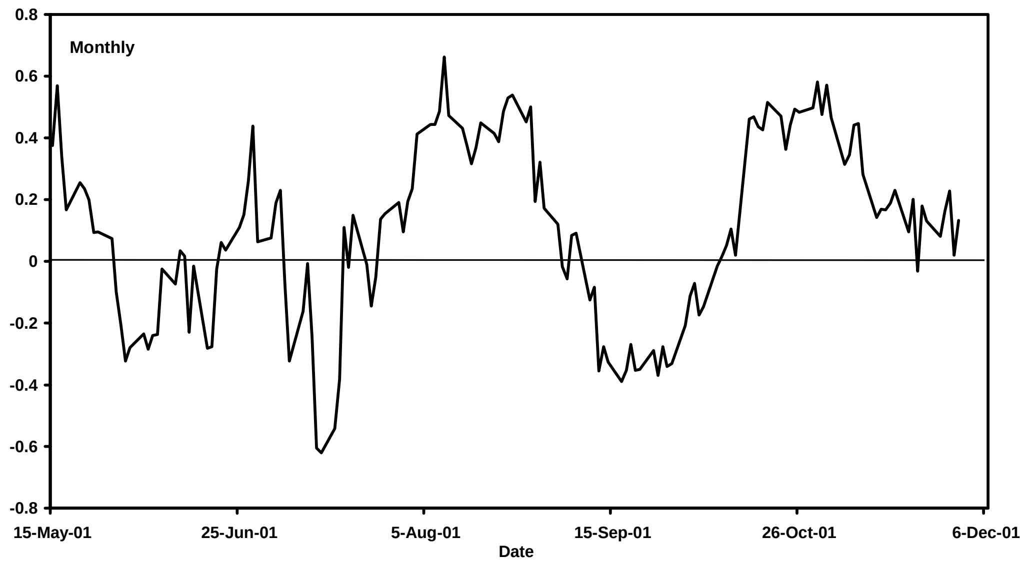

Figure 16.5: Monthly Risk Appetite Index

Source: IMF, World Economic Outlook, December 2001

The idea behind the index is that if investors become willing to bear more risks, they will bid up the prices of assets that have been risky in the past, and if they become more risk averse they will drive down the prices of risky assets by selling them. Hence the graph in Figure 16.5 assumes that movements in current asset prices which are systematically correlated with the observed past riskiness (the past volatility of returns) of those assets indicate a change in investor appetite for risk. The figure suggests that the terrorist attacks on September 11, 2001 caused a sharp temporary drop in investor appetite for risk. Following a recovery, risk appetite again began to weaken towards the end of 2001, perhaps as a reaction to the series of bad news about the state of the world economy announced at that time.

It is often questioned whether the observed movements of the stock market are really consistent with rational investor behaviour. In the short run the stock market sometimes seems to overreact to news, showing signs of 'herd behaviour'. One may also wonder how the large longer term stock market fluctuations observed during the last decade can be reconciled with realistic changes in the long-term earnings potential of firms[1]. Notice, however, that the theory presented above is compatible with the observation of frequent changes in stock prices, if investors frequently revise their forecasts of future dividends and their appetite for risk, and if they are often faced with unanticipated changes in real interest rates. Notice also that our theory only assumes a weak form of rationality: all we assume is that investors require the holding of shares to be just as attractive as the holding of interest-bearing assets. We have *not* excluded the possibility that financial investors may at times hold unduly optimistic or pessimistic expectations about future dividends, and that they may sometimes require 'unreasonably' high or low risk premia due to an inability to make realistic forecasts of the riskiness of business investment. In short, our equations (6) and (9) make no assumptions regarding the formation of expectations and risk premia;

[1] For a fascinating account of the less rational aspects of stock market behaviour, placing the bull market of the 1990s in historical perspective, see Robert J. Shiller: *Irrational Exuberance*, Broadway Books, New York, 2001 (paperback edition).

it only assumes that the expected return on shares is systematically related to the return obtainable on bonds. According to Figure 16.6 this does not appear to be a bad assumption for the longer run, since the realized returns on bonds and stocks in Denmark do indeed seem to move together when the rates of return are calculated over a 5-year or a 10-year holding period.

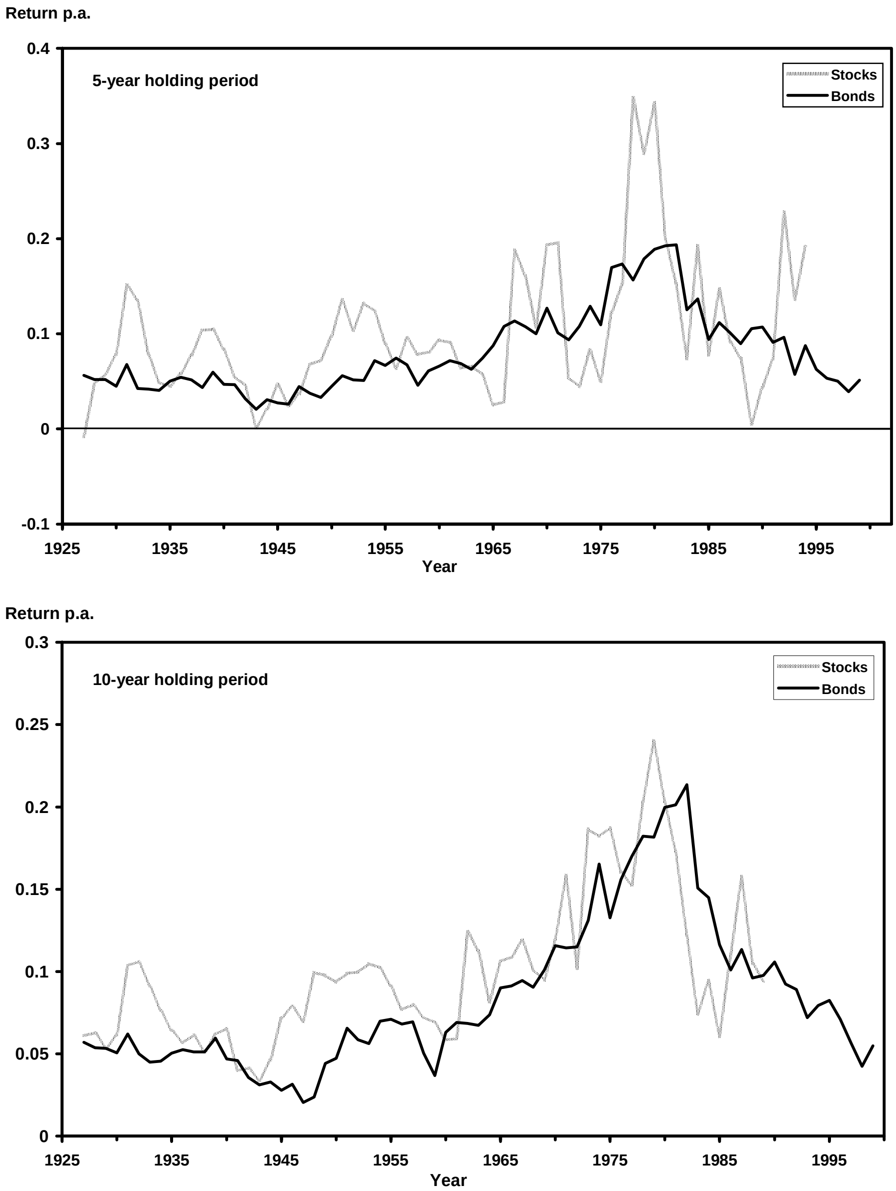

Figure 16.6: Rates of return on stocks and bonds in Denmark

Source: Jan Overgaard Olesen: "Empirical Studies of Price Behavior in the Danish Stock Market", Ph.D. dissertation, Ph.D serie 2.2001, Copenhagen Business School.

Although Figure 16.6 suggests that the price of stock is in fact linked to fundamentals over the long run, many observers of the stock market believe that stock prices can sometimes deviate from the fundamental value of firms. During such periods of 'speculative bubbles', stocks become objects of pure short-term speculation, and their prices cease to be pinned down by the discounted value of expected future dividends. In Chapter 22 we shall see that, somewhat surprisingly, speculative bubbles are not necessarily inconsistent with rational investor behaviour. But in this chapter we will assume that stock prices reflect fundamental values and show that the theory of the stock market outlined above can take us a long way towards understanding business investment.

3 Business Investment

Stock Prices and Investment

Our working hypothesis is that the firm chooses its level of investment with the purpose of maximizing its market value V_t. From equation (2) we see that maximization of V_t is equivalent to maximizing the sum of its owners' expected dividends and expected end-of-period wealth, $D_t^e + V_{t+1}^e$, since the individual firm has no influence on r and ε. The question is: what level of investment will maximize $D_t^e + V_{t+1}^e$?

To answer this, let us introduce the variable q to indicate the *ratio between the market value and the replacement value of the firm's capital stock*. By definition, we thus have $V_t \equiv q_t K_t$, where K_t is the real capital stock, and where the acquisition price of a unit of capital has been normalized to one so that the replacement value of the capital stock is simply K_t. Note the direct link between stock prices and our q-variable: if the market price of shares in the firm goes up, the value of q increases correspondingly. The advantage of specifying our theory of investment in terms of q is that this variable can be measured empirically (since stock market values as well as replacement values of business assets can in principle be observed), whereas the expected future dividends underlying V and q are very hard to measure. Introducing q therefore helps to make our theory of investment empirically testable.

Assuming that the firm communicates its investment plans for the current period to its owners, the shareholders will know the size of the firm's capital stock at the start of the next period (K_{t+1}), but they cannot know for sure what the level of the stock price q_{t+1} at that time will be. However, assume they expect the share price per unit of capital one period from now to be the same as the current share price so that $q^e_{t+1} = q_t$. We then have

$$V^e_{t+1} = q_t K_{t+1} \tag{7}$$

We have now expressed V^e_{t+1} in a form that will turn out to be convenient. Let us next consider the other determinant of current market value $V_t = \left(D^e_t + V^e_{t+1}\right) / (1 + r + \varepsilon)$, that is, the expected dividend D^e_t for period t. Suppose for the moment that the firm finances all of its current investment spending I_t via retained profits (we shall consider external financing later on). Furthermore, suppose realistically that increases in the firm's capital stock imply *adjustment costs*, including costs of installing new machinery, costs of training workers to use the new equipment, and possibly costs of changing the firm's organization. For convenience, all such costs will be called 'installation costs' and will be denoted by $c(I_t)$ to indicate that they are a function of investment spending.

With these assumptions, the expected dividend for period t will be equal to the expected profit in period t, Π^e_t (measured before deduction of installation costs), minus that part of profit which is retained in order to finance the expenditure $I_t + c(I_t)$ associated with new investment:

$$D^e_t = \Pi^e_t - I_t - c(I_t), \qquad c(0) = 0, \qquad c' > 0\ . \tag{8}$$

It seems reasonable to assume that installation costs will rise more than proportionately with investment spending. If investment is low, the changes in the capital stock are small and can be accommodated without significant changes in the firm's organization. But when investment is high, the firm may have to undertake significant organizational changes and extensive training of employees, and the attention of managers will be diverted from the firm's day-to-day business. Such an organizational overhaul is typically very expensive. A simple installation cost function capturing this assumption is

$$c\left(I_t\right) = \frac{a}{2} I_t^2 \ , \tag{9}$$

where a is a positive constant. Equation (9) implies that $dc/dI_t = aI_t$, that is, the *marginal* installation cost increases in proportion to the level of investment, reflecting that large changes in the capital stock are disproportionately more costly than small changes.

To derive our investment schedule, we finally need the bookkeeping identity

$$K_{t+1} = K_t + I_t \ , \tag{10}$$

stating that the capital stock at the beginning of period $t+1$ equals the capital stock existing at the beginning of the previous period plus the level of investment during period t. For simplicity, equation (10) abstracts from depreciation of the existing capital stock, but as you will learn from Exercise 16.2, our theory of investment can easily be generalized to allow for depreciation.

Starting from equation (2), and inserting all of the equations (7) through (10) into (2), we now find that the firm's market value at the start of period t can be written as follows:

$$V_t = \frac{D_t^e + V_{t+1}^e}{1+r+\varepsilon} = \frac{\overbrace{\Pi_t^e - I_t - \frac{a}{2} I_t^2}^{D_t^e} + \overbrace{q_t \left(K_t + I_t\right)}^{V_{t+1}^e}}{1+r+\varepsilon}. \tag{11}$$

The firm chooses its level of gross investment I_t so as to maximize the initial wealth of its owners, V_t, *taking the stock market's valuation of a unit of capital (q_t) as given.* The first-order condition $\partial V_t/\partial I_t = 0$ for the solution to this maximization problem yields

$$\overbrace{q_t}^{\substack{\text{expected}\\\text{capital}\\\text{gain}}} = \overbrace{1 + \frac{dc}{dI_t}}^{\substack{\text{foregone}\\\text{dividend}}},$$

and then, using $dc/dI_t = aI_t$:

$$I_t = \frac{q_t - 1}{a}. \tag{12}$$

The investment rule $q_t = 1 + dc/dI_t$ may be explained as follows: to finance the acquisition and installation of an extra unit of capital in period t, the firm must reduce its dividend payout in period t by an amount equal to the acquisition cost of a unit of capital - which we have set equal to 1 - *plus* the marginal installation cost dc/dI_t. This foregone dividend $1 + dc/dI_t$ is the shareholder's marginal opportunity cost of allowing the firm to undertake an extra unit of investment. The shareholder's marginal benefit from investment is the gain q_t in the value of shares resulting from the installation of an extra unit of capital. At the optimal level of investment, the marginal dividend foregone is just compensated by the extra capital gain on shares. Clearly, the higher the market valuation q_t of an extra unit of capital, the further the firm can push its level of investment before the marginal installation cost reaches the threshold where the shareholder's additional capital gain is offset by the extra dividend foregone. Hence we obtain the simple investment schedule in (12) which says that investment will be higher the higher the level of the stock price q. Equation (12) also shows that high marginal installation costs (reflected in a high value of the cost parameter a) reduce the optimal level of investment, as one would expect.

This is also clear from the graphical illustration of the determination of investment in Figure 16.7: a higher value of a increases the slope of the curve $1 + dc/dI_t = 1 + aI_t$ and thereby reduces the value-maximizing level of investment where $1 + dc/dI_t = q_t$.

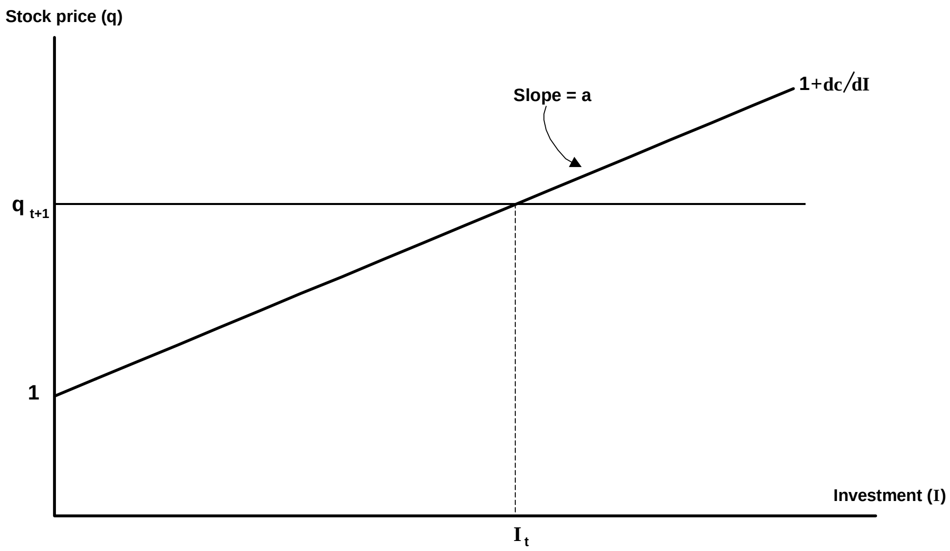

Figure 16.7: The optimal level of investment

The investment schedule (12) also holds when investment outlays are financed by issuing new debt or new shares rather than by retaining profits. Regardless of the mode of finance, the installation of an extra unit of capital will increase the expected market value of the firm's assets by the amount q_t, still assuming that the current stock price gives an indication of the expected value. If the cost of buying and installing the extra unit of capital $(1 + dc/dI_t)$ is financed by an increase in the firm's outstanding debt or by the issue of new shares, the expected increase in the market value of the shares owned by the firm's *existing* shareholders will be equal to the rise in total market value q_t *minus* the value of the newly issued debt or equity, $1 + dc/dI_t$. Of course it is optimal for existing shareholders to let the firm expand its investment until the expected marginal gain in the value of their shares is driven down to zero, that is, until $q_t - (1 + dc/dI_t) = 0$. But this is exactly the investment rule leading to the investment schedule in (12)! Hence we obtain the important result that the investment function $I_t = (q_t - 1)/a$ is valid *regardless of the method of investment finance.*

The theory outlined above is known as Tobin's q-theory of investment, named after

Nobel Laureate James Tobin who was the first economist to give a systematic formal account of the link between stock prices and business investment.[2] Figure 16.8 plots total fixed business investment in the U.S. against an estimate of Tobin's q, defined like above as the ratio of the stock market value of firms to the replacement value of their capital stock.

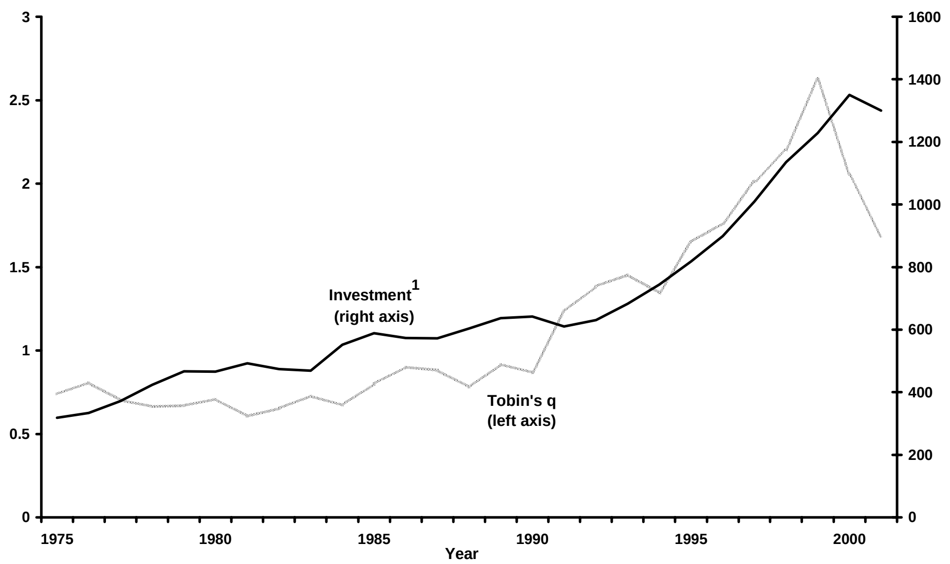

Figure 16.8: Investment and Tobin's q in the United States

[1] Real private non-redidential fixed capital formation measured in 1996 U.S. dollars.

Source: Board of Governors of Federal Reserve System (1999), Flow of Funds Accounts of the United States: Flows and Outstanding Stocks (fourth quarter 1998)

We see that although total investment and the q-ratio do tend to move in line over the longer run, they do not always change in the same direction in the short run. Part of the problem may be that, in practice, stock prices reflect many 'intangible' business assets besides physical capital, for example patents and know-how. Another part of the explanation for the sometimes weak relationship between investment and Tobin's q may be that the estimated value of q reflects the *average* ratio between the *total* market value and

[2] The classic statement of the theory was given in James Tobin: "A General Equilibrium Approach to Monetary Theory", *Journal of Money, Credit, and Banking* 1, February 1969, pp. 15-29. The theory was later refined and extended by Fumio Hayashi: "Tobin's Marginal q and Average q : A Neoclassical Interpretation", *Econometrica* 50, January 1982, pp. 213-224.

the *total* replacement value of the capital stock, whereas in theory investment decisions depend on the *marginal* value of q, that is, on the *increase* in market value relative to the acquisition price of an *additional* unit of capital. In our analysis above the marginal and average values of q were identical, because our simplifying assumptions implied proportionality between the firm's future capital stock and expected future profits. But if this proportionality breaks down, the marginal q will no longer coincide with the average q, and since only the latter can be measured empirically, this may make it difficult to test the q-theory of investment.

The Role of Interest Rates, Profits, and Sales

How does the q-theory of investment square with the conventional assumption that investment depends negatively on the real interest rate? The claim that investment varies positively with stock prices is fully consistent with the hypothesis that it varies negatively with the real interest rate.

To see this, let us go back to equation (6) and let us assume for simplicity (since this will not affect our qualitative conclusion) that real dividends are expected to stay constant at the level D_t^e from period t and onwards. Equation (6) then becomes:

$$V_t = D_t^e \left[\frac{1}{1+r+\varepsilon} + \frac{1}{(1+r+\varepsilon)^2} + \frac{1}{(1+r+\varepsilon)^3} + \cdots \right]. \tag{13}$$

If we multiply both sides of (13) by $1 + r + \varepsilon$ and subtract (13) from the resulting equation, we get:

$$V_t = \frac{D_t^e}{r+\varepsilon}. \tag{14}$$

Equation (14) is just a special version of the general formula stating that the value of the firm equals the present discounted value of expected future dividends. Now recall that, by definition, $V_t = q_t K_t$. From this relationship and (14) it follows that

$$q_t = \frac{D_t^e / K_t}{r+\varepsilon}, \tag{15}$$

According to (15) the market value of a unit of the firm's capital stock (q_t) equals the discounted value of the expected future dividends per unit of capital. Hence a rise in the real interest rate r will ceteris paribus reduce the stock price q_t, and this will reduce investment. From Figures 16.6 and 16.8 we have seen that stock prices do tend to adjust to keep the return on stocks in line with the return on bonds, and that investment tends to move in line with stock prices. This is indirect evidence of a negative impact of interest rates on investment.

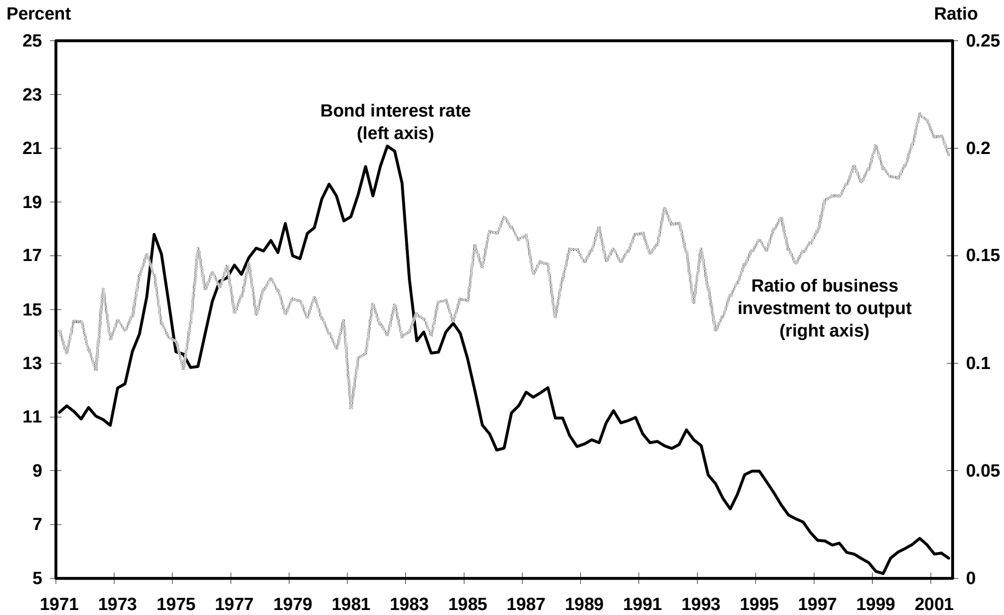

Figure 16.9: Investment ratio and bond interest rate in Denmark

Source: MONA database, Danmarks Nationalbank.

Figure 16.9 shows direct evidence on the relationship between the bond interest rate and the ratio of business investment to output in Denmark.[3] We see that there has been a clear negative relationship between the interest rate and investment in the second half of the 1990s. In other periods the negative correlation has been less clear, but this is not surprising, since our equation (15) implies that q will fluctuate not only with interest rates,

[3]Ideally, Figure 17.9 should plot the *real* interest rate, but since the expected inflation rate is unobservable, we have pragmatically chosen to show the nominal interest rate as a proxy for the real rate.

but also with changes in the risk premium ε and in the expected dividend ratio, D^e/K. Let us take a closer look at the likely determinants of the latter variable.

It seems reasonable to assume that expected dividends are positively related to the observed current profits of the firm, Π_t. For concreteness, suppose shareholders expect that the firm will pay out a fraction θ of its profits as dividends at the end of the period, so $D_t^e = \theta\Pi_t$. In that case the numerator of (15) may be written as θz_t, where $z_t \equiv \Pi_t/K_t$ is the firm's current *rate* of profit (the profit to capital ratio). From (15) we would then expect to observe a positive correlation between (changes in) the current profit rate and (changes in) current investment. Figure 16.10 suggests that such a positive relationship does in fact exist.

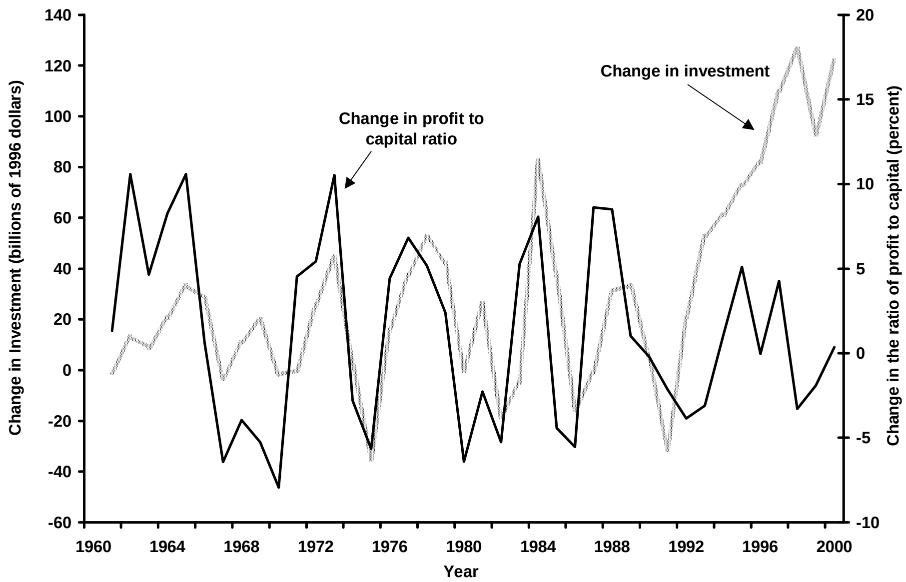

Figure 16.10: Changes in investment and profit in the United States 1960-2000

Source: National Income and Products Accounts (Bureau of Economic Analysis)

We would also expect a positive relationship between the profit rate and the output-capital ratio. For example, we know from growth theory that if output Y is given by the Cobb-Douglas production function $Y = AK^{\alpha}L^{1-\alpha}$ (where L is labour input), and if markets are competitive, total profits will be equal to αY. In that case the *rate* of profit is $\alpha Y/K$ which is directly proportional to the output-capital ratio Y/K. Even if markets are not competitive, it is still reasonable to assume that the more firms can produce and sell on the basis of a given capital stock, the higher their profit rate will be. Figure 16.11 roughly confirms this expectation.

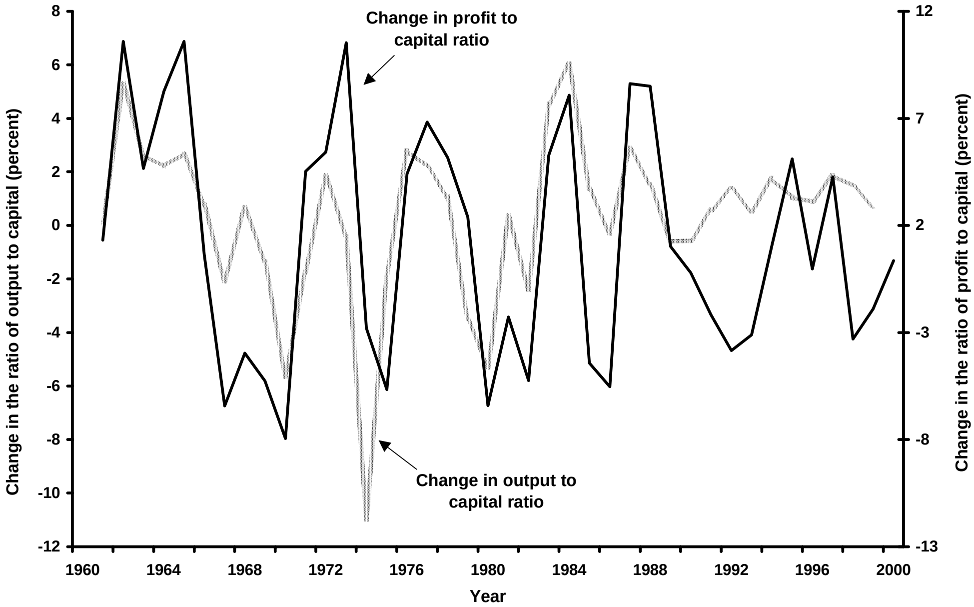

Figure 16.11: Changes in the ratio of output to capital and profit to capital in the United States 1960-2000

Source: National Income and Products Accounts (Bureau of Economic Analysis)

Although a higher current profit is likely to boost expected future dividends, it is too primitive to expect a mechanical one-to-one impact of the former on the latter variable. Firms and investors may sometimes have good reasons to expect that future profitability will deviate from realized current profits. Indeed, the fact that the economy moves up and down in cycles suggests that intelligent investors will not mechanically extrapolate current earnings into the future. Instead they will revise their expectations regarding future sales and profits as they receive new information on relevant economic and political events. Most advanced countries publish indices of 'business confidence' to measure business expectations regarding the near future. Usually these indices build on survey data where a sample of business managers report current and expected movements in their future output, sales, employment, investment, etc. Figure 16.12 shows the evolution of one such index of business confidence in the U.S. We see that business confidence can fluctuate quite a lot and that it is sometimes significantly affected by unanticipated political events.

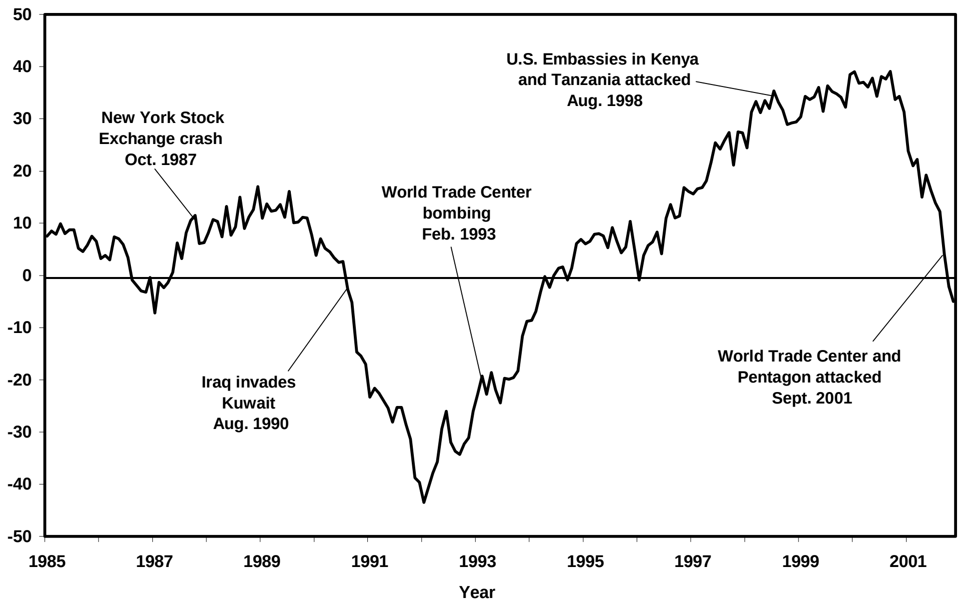

Figure 16.12: Business confidence in the United States.

Source: IMF, World Economic Outlook, December 2001.

Hence, to understand the high volatility of investment spending, we have to allow for changes in expectations regarding the future. Our earlier discussion of Figure 16.5 suggests that part of the impact of a change in expectations on investment may go via a change in the required risk premium ε.

To summarize, we have seen that the q-theory of investment is quite consistent with the hypothesis that business investment varies positively with the level of output (sales), whereas the existing capital stock and the real interest rate both have a negative impact on investment. Specifically, if E is an index of the 'state of confidence', we have argued that the expected dividend ratio D_t^e/K_t will be given by a function like $g(\frac{Y_t}{K_t}, E_t)$, where both of the first derivatives of the $g\,()$-function are positive. Using this relationship along with (15), we may thus write our investment function as

$$I_t = \left(\frac{1}{a}\right)\left(\frac{D_t^e/K_t}{r+\varepsilon} - 1\right) = \left(\frac{1}{a}\right)\left(\frac{g(\frac{Y_t}{K_t}, E_t)}{r+\varepsilon} - 1\right),$$

or, in more general form, and dropping time subscripts for convenience:

$$I = f\left(\underset{(+)}{Y}, \underset{(-)}{K}, \underset{(-)}{r}, \underset{(+)}{E}\right), \tag{16}$$

where the signs below the variables indicate the signs of the corresponding partial derivatives of the $f\,()$-function. In terms of the q-theory, an increase in Y or E will stimulate investment by raising q_t through an increase in the expected dividend ratio $D_t^e/K_t = \theta g(\frac{Y_t}{K_t}, E_t)$ (and possibly through a fall in the risk premium ε). An increase in the current capital stock K reduces investment by driving down D_t^e/K_t, and an increase in the real interest rate r likewise discourages investment via a negative impact on q_t.

In the case of firms whose market value is not directly observable because they are not quoted on the stock exchange, it is inappropriate to interpret equation (16) literally in terms of the q-theory. Nevertheless, as Exercise 16.3 will make clear, the investment behaviour of such firms may still be described by an equation like (16) if they invest with the purpose of maximizing the present value of the net cash flow to their owner (thereby maximizing his wealth). Equation (16) therefore summarizes our general theory of business investment.

Econometric research has confirmed that changes in Y, K and r influence investment in the manner indicated in (16). However, researchers have also found that it is quite difficult to fully explain all of the observed movements in investment. To illustrate, Figure 16.13 plots actual investment against the predicted level of investment estimated on the basis of a sophisticated version of the investment function (16). To a large extent, the difficulties of predicting investment undoubtedly stem from the difficulty of finding reliable quantifiable proxies for 'the state of confidence', E. Because expectations are so hard to measure and may sometimes change abruptly, it is inherently difficult to forecast investment.

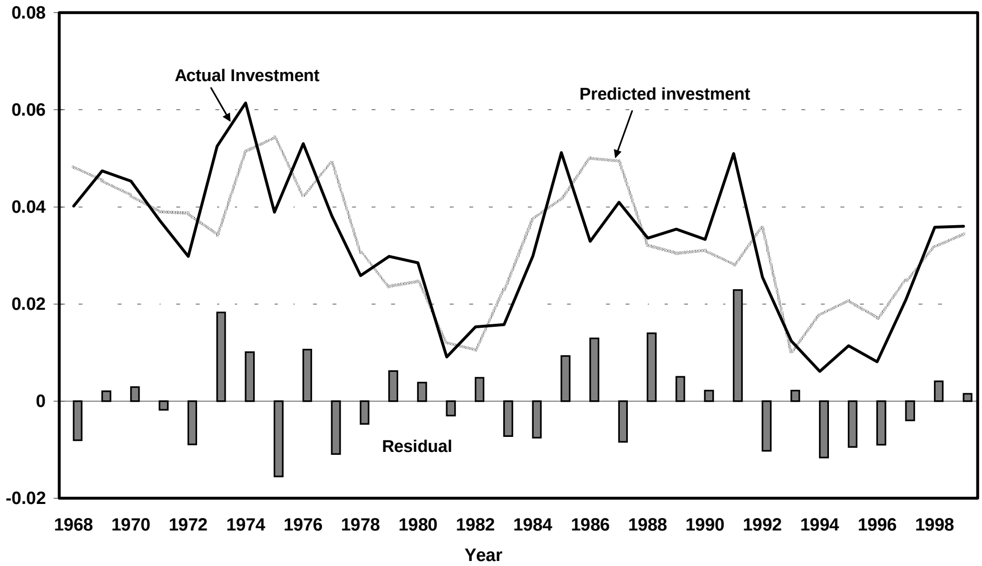

Figure 16.13: Actual and predicted values of net investment ratio in Denmark[1]

[1] Net business investment in machinery and equipment relative to value added.

Source: SMEC database, Secretariat of the Danish Economic Council.

4 The Housing Market and Housing Investment

A q-Theory of Housing Investment

Housing investment is an important component of total private investment, and as we saw in the previous chapter, it is even more volatile than business investment. Hence fluctuations in residential investment often play an important role during business cycles. A basic factor contributing to the volatility of housing investment is the fact that housing capital is highly durable. In any year the construction of new housing is only a very small fraction of the existing housing stock. To accommodate even small percentage changes in consumer demand for housing capital, construction activity may therefore have to undergo large relative changes.

In this section we will show that housing investment may be explained along lines which are similar to the q-theory of business investment. The present section may therefore be seen as an illustrative special version of the q-theory, adapted to fit the housing market. As a byproduct of our theory of housing investment, we will develop a theory of the formation of housing prices and identify the factors which may cause fluctuations in the market value of the housing stock. Since the stock of housing capital is an important component in total household wealth, and since the next chapter will show that private consumption depends on private wealth, the theory of the housing market developed below will also help us to understand fluctuations in private consumption.

We start by considering the production function of the construction sector. For concreteness, suppose that the construction of new housing, I^H, is given by the production function

$$I^H = A \cdot X^\beta, \qquad 0 < \beta < 1, \tag{17}$$

where X is a composite input factor (to be specified below), and A is a constant which depends on the productive capacity of the construction sector. The assumption that the parameter β is less than one implies that, over the time horizon we are considering, production is subject to diminishing returns to scale.

For simplicity, we assume that construction firms combine labour L and building materials Q in fixed proportions. Specifically, each unit of the composite input X includes a units of labour and b units of materials:

$$L = aX, \qquad\qquad Q = bX \ . \tag{18}$$

If W is the wage rate and p^Q is the price of materials, it follows from (18) that the price P of a unit of the composite input X is equal to

$$P = aW + bp^Q. \tag{19}$$

We will refer to P as 'the construction cost index'. If p^H is the market price of a unit of housing, the sales revenue of the representative construction firm will be $p^H I^H$, and its profits Π will be

$$\Pi = p^H I^H - PX = p^H I^H - P\left(I^H/A\right)^{1/\beta}. \tag{20}$$

In deriving the second equality in (20), we have solved (17) for X and substituted the solution into the expression for profits. Taking the housing price p^H and the input price P as given, the construction firm chooses its level of activity I^H with the purpose of maximizing its profit. (We might also assume that the firm maximizes its market value. This would give the same results, but via a more cumbersome procedure). According to (20), the first-order condition for profit maximization $d\Pi/dI^H = 0$ implies:

$$p^H - \overbrace{\frac{P}{\beta A}\left(\frac{I^H}{A}\right)^{\frac{1-\beta}{\beta}}}^{\substack{d(PX)/dI^H = \text{marginal} \\ \text{construction cost}}} = 0 \qquad \Longleftrightarrow$$

$$I^H = k \cdot \left(\frac{p^H}{P}\right)^{\frac{\beta}{1-\beta}}, \qquad k \equiv \beta^{\frac{\beta}{1-\beta}} A^{\frac{1}{1-\beta}}. \tag{21}$$

Equation (21) is the *supply curve* for the construction sector. It is seen to be derived from the fact that profit-maximizing construction firms will push construction activity to the point where the marginal construction cost equals the market price of a housing unit. The relative price variable p^H/P is an analogue of Tobin's q. Thus, since $0 < \beta < 1$, equation (21) says that housing investment I^H will be larger the higher the q-ratio of the housing price to the construction cost index is.[4] Figure 16.14 shows that this theory of housing investment fits the facts very well.

Housing Investment, Interest Rates and Income

Like the q-theory of business investment, our theory of housing investment is consistent with the hypothesis that investment varies negatively with interest rates and positively

[4] In this theory of housing investment the assumption of diminishing returns to the composite input X has taken the place of the installation costs which we included in our model of business investment.

with total income. To demonstrate this, we will now develop a theory of housing demand in order to explain the housing price p^H.

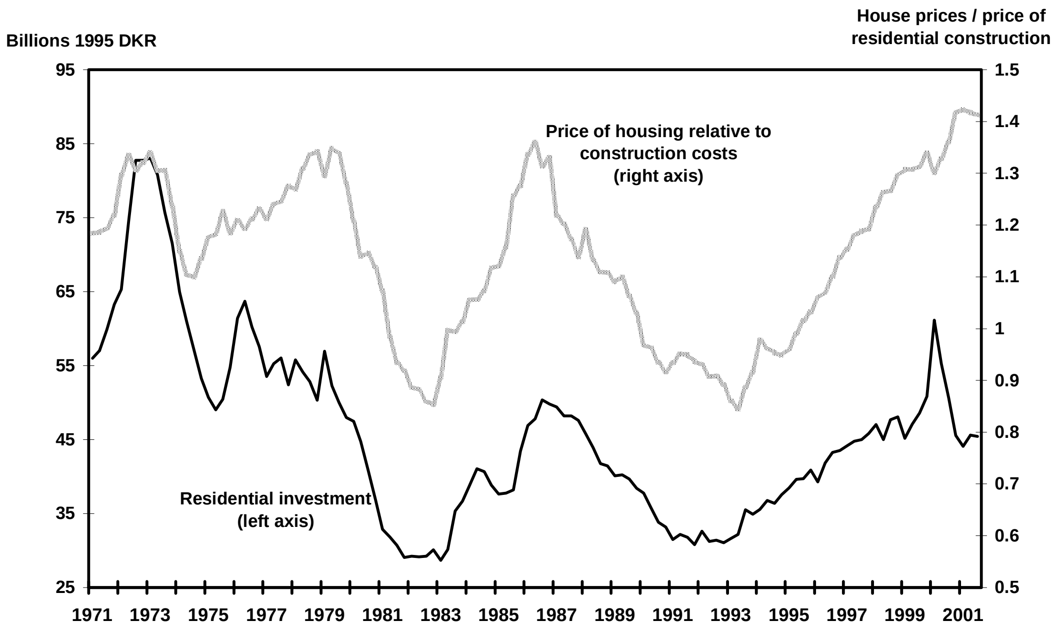

Figure 16.14: Residential investment and the price of housing relative to construction costs in Denmark

Source: MONA database, Danmarks Nationalbank.

Consider a consumer who starts out at the beginning of period 1 with a predetermined level of initial financial wealth V_1. During period 1 he earns labour income Y^L and purchases non-durable consumption goods Z. At the start of the period he also acquires a stock of owner-occupied housing H at the going market price p^H. A fraction d^H of the cost $p^H H$ of acquiring the house is financed by mortgage debt, and the remaining acquisition cost $\left(1-d^H\right)p^H H$ is financed out of his initial financial wealth. The remainder of his financial wealth $V_0 - \left(1-d^H\right)p^H H$ is invested in the bond market earning the real interest rate r. At the end of period 1 the consumer wishes to have a total amount of real wealth V_2 available, and he expects that the nominal housing price will follow the general rate of inflation so that the real housing price stays unchanged at p^H. Measured in real terms, the consumer is therefore subject to the budget constraint

$$V_2 = (1+r)\overbrace{\left[V_1 - \left(1-d^H\right)p^H H\right]}^{\substack{\text{share of initial wealth}\\ \text{invested in bonds}}} + \overbrace{\left(1-d^H\right)p^H H}^{\substack{\text{home}\\ \text{equity}}} - \overbrace{rd^H p^H H}^{\substack{\text{interest on}\\ \text{mortgage debt}}} + Y^L - Z \Longleftrightarrow$$

$$V_2 = (1+r)V_1 - rp^H H + Y^L - Z, \tag{22}$$

where we have assumed that labour income Y^L is received and spending on non-durables Z is made at the end of the period. In the next chapter we shall study how an optimizing consumer will wish to allocate his total consumption and wealth over time. However, for the moment we need not concern ourselves with the determination of V_2, since we are only interested in studying the allocation of consumption between housing and non-durables within each time period, for any given time path of wealth, including the optimal one. To focus attention on housing demand, we will therefore simplify our expressions by setting $V_2 = V_1$, that is, we will assume here that the consumer simply wants to keep his total real wealth constant over time. The budget constraint (22) may then be rearranged to give

$$Z = Y^L + rV_1 - rp^H H. \tag{23}$$

The consumer wishes to allocate his total consumption between housing H and non-durables Z so as to maximize his utility U which we assume to be given by the Cobb-Douglas function

$$U = H^\eta Z^{1-\eta}, \qquad 0 < \eta < 1. \tag{24}$$

In practice, the consumer will derive utility from the housing *service* flowing from the housing stock H, and not from the housing stock as such. The specification in (24) just assumes that the housing service is proportional to the housing stock. Using the budget constraint (23) to eliminate Z from (24), we get

$$U = H^\eta \left(Y^L + rV_1 - rp^H H\right)^{1-\eta}. \tag{25}$$

The consumer's optimal level of housing demand is found by maximizing the utility function (25) with respect to H. The first-order condition $dU/dH = 0$ for this is:

$$\eta H^{\eta-1}\left(Y^L + rV_1 - rp^H H\right)^{1-\eta} - rp^H(1-\eta)H^{\eta}\left(Y^L + rV_1 - rp^H H\right)^{-\eta} = 0, \qquad (26)$$

where the first term is $\partial U/\partial H$ (the drivative holding Z fixed), and the second is $\partial U/\partial Z$:

$$\frac{\partial U/\partial H}{\partial U/\partial Z} = rp^H. \qquad (27)$$

Equation (27) says that, in the consumer's optimum, the marginal rate of substitution between housing and non-durables (the left-hand side) must equal the relative price of housing, rp^H. If we solve (26) for H, we get:

$$H = \frac{\eta}{p^H}\left(\frac{Y^L}{r} + V_1\right). \qquad (28)$$

The variable Y^L/r is sometimes referred to as *human wealth* since it measures the present value of a perpetual stream of labour income Y^L. The magnitude $\left(\frac{Y^L}{r} + V_1\right)$ may thus be seen as a measure of the consumer's total wealth at the beginning of the period, consisting of the sum of his human and financial wealth. Equation (28) then says that housing demand varies positively with total wealth and with the preference for housing (η), and negatively with the housing price.

At the aggregate level, total labour income will vary positively with total output Y. Denoting the wage share of GDP by ω, we may therefore insert $Y^L = \omega Y$ and rearrange (28) to get:

$$p^H = \frac{\eta}{H}\left(\frac{\omega Y}{r} + V_1\right). \qquad (29)$$

Equation (29) is a short-run theory of price determination in the housing market. In the short run the housing stock H is predetermined by the accumulated historical levels of housing investment. Ceteris paribus, a higher pre-existing housing stock will imply a lower current housing price, according to (29). We also see that the housing price will be lower the higher the real interest rate r, the lower the level of current GDP, and the lower the level of financial wealth in the economy. Since we know from (21) that current construction activity varies positively with the housing price, we may therefore combine (29) and (21) to get a housing investment function of the form

$$I^H = k\left[\left(\frac{\eta}{PH}\right)\left(\frac{\omega Y}{r} + V_1\right)\right]^{\frac{\beta}{1-\beta}},$$

or more generally:

$$I^H = h\left(\underset{(+)}{Y}, \underset{(-)}{H}, \underset{(-)}{r}, \underset{(+)}{V_1}\right). \quad (30)$$

The negative impact of the interest rate on housing investment in (30) is based on the theory that a higher interest rate will ceteris paribus reduce the market price of housing. The negatively-sloped regression line in Figure 16.15 confirms that housing prices do in fact tend to fall when the bond interest rate goes up, and vice versa.

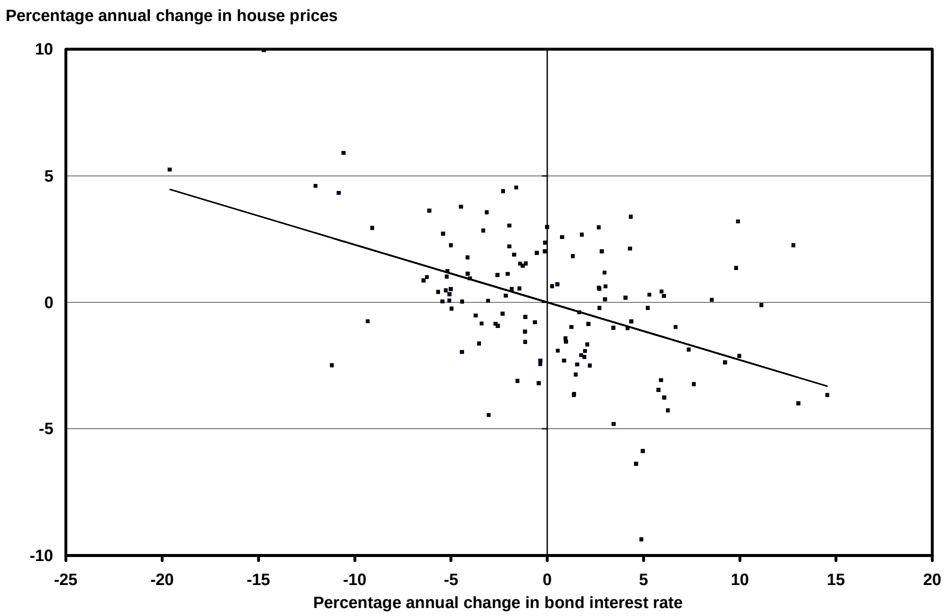

Figure 16.15: The long-term bond interest rate and house prices in Denmark

Note: Percentage changes are normalized by subtracting the average change from all observations.

Source: MONA database, Danmarks Nationalbank.

Housing Market Dynamics

Finally, we may note that the current and the future housing stocks are linked by the identity

$$H_{t+1} = H_t + I_t^H \tag{31}$$

where we have ignored depreciation of the existing housing stock for simplicity. Equations (21), (29) and (31) constitute a simple dynamic model of the housing market. For given values of Y, r and V_1, the predetermined housing stock H_t determines the housing price for period t via (29). Given the value of P, equation (21) then determines the current level of housing investment I_t^H which subsequently determines the next period's housing stock H_{t+1} via (31). We then get a new housing price p_{t+1}^H via (29) which enables us to determine I_{t+1}^H by use of (21), giving a new housing stock H_{t+2} via (31), and so on. This dynamic process means that whereas an upward shift in housing demand is fully absorbed by a rise in housing prices in the short run, over the longer run it will cause an increase in the housing stock which will dampen the initial price increase. Exercise 16.5 will ask you to explore the dynamics of the housing market in an extended model allowing for property taxes and for depreciation of the existing housing stock.

5 Summary

The main points of this chapter may be summarized as follows:

1. Empirically changes in stock prices and in housing prices tend to be followed by changes in output in the same direction. In part this reflects that higher asset prices lead to higher investment. This chapter explains the links between asset prices and investment.

2. A firm seeking to maximise the wealth of its owners will choose an investment plan which maximises the market value of the firm's assets. The value of the firm, referred to

as the fundamental stock value, is the present discounted value of the expected future dividends paid out by the firm. This follows from the shareholder's arbitrage condition which says that the expected return to shareholding, consisting of dividends and capital gains on shares, must equal the return to bondholding plus an appropriate risk premium.

3. When share prices reflect the fundamental value of firms, there are three possible reasons for the observed volatility of stock prices: i) fluctuations in (the growth rate of) expected future dividends, ii) fluctuations in the real interest rate, and iii) fluctuations in the required risk premium on shares. There is indirect evidence that the required risk premium fluctuates quite a lot.

4. The evidence suggests that the rate of return on stocks is tied to the rate of return on bonds over the long term. This accords with the view that stock prices reflect the fundamental value of firms. However, many observers believe that stock prices can sometimes deviate from fundamentals. The analysis in this chapter abstracts from such 'bubbles' in stock prices.

5. Increases in the firm's capital stock imply adjustment costs (installation costs), including costs of installing new machinery, costs of training workers to use the new equipment, and perhaps costs of adapting the firm's organization. These installation costs will typically increase more than proportionally with the firm's level of investment.

6. The value-maximising firm will push its investment to the point where the shareholder's capital gain from a unit increase in the firm's capital stock is just offset by the dividend he must forego to enable the firm to purchase and install an extra unit of capital. Because the marginal installation cost is increasing in the volume of investment, this investment rule implies that the firm's optimal level of investment will be higher, the higher a unit increase in the firm's capital stock is valued by the stock market. An increase in the ratio of stock prices to the replacement cost of the firm's assets will therefore stimulate its investment.

7. The market value of stocks relative to the replacement value of the underlying business assets is referred to as Tobin's q. Our theory of investment may be summarized by saying that business investment is an increasing function of Tobin's q.

8. Stock prices reflect expected future dividends which tend to be positively affected by a rise in current corporate profits. The value of Tobin's q therefore tends to vary positively with current profits, which in turn vary positively with the output-capital ratio. Hence investment is an increasing function of current output and a decreasing function of the existing capital stock.

9. Ceteris paribus, a rise in the real interest rate implies that expected future dividends are discounted more heavily, leading to a fall in Tobin's q via lower stock prices. Thus a higher real interest rate tends to depress investment. A rise in the required risk premium on shares, generated by more uncertainty about the future, will have a similar negative impact on investment.

10. A version of the q-theory can explain investment in owner-occupied housing. When the market price of residential property increases relative to the cost of housing construction, it becomes profitable for firms in the construction sector to increase the supply of new housing units. As a consequence, housing investment (construction activity) goes up. There is strong empirical evidence in favour of this hypothesis.

11. In the short run, the market price of housing varies positively with current income and financial wealth and negatively with the real interest rate and with the existing housing stock. Since construction increases with the market price of housing, it follows that housing investment is an increasing function of income and wealth and a decreasing function of the real interest rate and the current housing stock.

6 Exercises

Exercise 1: Stock market valuation and 'fundamentals'

The purpose of this exercise is to illustrate how our equation (6) for the fundamental stock price may be used to evaluate whether stock prices are unrealistically high or low, that is, whether the stock market is 'overvalued' or 'undervalued'. Suppose that real dividends are expected to grow at the constant rate g^e. If the actual real dividend for period t is D_t, the expected real dividend for future period n will then be given by:

$$D_n^e = D_t \left(1 + g^e\right)^{n-t} \qquad \text{for} \qquad n = t+1,\ t+2, \cdots \tag{1}$$

1. Use equation (6) in the text to demonstrate that, when expected future dividends are given by (1) above, the value of shares will be given by:

$$V_t = \frac{D_t}{r + \varepsilon - g^e}. \tag{2}$$

Stock market analysts often focus on the so-called 'trailing dividend yield' D_t/V_t, defined as the current dividend relative to the current market value of shares. We will simply refer to this ratio as the 'dividend yield'.

2. Suppose you have information on the current dividend yield D_t/V_t, the real interest rate r, and the required risk premium on shares, ε. Use equation (2) above to solve for the value of the expected real growth rate g^e which is necessary to justify current stock prices.

3. Suppose alternatively that, in addition to information on the current dividend yield and the current real interest rate, you have somehow obtained information on the expected future growth of real dividends whereas you do not know the required risk premium on shares. Use equation (2) to derive the value of the risk premium which will justify current stock prices.

4. In the United States in 1999 the average dividend yield was 1.2 percent and the real interest rate on (approximately) risk-free 10-year government bonds was 3.4 percent. Moreover, the average historical risk premium on shares in the period 1980-99 was 2.8 percent per annum (all of these figures are taken from the IMF's World Economic Outlook, May 2000). What was the annual growth rate of real dividends which U.S. financial investors expected in 1999 if they required a risk premium equal to the historical average? On the basis of your result, would you say that the U.S. stock market was overvalued or undervalued in 1999? Motivate your answer.

5. Over the period 1980-1999 the average growth rate of U.S. real GDP was 3.0 percent. Suppose now that U.S. investors in 1999 expected future real dividends to grow in line with historical GDP growth so that $g^e = 0.03$ (discuss whether this might be a reasonable assumption). Given the other pieces of information in the previous question, what was the risk premium on shares required by U.S. investors in 1999? Would you say that this risk premium was 'reasonable'? Can you imagine any reasons why U.S. investors in 1999 should require a lower or a higher risk premium than the average historical premium?

Exercise 2: A generalized q-theory of investment

Suppose that instead of equation (9) in the text, the installation cost function takes the more general form:

$$c(I_t) = \frac{a}{\eta+1} I_t^{\eta+1}, \qquad \eta > 0. \tag{1}$$

1. Derive an expression for investment as a function of q_t (a generalized version of equation (12)), using the procedure described in the text. How does the value of the parameter η affect investment (assume that $q_t - 1 > a$)? Give an intuitive explanation for your result.

Suppose that during each period, a fraction δ of the capital stock has to be scrapped because of wear, tear, and technical obsolescence, so the change in the capital stock is given by

$$K_{t+1} - K_t = I_t - \delta K_t, \tag{2}$$

where I_t indicates *gross* investment, including the replacement investment which just serves to compensate for depreciation. Suppose that only *net* additions to the capital stock generate adjustment costs. In that case installation costs will be determined by *net* investment $I_t - \delta K_t$ so that (1) must be replaced by

$$c(I_t) = \frac{a}{\eta + 1} (I_t - \delta K_t)^{\eta+1}. \tag{3}$$

2. Discuss whether it is reasonable to assume that only net investment (but not replacement investment) generates adjustment costs. Derive a revised expression for gross investment I_t, assuming that installation costs are given by (3) above.

In a stationary economy with no long-run growth, a long-run equilibrium requires that the capital stock be constant over time. In such a situation where net investment is zero there is no need for firms to retain any part of their net profit. Hence all net profits will be paid out as dividends. According to equation (15) in the text, this implies $q_t = (\Pi_t / K_t) / (r + \varepsilon) = z_t / (r + \varepsilon)$, where you recall that z is the profit rate.

3. What is the ratio of the market value to the replacement value of the firm's capital stock in a stationary long-run equilibrium? Furthermore, what is the relationship between the profit rate and the required return on shares in such a long-run equilibrium? Try to provide some economic intuition for your results.

Exercise 3: Tax policy and investment (I)

This exercise serves two purposes. First, you are asked to demonstrate that the investment behaviour of unincorporated firms is similar to the investment behaviour of corporations which are quoted on the stock market. Second, you are invited to study how various forms of capital income taxation will influence investment.

We consider an entrepreneur who owns a private unincorporated business firm. We divide the entrepreneur's time horizon into two periods which may be thought of as 'the present' (period 1) and 'the future' (period 2). At the beginning of period 1 the entrepreneur has accumulated a predetermined capital stock K_1 which is invested in his firm. During period 1 he incurs gross investment expenditure I with the purpose of replacing and increasing his capital stock. In each of the two periods, the capital stock depreciates at the rate δ, so at the beginning of period 2 the entrepreneur's capital stock will be given by

$$K_2 = K_1 (1 - \delta) + I, \qquad 0 < \delta < 1. \tag{1}$$

The entrepreneur undertakes investment and employs labour with the purpose of maximizing the present value V of the net cash flows Π_1 and Π_2 withdrawn from the firm during periods 1 and 2, respectively. In other words, the entrepreneur wants to maximize

$$V = \frac{\Pi_1}{1+r} + \frac{\Pi_2}{(1+r)^2}. \tag{2}$$

At the end of period 2 the entrepreneur plans to liquidate the firm and to sell its remaining assets at their replacement value $(1 - \delta) K_2$. For simplicity, we assume that there are no adjustment costs associated with changing the firm's capital stock. If Y_t is the firm's output, L_t is labour input, and w_t is the real wage rate, the cash flows from the firm to the owner during the two periods will then be

$$\Pi_1 = Y_1 - w_1 L_1 - I, \tag{3}$$

$$\Pi_2 = Y_2 - w_2 L_2 + (1 - \delta) K_2, \tag{4}$$

where (4) includes the revenue from the sale of the firm's remaining capital stock at the end of period 2. Output in the two periods is given by the Cobb-Douglas production function

$$Y_t = A_t K_t^{\alpha} L_t^{\beta}, \qquad 0 < \alpha < 1, \quad 0 < \beta < 1, \quad 0 < \alpha + \beta \leq 1, \qquad t = 1, 2. \tag{5}$$

1. Demonstrate that the entrepreneur's optimal gross investment during period 1 can be written as:

$$I = \frac{\alpha Y_2}{r + \delta} - (1 - \delta) K_1. \tag{6}$$

(Hint: use (1) to eliminate K_2 before you derive your first-order condition). Does the investment function (6) have the same qualitative properties as the business investment function derived in the main text of the chapter? (Hint: note that the variable Y_2 must be interpreted as *expected* output in period 2).

We will now study the effects of a profits tax. In accordance with existing tax rules, we assume that the firm is allowed to deduct its labour costs and the depreciation on its capital stock from taxable profits. If the profits tax rate is τ, the firm's tax bill T then becomes

$$T_t = \tau (Y_t - w_t L_t - \delta K_t), \qquad 0 < \tau < 1, \qquad t = 1, 2, \tag{7}$$

and the after-tax cash flows from the firm to the entrepreneur become equal to

$$\Pi_1 = Y_1 - w_1 L_1 - I - T_1, \tag{8}$$

$$\Pi_2 = Y_2 - w_2 L_2 + (1 - \delta) K_2 - T_2. \tag{9}$$

2. Derive the analogue of the firm's investment function (6) in the presence of the profits tax. Explain how the profits tax affects investment.

We have so far assumed that investment is financed by retained profits. Consider now the alternative case where only replacement investment δK is financed by retained earnings whereas net investment expenditure $I - \delta K$ is financed by debt. In that case the firm's stock of debt B will always be equal to its capital stock, that is

$$B_t = K_t, \qquad t = 1, 2, \tag{10}$$

and the firm's revenue ΔB_1 from new borrowing during period 1 will be equal to its net investment during that period:

$$\Delta B_1 = I - \delta K_1. \tag{11}$$

Using (10) and (11) and noting that the firm's expenses on interest payments will be $rB = rK$, we may then write the net cash flows to the entrepreneur as

$$\begin{aligned} \Pi_1 &= Y_1 - w_1 L_1 - rB_1 - T_1 - I + \Delta B_1 \\ &= Y_1 - w_1 L_1 - (r + \delta) K_1 - T_1, \end{aligned} \tag{12}$$

$$\begin{aligned} \Pi_2 &= Y_2 - w_2 L_2 + (1 - \delta) K_2 - T_2 - (1 + r) B \\ &= Y_2 - w_2 L_2 - (r + \delta) K_2 - T_2, \end{aligned} \tag{13}$$

assuming that the entrepreneur must repay all of its debt with interest at the end of the second period. Since the tax code allows interest payments as well as depreciation to be deducted from taxable profits, the tax bills for the two periods will be

$$T_t = \tau \left[Y_t - w_t L_t - (r + \delta) K_t \right], \qquad 0 < \tau < 1, \qquad t = 1, 2. \tag{14}$$

3. Derive the firm's investment function on the assumption that net investment is fully financed by debt (hint: remember to use (1) and (5)). Does the profits tax affect investment? Does it yield any revenue? Is the tax system neutral towards the firm's choice of financing method? Explain your results.

Exercise 4. Tax policy and investment (II)

This exercise builds on the model set up in Exercise 16.3 and proceeds to study the effects on investment of alternative forms of capital income taxation. We return to the assumption made in the beginning of Exercise 16.3 that investment is financed by retained earnings and we recall that

$$K_2 = K_1 (1 - \delta) + I, \qquad 0 < \delta < 1, \tag{1}$$

$$Y_t = A_t K_t^{\alpha} L_t^{\beta}, \qquad 0 < \alpha < 1, \quad 0 < \beta < 1, \quad 0 < \alpha + \beta \leq 1, \qquad t = 1, 2, \tag{2}$$

$$\Pi_1 = Y_1 - w_1 L_1 - I - T_1, \tag{3}$$

$$\Pi_2 = Y_2 - w_2 L_2 + (1 - \delta) K_2 - T_2, \tag{4}$$

$$T_t = \tau (Y_t - w_t L_t - \delta K_t), \qquad 0 < \tau < 1, \qquad t = 1, 2. \tag{5}$$

Our notation is the same as in Exercise 3: K is the capital stock, I is gross investment, L is labour input, Y is output, Π is the net cash flow from the firm to the owner, w is the real wage, δ is the depreciation rate, T is the tax bill, and τ is the tax rate. Suppose now that the tax rate τ is levied not only on business profits, but on all income from capital, including interest income. The relevant discount rate for the entrepreneur will then be the *after-tax* interest rate $r(1 - \tau)$, and he will now strive to maximize

$$V = \frac{\Pi_1}{1 + r(1 - \tau)} + \frac{\Pi_2}{[1 + r(1 - \tau)]^2}. \tag{6}$$

1. Demonstrate that the uniform tax on all capital income will be *neutral* towards investment in the sense that I will be unaffected by τ. Try to explain intuitively why the uniform capital income tax does not discourage business investment.

In practice, the tax code often allows firms to write off their assets at a faster rate than the true depreciation rate δ. In this case we say that the tax code allows *accelerated depreciation.* If D is the amount of depreciation allowed in the firm's tax accounts, we may define the rate of depreciation for tax purposes as $\widehat{\delta} \equiv D/K$. We may then specify the firm's tax bill as

$$T_t = \tau \left(Y_t - w_t L_t - \widehat{\delta} K_t \right), \qquad t = 1, 2. \tag{7}$$

2. Demonstrate that a uniform capital income tax will *stimulate* investment if the tax code allows accelerated depreciation so that $\widehat{\delta} > \delta$ (Hint: when you derive your investment function, remember that under a uniform capital income tax the entrepreneur wishes to maximize expression (6)). Try to give an intuitive explanation for this 'taxation paradox'.

In recent years many governments in the OECD area have tried to broaden the business income tax base by removing various accelerated depreciation schemes so as to bring depreciation for tax purposes more into line with the true economic depreciation of business assets. At the same time many countries tax business profits at a rate which differs from the tax rate on interest income. Against this background, we will now assume that $\widehat{\delta} = \delta$ but that the tax rate on business profits (τ) differs from the personal tax rate on interest income (m).

3. Derive the firm's investment function on the assumption that the tax rate on business profits differs from the personal tax rate on interest income (hint: note that the entrepreneur's discount rate is now equal to $r(1-m)$). What determines whether the tax system encourages or discourages investment? Explain your result.

Some tax economists have argued that, just as firms are allowed to deduct the interest on their debt from taxable profits, they should also be allowed to deduct an imputed market return on their equity capital in order to avoid tax discrimination against equity finance. Under such a tax on 'pure' profits where the firm may deduct a 'normal' return rK from the profits tax base, its tax bill becomes

$$T_t = \tau \left[Y_t - w_t L_t - (r + \delta) K_t \right], \qquad t = 1, 2, \tag{8}$$

assuming that depreciation for tax purposes corresponds to the true economic depreciation. Suppose now that the entrepreneur is a wealthy person who has the possibility to deposit his financial wealth in a foreign bank account which cannot be monitored by the domestic tax authorities. In that case his interest income cannot be taxed, and his discount rate will then be equal to the pre-tax interest rate r.

4. Derive the firm's investment function when the tax bill is given by (8) and the entrepreneur's interest income escapes personal income tax. Does the tax system affect investment? Does the profits tax yield any revenue? Explain your results.

Over the years many tax economists have proposed that the conventional tax on business profits be replaced by a so-called *cash flow tax* levied on the net cash flow from the firm to its owner. Under a cash flow tax the firm's tax bill in the two periods will be

$$T_1 = \tau \left(Y_1 - w_1 L_1 - I \right), \tag{9}$$

$$T_2 = \tau \left[Y_2 - w_2 L_2 + (1 - \delta) K_2 \right]. \tag{10}$$

5. Derive the firm's investment function under a cash flow tax, assuming that interest income goes untaxed. Does the cash flow tax affect investment? Give an intuitive explanation for your result.

6. Tax economists sometimes say that the cash flow tax makes the government a 'silent partner' in the firm. What is the justification for this claim? What is the relationship between the value of the firm and the present value of the cash flow tax revenue?

7. As a summary of your findings in the present and the previous exercise, give a brief restatement of the alternative conditions under which the tax system will be neutral towards business investment.

Exercise 5: Tax policy and the housing market

In this exercise we will extend our model of the housing market to allow for property taxes and depreciation. You are then asked to study how the housing market reacts to tax policy in the short run and in the long run.

We assume that the government levies a proportional property tax at the rate τ on the current value $p^H H$ of the consumer's housing stock H. We also assume that the government imposes a proportional income tax at the rate m but that it allows interest expenses to be deducted from taxable income. Finally, we assume that the consumer has to spend an amount $\delta p^H H$ on repair and maintenance during each period to maintain the value of his house. The parameter δ may thus be interpreted as the depreciation rate for housing capital.

We consider a young consumer who starts out with zero financial wealth at the beginning of period t and who must therefore borrow an amount $p_t^H H_t$ to acquire the housing stock H_t at the going market price p_t^H. For simplicity we assume that the consumer's planned net saving is zero, so his current spending on non-durable consumption goods Z_t is given by the budget constraint

$$Z_t = (1-m)\,Y_t^L - [r\,(1-m) + \delta + \tau]\,p_t^H H_t, \qquad 0 < m < 1, \tag{1}$$

where Y^L is pre-tax labour income and where the term $[r\,(1-m) + \delta + \tau]\,p^H H$ reflects net interest payments on mortgage debt plus expenses on housing repair and property tax. Note that because interest expenses are deductible, it is the *after-tax* interest rate $r\,(1-m)$ which appears in the budget constraint (1).

The consumer's preferences are given by the Cobb-Douglas utility function

$$U = H_t^{\eta} Z_t^{1-\eta}, \qquad 0 < \eta < 1. \tag{2}$$

1. Demonstrate that the consumer's housing demand will be given by

$$H_t = \frac{\eta (1-m) Y_t^L}{[r(1-m) + \delta + \tau] p_t^H}, \tag{3}$$

and give your comments on this expression.

We will now set up a complete partial equilibrium model of the housing market. We start out by rearranging (3) to get an expression for the housing price:

$$p_t^H = \frac{\eta (1-m) Y_t^L}{[r(1-m) + \delta + \tau] H_t}. \tag{4}$$

The supply side of the housing market is modeled in the same manner as in the main text of the chapter. The construction of new houses I^H is therefore given by equation (21) in the text, but for convenience we may set the exogenous construction cost index P equal to 1 by appropriate choice of our units of measurement. We then get

$$I_t^H = k \cdot \left(p_t^H\right)^{\frac{\beta}{1-\beta}}, \qquad 0 < \beta < 1, \tag{5}$$

where we remember that k is an exogenous constant. Finally, since a fraction δ of the existing housing stock must be replaced in each period, our previous bookkeeping identity (31) for the evolution of the housing stock must be modified to

$$H_{t+1} = (1-\delta) H_t + I_t^H, \qquad 0 < \delta < 1. \tag{6}$$

Equations (4) through (6) constitute our model of the housing market where the exogenous variables and parameters are Y^L, r, η, β, k, δ, m and τ. At the beginning of each period the existing housing stock H_t is predetermined by the accumulated historical levels of housing investment. Given the initial housing stock, equation (4) may therefore be used to find the short run equilibrium housing price p_t^H which may then be inserted into (5) to give the current level of housing investment I_t^H. Once I_t^H is known, we may insert its value into (6) along with the predetermined value of H_t to find the housing stock H_{t+1} at the beginning of the next period, which then determines p_{t+1}^H via (4), and so on.

2. Condense the housing market model (4) through (6) into a single non-linear first-order difference equation in the housing stock H. State the condition under which this difference equation is locally stable so that the housing market will converge on a long-run equilibrium characterized by a constant housing stock (hint: the difference equation will be locally stable if dH_{t+1}/dH_t is numerically smaller than one. When you derive and simplify your expression for dH_{t+1}/dH_t, use equations (4) and (5) plus the fact that, in a local stability analysis, the derivative dH_{t+1}/dH_t is calculated in the long-run equilibrium point where $I_t^H = \delta H_t$ initially, because the housing stock is constant in long-run equilibrium). Discuss whether the housing market is likely to be locally stable for plausible parameter values.

3. In a long-run equilibrium the housing stock is constant, $H_{t+1} = H_t$, implying from (6) that $I^H = \delta H$. Insert this condition for long-run equilibrium into (5) and invert the resulting equation to get a *long-run supply curve for the housing market.* Construct a diagram (with H along the horizontal axis and p^H along the vertical axis) in which you draw this long-run supply curve along with the housing demand curve given by (4). Identify the long-run equilibrium of the housing market and denote the long-run equilibrium housing stock by H^*. Insert a *short-run* housing supply curve for period 0 in your diagram, assuming that the housing market starts out in period 0 with a housing stock $H_0 < H^*$ (Hint: remember that in the short run the housing stock is predetermined. What does this imply for the slope of the short-run housing supply curve?). Provide a graphical illustration of the adjustment to a long-run housing market equilibrium. Give an intuitive verbal description of the adjustment process. How does the level of housing investment I_t^H evolve during the adjustment to long-run equilibrium?

4. Use the model (4) through (6) to derive expressions for the long-run equilibrium values of the housing price and the housing stock. Comment on the expressions.

5. Suppose now that the government raises the property tax rate τ. Use your expressions from Question 4 to derive the long-run effects on p^H and H of a marginal increase in τ. Use a diagram like the one you constructed in Question 3 to illustrate the effects of the property tax increase in the short run (where the housing stock is predetermined) and in the long run when the housing stock has fully adjusted to the tax increase. Illustrate the gradual adjustment of the market to the higher property tax and provide a verbal explanation of the adjustment process.

6. Suppose next that the government reduces the income tax rate m. How will this affect the housing market in the short run and in the long run? Give a graphical illustration and explain your results.

7. Consider again a cut in the income tax rate m, but suppose now that the tax code allows property taxes as well as expenses on housing repair to be deducted from taxable income (as was in fact once the case in Denmark). In these circumstances, how will the income tax cut affect the housing market? Explain your finding (Hint: start by considering how the consumer budget constraint must be modified to allow for deductibility of expenses on property taxes and housing repair. Then use the revised budget constraint to derive a revised expression for housing demand and the housing price).